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THE MASSES OF SPHERICAL GALAXIES M32. A LIKELY APPLICATION

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1. *Introduction.*—Of great importance for the understanding of the stellar content of galaxies is the accurate knowledge of the M/L ratio. Unfortunately both M and L are subject to large uncertainties; thus not very long ago Baade revised upward the luminosities of galaxies by a factor of 4 and we may still have to revise them by another large factor.** On the other hand, our knowledge of the masses of galaxies seems also to be very uncertain save for a few cases of spirals where rotational velocities permit the application of Mayall's method[1] and provided their distances are correctly estimated. For early E galaxies, where no rotation is expected or measurable, the only masses known seem to be that of M32 determined by Schwarzschild[2] and the statistical determination in double galaxies, by Page.[3] For the *E7* galaxy NGC 3115 where rotation has been measured, Oort[4] and later Schwarschild[2] have estimated the mass; since Oort's determination of the mass of NGC 3115 it appeared that there was great difference in the M/L ratio for spirals and the vecinity of the sun on the one hand, and the M/L ratio for galaxies composed of what Baade called population II on the other. Schwarschild[2] has recently summarized this state of affairs; it turns out from his study that M/L (pg) $\approx$ 4 for the vecinity of the sun, M33 and large Magellanic cloud, and M/L (pg.) $\approx$ 100-200 for the only E systems individually studied: M32 and NGC 3115. To explain this wide difference it has been suggested several times that E galaxies have an enormous proportion of main sequence dwarfs, and even a luminosity function has been computed by Roberts[5] which accounts for the difference in 6 colors between M32 and an average globular cluster and gives an M/L (pg.) $\approx$ 85. Less orthodox has been the suggestion by Kraft[6] that the difference in the M/L ratio is caused by large quantities of gas. Baum[7] has also discussed the content of elliptical galaxies based on their integrated properties.

In view of the great significance these considerations may have, we think that the case of the two galaxies where this difference appears deserves some additional analysis. For NGC 3115 the uncertainties in its distance are enormous since in addition to those involved in the scale of distances, the only criteria we seem to have are its radial velocity and its apparent diameter. Therefore, it does not look as if very reliable conclusions can be drawn from it. As for M32 the mass was derived by Schwarzschild[2] from an asymmetry in the radial velocities of M31 assuming it to be caused by the tidal action of M32 on the asymmetric arm of M31; in computing the mass of M32 the differences in velocities ΔV are quite uncertain and the distance d taken between the perturbed area and M32 is even less known since we do not know what the trajectory of M32 has been in the past; as the mass of M32 thus determined is $\sim d^2 \Delta V$ we see that it could be very easily uncertain by a factor of 10 or more.

We think it should be safer to estimate the mass of M32 from the dispersion of its star velocities as determined by Minkowski,[8] and with the help of the Virial Theorem. Since the distance to M32 ranks among the best known for galaxies, we should expect a reliable estimate of the mass and consequently of the M/L ratio.

2. *Discussion of the Method.*—For an stationary ensemble of particles subject only to Newtonian forces the Virial Theorem states that:

$$2\,T + \Omega = 0 \tag{1}$$

where T is the total kinetic energy of the system and Ω its total potential energy. A galaxy composed only of stars satisfies the conditions of the theorem; furthermore, since we are dealing with an evolved system we can introduce the not very unrealistic assumption that all the stars have the same mass. We will have therefore:

$$M\,\overline{V^2} + \Omega = 0 \tag{2}$$

where $\overline{V^2}$ is the average of the square of the space velocities of the stars referred, say, to the center of mass of the galaxy; M is the total mass of the galaxy. To establish a relationship between $\overline{V^2}$ and the observed velocity dispersion one must make an additional assumption about the statistics of the orbits of the stars in the galaxy; if we restrict ourselves to the case of a spherical configuration of stars, we can assume with Belzer, Gamow and Keller,[9] and based on the analysis of Von Hoerner[10] on the orbits of globular clusters, that the components of the spherical system move in so highly eccentric orbits that we can think of their stars as performing anharmonic oscillations through the center of the gal-

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** Data communicated to me by Dr. W. Baum, suggests this possibility.

axy. In this case, when analyzing spectroscopically the light of the center of the image of an spherical galaxy, we will be obtaining the true picture of the dispersion of the space velocities, i. e.

$$\sigma^2_{obs} = Estimate\ \overline{V^2}$$

Next we need to estimate the potential energy of the system; this is:

$$\Omega = -G \int_0^R \frac{M(r)\, d\, M}{r}$$

where $M(r)$ is the mass inside a sphere concentric with the galaxy and of radius r; R is the "radius" of the galaxy. We see that in order to estimate the potential, energy, a knowledge of the mass distribution within the galaxy is needed. Here one can proceed in a number of ways: we can say that the mass distribution corresponds to that of a polytropic gas sphere of a given index, for instance $n = 5$, as has been done by Spitzer.[11] However the potential energy could be too sensitive to the mass distribution; therefore, it is preferable to follow the more realistic possibility. This we think is given by the light distribution in space, as it can be inferred from the projected densities (i. e. in the focal plane of the telescope). In other words, that the M/L ratio is constant throughout the galaxy. The reasons for thinking that this is so are the following:

1) Schwarzschild[2] has found that the data on the rotational velocities in M31 and M33 are consistent with the assumption of constant M/L.

2) Baum has pointed out that the distribution of globular clusters in M87 follows the same trend as the luminosity and that as a consequence no equipartition of energies has been reached.[12]

3) Photoelectric color data also by Baum[13] for the same object from $d = 11''$ to $712''$ strengthens this supposition since in the case of complete equipartition de Vaucouleurs[14] has estimated that the color index will change from 0.7 in the center to 1.23 at the periphery, contrary to Baum's observations.

4) Morgan and Mayall in analyzing the spectra of M31, do not find any appreciable change from the inner nucleus to the main body.[15]

3. *Derivation of the formula.*—To compute the potential energy we shall use two independent sets of data:

a) De Vaucouleurs[14] has computed the light densities in space for spherical galaxies from his interpolation formula representing the projected intensities; if we multiply these light densities by $K\left(=\frac{M}{L}\right)$ a constant throughout the galaxy as shown above, we obtain the mass density; We thus have:

$$\Omega = -G\, K^2 \int_0^{R/a} \frac{a^5\, L(\alpha) dL}{\alpha}$$

From de Vaucouleurs' standard $B(\alpha)$ one can derive the particular run of density of a given galaxy multiplying the light densities* $B(\alpha)$ by a constant; the radius in centimeters to which this density corresponds is equal to the adimensional radius α times a characteristic length a, which is the radius in centimeters of the circle which contains ½ of the light of the "projected" galaxy on a plane normal to the line of sight and passing through the center of the galaxy in question.

Obviously, the radius of this circle depends on the distance between the galaxy and the observer for a fixed radius on the telescopic image. For a particular galaxy we will include the factor which gives, the actual light densities in the factor K of conversion from light to mass; let us call it now K'.

$$\Omega = -G\, K'^2\, a^5 \int_0^{R/a} \frac{L'(\alpha)\, dL'}{\alpha} \tag{3}$$

and where $L'(\alpha) = \int_0^\infty 4\pi\alpha^2\, B(\alpha) d\alpha$. The integral in equation (3) does not depend on the particular galaxy we may be investigating, and can be computed from de Vaucouleurs' data once and for all; a is obtainable from the photometry of the galaxy in question, and K'^2 we obtain as follows:

$$M = K'\, a^3\, L'\, (R/a)$$

$$\Omega = -G \frac{M^2}{a\, L'^2(R/a)} \int_0^{R/a} \frac{L'(\alpha)\, dL'}{\alpha} \tag{4}$$

* In space.

and from equation (2)

$$M \overline{V^2} = G \frac{M^2}{a\, L'^2(R/a)} \int_0^{R/a} \frac{L'(\alpha)\, dL'}{\alpha}$$

finally

$$M = \frac{a \overline{V^2}}{G} \frac{L'^2(R/a)}{\int_0^{R/a} \frac{L'(\alpha)\, dL'}{\alpha}} \qquad (5)$$

This equation gives the mass in grams of any spherical galaxy for which a knowledge of the average square velocities is available and also for which a has been determined as described previously. The factor $\frac{L'^2(R/a)}{\int_0^{R/a} \frac{L'(\alpha)\, dL'}{\alpha}}$ was determined by graphical integration taking as the upper limit $R/a \equiv \alpha = 24$ which is indeed an extrapolation of the observations. However, the contribution to the integral of the extrapolated part is about 10% of the total. It came out:

$$\frac{L'^2(R/a)}{\int_0^{R/a} \frac{L'(\alpha)\, dL'}{\alpha}} = 3.11$$

b) The second estimate of the potential energy is obtained from Baum's data who kindly supplied me with his unpublished photometry of M87,* which extends to the amazing distance of 712″. The method we follow is the one advocated by Schwarzschild[2] which seems best suited when dealing with empirical data. Since it is well known from the work of Hubble,[16] de Vaucouleurs[17] and Baum[12] that all spheroidal galaxies seem to be built on the same model, individual galaxies differing only in scale and total luminosity, one can compute the potential energy from the data of Baum and Van Houten[18] and then to establish the type of transformation one will have to develop in order to estimate the potential for any other galaxy.

Let

$$W = \int_0^R \frac{L(r)\, dL}{r}$$

Integrating it graphically via Schwarzschild's method we obtained $\frac{L^2(R)}{W} = 3.2 \times 10^2$, in the units in which the photometry was expressed, i. e. 25th magnitude per square second of arc. The integration was carried to the limit where the photometry extended i. e. $R = 712''$. It can be shown that if we introduce another system of units where the unit of length is a as defined above, and any other unit of intensity, and calling with primes the quantities in the new system of units, the following relation holds:

$$\frac{L^2(R)}{W} = \lambda \frac{L'^2(R')}{W'}$$

where λ is the number of times the old unit of length is contained in the new. We see first that $\frac{L^2(R)}{W}$ depends only on the scale and not on the units of intensity; second, $\frac{L'^2(R)}{W'}$ is the same for all galaxies since expressed in these units all galaxies have the same light distribution. Thus from the data for M87 we can get $\frac{L'^2(R')}{W'}$. Since λ came out to be 113″ therefore $\frac{L'^2(R')}{W'} = 2.8$ but this quotient is expressed in de Vaucouleurs' units and hence is directly comparable with the results computed with de Vaucouleurs' figures; as we see they are very similar. Since the galaxy certainly extends beyond 712″ we should expect $\frac{L'^2(R')}{W'}$ to be somewhat larger. We therefore take a compromise value closer to that found with de Vaucouleurs' extrapolations which extend much farther.

* As well as Van Houten's photometry for the inner *($d \leqslant 10''$)* part of this galaxy.

Finally, the expression for the mass in grams is: (everything else in c. g. s. units).

$$M = 3.0 \frac{a \sigma^2_{obs}}{G} \qquad (6)$$

or if we prefer:

$$M = 0.2 \; a \; d \; \sigma^2_{obs} \qquad (6a)$$

where M is given here is units of solar mass, a in minutes of arc; d, the distance to the galaxy in question, in parsecs, and finally σ_{obs} in km/sec.

Using the data for M87 we computed $G \int_0^\infty \frac{M(r)dr}{r^2}$ which can be transformed to the integral for any other spherical galaxy of mass M and scale a. Equating the above integral to $\frac{1}{2} V_e^2$ where V_e is the velocity of escape from the center of a galaxy, and introducing expression (6) for the mass we finally derive:

$$V_e = 3.7 \; \sigma_{obs} \qquad (7)$$

hence the maximum relative velocity with which two spherical galaxies will collide, assuming zero initial velocities, is:

$$V_{ch} = 7.4 \; \sigma_{obs} \qquad (8)$$

where σ_{obs} is the same for both galaxies.

4. *Numerical Applications.*—On the assumption that M32 is a spherical galaxy we can compute the mass with Minkowski's[8] value for the dispersion $\sigma_{obs} = 100$ km/sec. and de Vaucouleurs'[14] $a = 30''$: with a distance of 5×10^5 pc the mass we obtain is: $M = 5 \times 10^8 M_\odot$ and the velocity of escape is: 370 km/sec.

Finally with the modulus associated to that distance and Holmberg's[19] magnitudes for M32 we obtain:

$$\left.\frac{M/M_\odot}{L/L_\odot}\right|_{pg} = 5 \; ; \qquad \left.\frac{M/M_\odot}{L/L_\odot}\right|_{vis} = 3.3 \; ; \qquad \left.\frac{M/M_\odot}{L/L_\odot}\right|_{Bol} = 1.2$$

where $M/L|_{Bol}$ was obtained by extrapolating beyond $\lambda = 10\,000$ Å Stebbins and Whitford's[20] six color photometry of M32.

As a check on the validity of the expression (6), as well as its dependence on the true eccentricity, we have applied it to M31 which certainly is an extreme case and for which Minkowski[8] has estimated the velocity dispersion to be $\sigma_{obs} = 225$ km/sec; therefore σ^2_{obs} (M31) is about five times the value for M32. Besides, from the photometry of M31 by Redman and Shirley,[21] we obtain $a = 40'$: if M31 is at the same distance as M32 then a (M31) is about 80 times a for M32 hence:

$$\frac{M\ (\text{M31})}{M\ (\text{M32})} = 400 \quad ; \quad M\ (\text{M31}) = 2 \times 10^{11} M_\odot \quad ; \quad V_e \approx 800 \; km/sec$$

This value for the mass is in fairly good agreement with Wyse and Mayall's estimate 1.9×10^{11} $M_\odot$ (corrected by a factor of 2 on account of the new distance to M31 used in estimating the mass) as well as with Schwarzschild's estimate of 1.4×10^{11} $M_\odot$, and Schmidt's[22] mass of 3.4×10^{11} $M_\odot$ and velocity of escape (from the center) of 700 Km/Sec. This mass is somewhat different; however the distance used by Schmidt is about 40% larger than that we took. With this distance our formula will give a mass of 2.8×10^{11} $M_\odot$.

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LA MASA DE LAS GALAXIAS ESFERICAS; M32, UNA POSIBLE APLICACION

Es de gran importancia para la comprensión del contenido estelar de las galaxias el conocimiento preciso de sus propiedades integradas; destacándose entre éstas la relación M/L. Según un trabajo reciente de Schwarzschild, la relación M/L para las galaxias de tipo E es considerablemente mayor que la correspondiente a las galaxias espirales, irregulares y la vecindad del sol.

En este trabajo hacemos notar cuán débil es la evidencia sobre la que descansa la aparente diferencia en las relaciones M/L. Una fuente de incertidumbre depende de la distancia de los objetos estudiados, y otra de la determinación de las masas de las galaxias esferoidales.

Proponemos entonces determinar las masas de las galaxias esferoidales por medio del teorema virial (ecuación 1). Para ello es necesario conocer $\overline{V^2}$ y la energía potencial total de la configuración; afortunadamente, para algunas galaxias Minkowski ha determinado espectroscópicamente la dispersión de velocidades. Si como es razonable suponer las estrellas se mueven —en estas galaxias— en órbitas muy excéntricas, las dispersiones de velocidades observadas por Minkowski son una estimación directa de $\overline{V^2}$.

Para determinar la energía potencial supusimos que la relación M/L es constante dentro de una galaxia; suposición que consideramos fundada en vista de las razones que se exponen.

Se han realizado dos determinaciones independientes de la energía potencial.

a) De Vaucouleurs ha calculado en general las densidades luminosas tridimensionales para una galaxia esférica a partir de su fórmula de interpolación para los brillos proyectados. Con los datos tabulados por de Vaucouleurs calculamos gráficamente la energía potencial, la cual nos queda en función de la masa de la configuración y de la escala a; esta última la podemos definir como el producto de la distancia a la galaxia —en centímetros— por el radio, en radianes, del círculo concéntrico a la imagen de la galaxia y que contenga la mitad de la luz total.

b) Con la fotometría de Baum y Van Houten para M87, es posible también calcular la energía potencial para esta galaxia, y sabiendo que todas las galaxias esféricas están construidas con arreglo al mismo plan, generalizamos el resultado.

Combinando estas dos determinaciones de la energía potencial con la dispersión de velocidades, se obtienen las fórmulas (6) y (6a) que nos dan la masa en función de la escala a y la dispersión de velocidades σ_{obs}.

También podemos encontrar por integración gráfica la energía necesaria para llevar una partícula desde el centro de una galaxia esférica hasta su periferia; de aquí concluimos que la velocidad de escape desde el centro de una galaxia esférica es $Ve = 3.7\ \sigma_{obs}$ y que la máxima velocidad relativa con que chocarían dos galaxias inicialmente en reposo es: $V_{ch} = 7.4\ \sigma_{obs}$ donde hemos supuesto la misma σ_{obs} para ambas galaxias.

Con los datos de Minkowski y la fotometría de Vaucouleurs podemos calcular la masa de M32 bajo la hipótesis de que las consideraciones anteriores le sean aplicables. Así la fórmula (6) nos dá:

$$M(\text{M32}) = 5 \times 10^{8}\ M_{\odot} \quad ; \quad Ve = 370\ km/seg.$$

$$\left.\frac{M/M_{\odot}}{L/L_{\odot}}\right|_{pg} = 5\ ; \qquad \left.\frac{M/M_{\odot}}{L/L_{\odot}}\right|_{vis} = 3.3 \qquad \left.\frac{M/M_{\odot}}{L/L_{\odot}}\right|_{Bol} = 1.2$$

Como una prueba de la validez de la fórmula (6) y su aplicabilidad a M32, ofrecemos el cálculo de la masa de M31; este compara muy satisfactoriamente con las determinaciones independientes basadas en el análisis de la rotación de esta galaxia por Wyse y Mayall, Schwarzschild, y Schmidt. Con los datos de Minkowski sobre la dispersión de velocidades y la fotometría de Redman y Shirley encontramos:

$$M(\text{M31}) = 2 \times 10^{11}\ M_{\odot} \quad ; \quad V_e \approx 800\ km/seg.$$