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AN ELEMENTARY THEORY FOR THE FORMATION OF ARMS IN SPIRAL GALAXIES

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Introduction.—In a previous paper, Pismis has found that individual stellar associations within several kiloparsecs of the sun have a tendency of moving away from the galactic center[1]. At the same time there is an unmistakable evidence from radio observations that hydrogen gas is expelled from the central region of our own galaxy[2]. These two empirical facts lead us to suggest that the arms in spiral galaxies may be formed, partly at least, out of ejected gas from the central region. From this suggestion we formulate, in what follows, an elementary theory for the formation of spiral arms based upon the gravitational interaction between the ejected matter and the central mass of the galaxy. Several theories for the formation of spiral arms based upon the Newtonian law of attraction have been proposed in the past[3]; the present one differs from them by (1) conception of ejection of mass from the central region and (2) simplicity of mathematical formalism. To be specific, we regard the spiral arm at any given instant as the locus of particles ejected at different times by a source which rotates with the galaxy.

Formulation.—We shall consider the attraction of the ejected masses by the galactic nucleus alone because the mass in the disk is very small and can therefore be neglected. In other words, we can treat the motion of the ejected particles simply as a two-body problem.

Let the ejection point or source of ejection be rotating around the center of the galaxy with an angular velocity ω_o in a circular path of radius r_o. For the convenience of calculation, we shall adopt the linear rotational velocity of the ejection point as the unit of velocity and its distance from the center (i. e., r_o) as the unit of distance. If the relative ejection velocity with respect to the rotating source of the ejection lies in the galactic plane, makes an angle α with the radius vector, and has a magnitude λ in our unit, the velocity u_o of the ejected particles with respect to the center of the galaxy at the time of ejection is (Cf. Figure 1).

$$u^2_o = 1 + 2\lambda \sin \alpha + \lambda^2 \qquad (1)$$

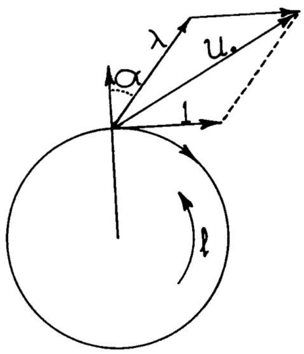

Fig. 1

Once ejected, the particles will follow an orbit of the form of a conic section according to the values of u_o and r_o. In the present paper we shall confine our investigation to elliptical orbits for which, as we shall see presently, there exists an especially easy short-cut for computing the locus of particles ejected at different times but seen at a given instant.

If the semi-major axis of the elliptical orbit is denoted by γ (in units of r_o), we have from the law of motion,

$$u^2_o = 2 - \frac{1}{\gamma} \qquad (2)$$

Combining (1) and (2) we obtain

$$\frac{1}{\gamma} = 1 - 2\lambda \sin \alpha - \lambda^2, \qquad (3)$$

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which determines γ from given values of λ and α. The eccentricity, e, of the elliptical orbit, also determined by these two parameters, is given by

$$1 - e^2 = \frac{1}{\gamma}(1 + \lambda \sin \alpha)^2 \tag{4}$$

which can be derived directly from a consideration of angular momentum of the ejected particles.

Let us denote by x the distance of a particle from the center of galaxy in units of r_o. We then have for the equation of the orbit

$$x = \frac{(1 + \lambda \sin \alpha)^2}{1 + e \cos v} \tag{5}$$

where

$$v = \theta - \tilde{\omega} \tag{6}$$

is the true anomaly.

What interests us are not the orbits of the particles themselves but the locus of these particles ejected at different times by the same revolving source. In order to derive this locus, let us assume that the time of observations is t_o when the ejecting source is located at $x = 1,\ \theta = o$. Therefore at time $t = t_o - \Delta t$, it is located at $x = 1,\ \theta = \theta_o = -\omega_o \Delta t$. Thus it is our purpose to find out the position on the elliptical orbit at time $t = t_o$ of the particle which is ejected at $x = 1,\ \theta = \theta_o = -\omega_o \Delta t$ at time $t = t_o - \Delta t$. To be specific we determine for a given pair of values λ and α the true anomaly $v_o = \theta_o - \tilde{\omega}$ from (5) at the time of ejection, i. e. $t = t_o - \Delta t$ by the condition $x = 1,\ \theta = \theta_o$. The corresponding eccentric anomaly E_o and the mean anomaly M_o can be calculated from the well-known formulae. Thus, the mean anomaly M_o corresponds to time $t = t_o - \Delta t$. At time t_o the mean anomaly is

$$M = M_o + \frac{2\pi}{P}\Delta t \ , \tag{7}$$

where P is the period of orbital motion of the ejected particles. Here we see clearly why it is especially easy to compute the locus of spiral arms from the elliptical orbits of the ejected particles. For both parabolic and hyperbolic orbits more elaborate calculation is required in order to derive the locus of ejected particles at any given instant.

If P_o is the period of rotation of the ejected source, (7) can be rewritten as

$$M = M_o + \frac{P_o}{P}(\omega_o \Delta t) = M_o + \left(\frac{1}{\gamma}\right)^{3/2} \omega_o \Delta t \ , \tag{8}$$

from which we can compute the corresponding eccentric and true anomalies and consequently x and θ. In this way we determine the position at t_o of the ejected particle which leaves the source at $x = 1$, $\theta = \theta_o = \omega_o \Delta t$, and $t = t_o - \Delta t$. If we repeat the computation for different values of Δt, a locus of particles ejected at different times but observed at any given instant (i. e. at $t = t_o$) can be derived. Figures 2 and 3 illustrate two cases with $\alpha = o,\ \lambda = 0.99$ and $\alpha = o,\ \lambda = 0.90$ respectively. In both cases a locus of the form of spiral (denoted by ABCDE) is obtained. The broken lines AA', BB', etc. represent the individual orbits. In other words, the particles observed at A left the ejecting source at A', etc. The central circle represents the orbit of the ejecting source but does not necessarily mean the actual size of the central nucleus of the galaxy because the ejection may take place in its interior.

Discussion.—In this manner we derive a model for the spiral arms of galaxies. It predicts very well the form of spirals, but we do not know at present why does the ejection take place always in the opposite ends of a diameter of the central region so that one pair or sometimes several pairs of symmetric arms ar formed simultaneously as is observed in almost all spiral galaxies. Also, the model thus obtained predicts too large velocities along the radius vector. This difficulty is however not serious, because the deceleration of the outgoing particles by the prevailing gas and dust originally contained in the disk —a process which has not been included in the present preliminary study— will considerably reduce their velocities of expansion.

Actually the ejected matter must represent only a part of the mass that finally becomes a spiral arm .Inside the nucleus the gas is ionized and may be highly turbulent. Magnetic field may develop inside it at the expense of turbulent energy. Thus the magnetic lines of force are greatly twisted by the turbulent motion. When the gas is ejected out of the nucleus, it carries the magnetic lines of force with it and at the same time stretches and also straightens somewhat the twisted lines of force. The lines of

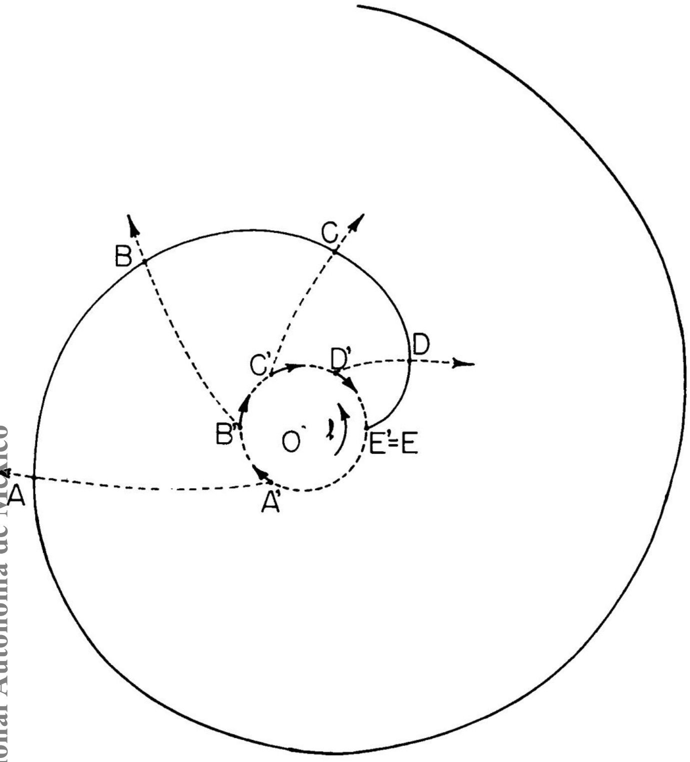

Fig. 2. *Locus of ejected particles for* $\lambda = 0.99$ *and* $\frac{P}{P_o} = 356$

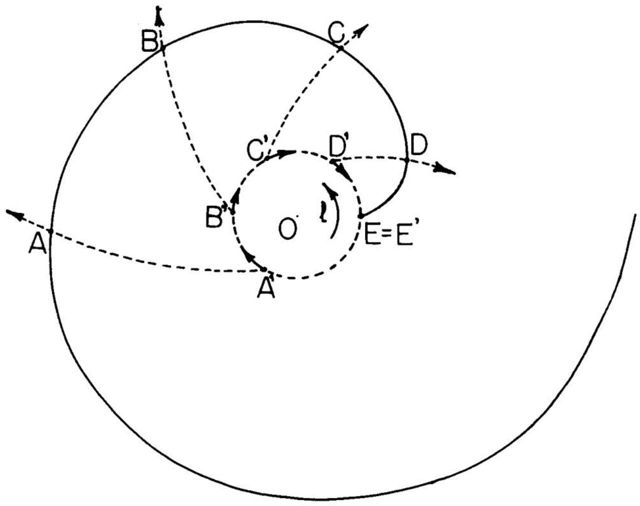

Fig. 3. *Locus of ejected particles for* $\lambda = 0.90$ *and* $\frac{P}{P_o} = 12.07$

force thus straightened may contribute dominatly to the magnetic field now existing in the spiral arms. To form the spiral arms, the ejected matter with the aid of the magnetic lines of force sweeps and collects gas and dust that are already in the disk. If this picture should be correct, our calculation, by neglecting electromagnetic interaction as well as the resistance of the medium in the disk, would represent only a first approximation. However, the general conclusion that spiral arms could be formed by the ejection of gas from the galactic nucleus will not be changed by the introduction of these forces. The mass ejected by the central region may contribute only a very small part to the total mass of the spiral arms, but it provides a trigger mechanism for the segregating of matter originally in the disk to become arms through the influence of its magnetic lines of force and its outward momentum.

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UNA TEORIA ELEMENTAL PARA LA FORMACION DE BRAZOS EN GALAXIAS ESPIRALES

Resumen

En el artículo anterior se ha señalado que las asociaciones estelares tienden a alejarse del centro galáctico[1]; por otra parte, las observaciones radio-astronómicas han puesto en evidencia que la región central de la galaxia expele hidrógeno gaseoso. Estos dos resultados empíricos sugieren que los brazos espirales de las galaxias, cuando menos parcialmente, pueden formarse del material lanzado por la región central. El presente trabajo delinea una teoría para la formación de los brazos espirales, teoría que difiere de las anteriores en: 1) su simplicidad matemática y 2) en el concepto de una expulsión de masa de la región central.

Se supone, que desde un punto de la región central en rotación se inicia una expulsión de material (véase Fig. 1). Para fines de simplificación se supone que las partículas lanzadas describen órbitas elípticas bajo la atracción de la masa central, despreciando así la atracción del material en el disco que es relativamente pequeña.

En un instante dado, no son las órbitas elípticas las que nos interesan, dado que no las podemos observar, sino el lugar geométrico en un instante de las partículas lanzadas desde el tiempo t_0 (o sea cuando principia este fenómeno de lanzamiento). El lugar geométrico mencionado, calculado para dos diferentes valores de la excentricidad, aparece en las figuras 2 y 3, en las cuales se ve claramente la tendencia espiral de las curvas.

La presente teoría predice así la forma espiral de los brazos, pero no explica por qué el fenómeno de la eyección simultánea ocurre en puntos diametralmente opuestos de la región central. Así mismo el modelo trazado aquí supone velocidades demasiado altas a lo largo del radio vector. Pero esta última dificultad no parece ser muy seria, pues durante su movimiento hacia afuera las partículas tenderán a disminuir su velocidad debido al material ya existente en el disco.

Es de esperar que el material lanzado representa tan sólo una fracción de la masa que llega a constituir un brazo espiral. Dentro del núcleo galáctico se formarán campos magnéticos a costa de la energía turbulenta. Las líneas de fuerza magnéticas estarán altamente distorcionadas. Al ser lanzado, el gas llevará consigo estas líneas magnéticas a las que "suavizará" parcialmente en su distorción. Tales líneas de fuerza contribuirán grandemente al campo magnético de los brazos espirales. La materia, aunada a las líneas de fuerza magnéticas, barre y junta el polvo y el gas del disco. Aunque este último mecanismo no está tratado en el presente trabajo, la introducción de las fuerzas magnéticas no alterará la conclusión de que los brazos espirales puedan formarse como consecuencia del lanzamiento del material de un núcleo galáctico en rotación.