

DENSIDADES, POTENCIALES Y FUNCIONES ASOCIADAS EN UNA GALAXIA ESFERICA REDUCIDA *

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Introducción.—El análisis de los movimientos estelares en una galaxia requiere el conocimiento de la densidad, el potencial, la fuerza, etc. como función de la distancia al centro. Actualmente la mejor tabla disponible, para galaxias esféricas, es la calculada por *de Vaucouleurs*⁽¹⁾ en la que se dan las densidades luminosas, las densidades materiales, los potenciales y funciones similares para 18 puntos de la galaxia reducida, bajo la hipótesis de que existe equipartición de energía para las estrellas, lo cual conduce a una distribución de masa completamente diferente de la distribución luminosa. Como esta tabla nos ha parecido insuficiente en algunos de los trabajos que tenemos en desarrollo, y como por otra parte creemos que la hipótesis de equipartición es incorrecta⁽²⁾, hemos decidido calcular otra tabla más extensa en cuanto al número de puntos y de variables, suponiendo que la relación M/L es independiente de la posición en la galaxia; además, investigamos el efecto que sobre las variables calculadas produce modificaciones en el parámetro b de la fórmula de interpolación de *de Vaucouleurs*.

Las variables se calcularon para una galaxia reducida, pero se indica cómo usar estos resultados generales para cualquier galaxia esférica particular cuya masa, escala y relación M/L , sean conocidas.

En las tablas 1 a 4 se listan, —para diferentes valores de la distancia s al centro de la galaxia reducida— la densidad, la masa interior a s , el brillo proyectado interior a la isofota de radio α , la fracción de la masa proyectada interior a α que corresponde a una esfera de radio $s = \alpha$ concéntrica con la galaxia; se listan también la fuerza, el potencial, la velocidad de escape y la energía potencial interior a s .

Finalmente se discute para una galaxia esférica la relación entre la dispersión de velocidades observada espectroscópicamente, y la dispersión de velocidades estelares espaciales.

Estas consideraciones son de interés para una mejor aplicación del Teorema Virial a la determinación de masas de galaxias.

Definiciones

s	:	distancia de un punto al centro de la galaxia reducida.
a_s	:	semi-eje mayor efectivo de una galaxia particular (en radianes).
d	:	distancia del observador a una galaxia particular (en centímetros).
$a = a, d$	:	escala de una galaxia particular (en centímetros).
$r = as$	:	distancia de un punto al centro de una galaxia particular (en centímetros).
$\varrho(r)$	:	densidad material a la distancia r de una galaxia particular.
$\varrho_R(s)$	:	densidad a la distancia s del centro de la galaxia reducida.
$M(r)$	:	masa interior a una esfera de radio r , concéntrica con una galaxia particular; $M(\infty) = M$ = masa total de una galaxia particular.
$F(r)$	:	fuerza por unidad de masa a la distancia r del centro de una galaxia particular.
$f(\alpha)$	:	brillo interior a la isofota de radio α , concéntrica con la imagen de una galaxia.
$V(r)$	:	potencial a la distancia r del centro de una galaxia particular.
$v(r)$	:	Velocidad de escape a la distancia r del centro de una galaxia particular.
$\Omega(r)$	:	Energía potencial interior a r , en una galaxia particular.
K	:	relación masa/luminosidad que consideramos independiente de la posición en la galaxia.

Cuando estas variables en lugar de referirse a una galaxia particular, se usen para un punto s de la galaxia reducida, se indicará este cambio con el índice R : Así, por ejemplo, $M_R(\infty) = f_R(\infty) K_R$: $K = K_R$.

Los valores de las variables calculadas para la galaxia reducida se transforman en valores (en unidades c. g. s.) para una galaxia particular por medio de un factor C de transformación; este factor depende únicamente de la masa, la relación M/L y la escala de la galaxia particular. El índice indica la variable que esta C transforma; ej. $M(r) = C_M M_R(s)$.

* Entendemos por galaxia reducida, al igual que *de Vaucouleurs*, aquella cuyo "brillo" superficial a la distancia α de su centro está dado por:

$$\log_{10} B(\alpha) = -A(\alpha^{1/4} - 1) \quad ; \quad \text{o su equivalente} \quad B(\alpha) = e^b e^{-b\alpha^{1/4}}$$

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Es relativamente simple encontrar los factores de transformación; a continuación se deduce C_p :

$$M = \int_0^{\infty} 4\pi r^2 \varrho(r) dr = C_p a^3 \int_0^{\infty} 4\pi s^2 \varrho_R(s) ds = C_p a^3 M_R(\infty)$$

$$M = C_p a^3 K f_R(\infty) \quad C_p = \frac{M}{a^3 K f_R(\infty)}$$

Las demás constantes de transformación se encuentran en forma similar, (véase la siguiente sección donde se presentan las ecuaciones usadas para la galaxia reducida) pudiéndose demostrar que:

$$C_M = \frac{M}{K f_R(\infty)} \quad (1) \quad C_v = \frac{C_M}{a} \quad (4)$$

$$C_p = \frac{C_M}{a^3} \quad (2) \quad C_v = \sqrt{C_v} \quad (5)$$

$$C_F = \frac{C}{a^2} \quad (3) \quad C_\Omega = \frac{C_M^2}{a} \quad (6)$$

Las Ecuaciones

a) Densidades. Las densidades espaciales se han calculado por medio de la ecuación desarrollada por *Von Zeipel*⁽³⁾ para un cúmulo estelar esférico, suponiendo que la densidad material proyectada $\sigma_R(\alpha)$ sea:

$$\sigma_R(\alpha) = K B(\alpha) \quad \sigma_R(\alpha) = K e^b e^{-b\alpha^{1/2}} \quad (7)$$

La densidad espacial está dada por:

$$\varrho_R(s) = -\frac{K}{\pi} \int_s^{\infty} \frac{B'(\alpha) d\alpha}{\sqrt{\alpha^2 - s^2}} \quad (8)$$

El cálculo numérico de esta integral presenta dificultades ya que el integrando tiende a ∞ cuando α tiende a s ; sin embargo, puede demostrarse que para toda $s > 0$ la integral existe. Necesitamos, entonces, transformar esta integral impropia en otra equivalente más fácil de trabajar numéricamente sea:

$$\alpha^{1/2} = t \quad j = s^{1/2} \quad (9)$$

Substituyendo (7) y (9) en (8) nos da:

$$\varrho_R(j^2) = \frac{e^b b K}{\pi} \int_j^{\infty} \frac{e^{-bt} dt}{\sqrt{t^2 - j^2}} = \frac{e^b b K}{\pi} \int_j^{\infty} \frac{e^{-bt} dt}{\sqrt{t-j} \sqrt{(t+j)(t^2+j^2)(t^4+j^4)}} \quad (10)$$

integrando por partes esta última integral, e introduciendo: $h(t) = \sqrt{(t+j)(t^2+j^2)(t^4+j^4)}$

nos queda:

$$\varrho_R(j^2) = \frac{2 e^b b K}{\pi} \int_j^{\infty} \frac{\sqrt{t-j} e^{-bt}}{h(t)} \left(b + \frac{h'(t)}{h(t)} \right) dt \quad (11)$$

$$\begin{aligned} \text{Además} \quad \frac{h'(t)}{h(t)} &= \frac{d}{dt} L_n h(t) = \frac{d}{dt} \frac{1}{2} [L(t+j) + L(t^2+j^2) + L(t^4+j^4)] = \\ &= \frac{1}{2(t+j)} + \frac{t}{t^2+j^2} + \frac{2t^3}{t^4+j^4} \end{aligned}$$

finalmente

$$\varrho_R(s) = \frac{2 e^b b K}{\pi} \int_{s^{1/4}}^{\infty} \frac{\sqrt{t-s^{1/4}} e^{-bt}}{\sqrt{(t+s^{1/4})(t^2+s^{1/2})(t^4+s)}} \left(b + \frac{1}{2(t+s^{1/4})} + \frac{1}{t^2+s^{1/2}} + \frac{2t^3}{t^4+s} \right) dt \quad (12)$$

El integrando en la ecuación (12) está definido para todos los puntos del intervalo de integración.

b) Las $M_R(s)$ se calcularon por medio de la ecuación

$$M_R(s + \Delta s) = M_R(s) + \int_s^{s+\Delta s} 4\pi s^2 \varrho_R(s) ds \quad s > 0.01 \quad (13)$$

$$M(s) = 4/3 \pi s^3 \varrho_R(0.01) \quad s \leq 0.01$$

Es decir, como se desconoce el comportamiento del brillo proyectado para $\alpha < 0.01$ hemos supuesto que la densidad es constante $= \varrho(0.01)$ para $s \leq 0.01$. En M32 a la distancia de 8×10^5 pcs, esto equivale a suponer un núcleo con densidad constante y de radio 1.2 parsecs.

c) Las fuerzas se calcularon con:

$$\frac{F_R(s)}{G} = \frac{M_R(s)}{s^2} \quad (14)$$

d) Los potenciales se calcularon por medio de la ecuación:

$$\begin{aligned} \frac{V_R(s + \Delta s)}{G} &= \frac{V_R(s)}{G} + \int_s^{s+\Delta s} \frac{M_R(s)}{s^2} ds = \\ &= \frac{V_R(s)}{G} + \frac{M_R(s)}{s} - \frac{M_R(s + \Delta s)}{s + \Delta s} + \int_s^{s+\Delta s} 4\pi s \varrho_R(s) ds \end{aligned} \quad (15)$$

e) Las velocidades de escape se calculan por medio de:

$$v_R(s) = \sqrt{2[V(\infty) - V(s)]} \quad (16)$$

f) Las energías potenciales con:

$$\frac{\Omega_R(s)}{G} = 1/2 \int_0^s \frac{V}{G} dM_R - 1/2 \frac{M_R^2(s)}{s} - \frac{M_R(s)V}{2G} \quad (17)$$

g) Finalmente, el brillo proyectado interior a α por la ecuación:

$$\begin{aligned} f(\alpha) &= 2\pi \int_0^{\alpha} \alpha B(\alpha) d\alpha = \frac{8! \pi e^b}{b^8} \left[1 - e^{-b\alpha^{1/4}} \sum_{i=0}^{i=7} \frac{b^i}{i!} \alpha^{i/4} \right] \\ &= f(\infty) \left[1 - \frac{B(\alpha)}{e^b} \sum_{i=0}^{i=7} \frac{b^i}{i!} \alpha^{i/4} \right] \end{aligned} \quad (18)$$

Al calcular estas funciones es interesante saber qué tan sensibles son a variaciones en el parámetro b .

En efecto, en el plano $[\log_e B(\alpha), \alpha^{1/4}]$ la fórmula de interpolación de *de Vaucouleurs* está representada por una línea recta que no describe con absoluta fidelidad las observaciones empíricas, por ejemplo las de *Baum*⁴. En vista de esto, hemos considerado 3 casos:

Caso 1. La fórmula de interpolación es válida para toda $\alpha \geq 0.01$, pero si esto es así entonces de la condición $\frac{f(1)}{f(\infty)} = \frac{1}{2}$ y de la ecuación (18) vemos que el parámetro b queda ya determinado. Así resulta que no hay propiamente un parámetro "libre" esta b teórica se le encontró igual a 7.6692494.

Caso 2. Como ya se mencionó, en el plano $[\log_e B(\alpha), \alpha^{1/4}]$ las observaciones a partir de cierto punto α_0 se desvían de la recta teórica, sin embargo a partir de α_0 también se las puede representar con una recta pero de diferente pendiente. Así tenemos 2 rectas, una en el intervalo $0.01 \leq \alpha \leq \alpha_0$ con la misma pendiente que en el Caso 1, y otra en $\alpha > \alpha_0$, con pendiente $b_1 = 6.2255$.

Caso 3. Como en el caso anterior, aquí consideramos también 2 rectas: en el intervalo $0.01 - \alpha_0$ la pendiente es nuevamente $b = 7.6692494$; para valores de $\alpha > \alpha_0$ usamos una recta con pendiente $b_2 = 10.6291$.

Pero α_0 no es una constante que podamos fijar arbitrariamente;* en efecto, una vez impuesta la condición de que en el plano $[\log_e B(\alpha), \alpha^{1/4}]$ el brillo proyectado está representado por 2 rectas que se intersectan en α_0 , se sigue que necesariamente $\alpha_0 = 1$.

Muy probablemente los Casos 2 y 3 acotan cualquier desarrollo futuro en la fotometría de las galaxias E_0 .

Los cálculos realizados.

Para 160 puntos adecuadamente distribuidos dentro de la galaxia reducida se calcularon:

$$\frac{\varrho_R(s)}{K} - \frac{M_R(s)}{K} \left| f_R(\alpha) \right|_{\alpha=s} - \frac{M_R(s)}{K f_R(\alpha)} \Big|_{\alpha=s} - \frac{M_R(s)}{M_R(\infty)} - \frac{F_R(s)}{GK} - \frac{V_R(s)}{GK} - \frac{v_R(s)}{\sqrt{GK}} - \frac{\Omega_R(s)}{GK^2}$$

Para calcular $M_R(s)$ se integró por regla de *Simpson* la integral en (13); se usaron 10 puntos para cada integración, así que en total hubo que calcular 1600 densidades. Las integrales requeridas en el cálculo de los potenciales, se calcularon también por regla de *Simpson*.

Las integrales en la fórmula para la densidad se calcularon por regla de *Simpson*. Con 90 puntos en el intervalo $0.01 - 1.00$, 70 en el intervalo $1 - 1.35$ y 55 puntos de 1.35 a 28 ; el límite superior de las integrales se hizo $t = 4$, lo cual equivale a $\alpha = 256$.

Todos estos cálculos se realizaron con la calculadora IBM 650 del Centro Electrónico de Cálculo de la Universidad Nacional de México. Ciertas partes del cálculo fueron programadas con la rutina de *Livermore*⁽⁶⁾, la cual ha sido modificada para incluir una pseudo instrucción más que integra por regla de *Simpson*; y el resto del programa se ensambló con S. O. A. P.

Las Tablas.

La Tabla 1 da las densidades con 7 cifras significativas en la cercanía del centro, donde aquella varía más rápidamente. En total se dan 345 densidades en el intervalo 0.01 a 1.00 .

La Tabla 2 lista para 160 puntos:

$$s \quad \frac{\varrho_R(s)}{K} - \frac{M_R(s)}{K} \left| f_R(\alpha) \right|_{\alpha=s} - \frac{M_R(s)}{K f_R(\alpha)} \Big|_{\alpha=s} - \frac{M_R(s)}{M_R(\infty)} - \frac{F_R(s)}{GK} - \frac{V_R(s)}{GK} - \frac{v_R(s)}{\sqrt{GK}} - \frac{\Omega_R(s)}{GK^2}$$

Estos resultados se transcriben con 5 cifras significativas, la quinta redondeada.

La Tabla 3 contiene la densidad y demás parámetros calculados con los coeficientes, según el caso 2.

La Tabla 4 contiene las mismas variables que la tabla anterior, pero con los coeficientes, según el caso 3.

* Agradecemos al Dr. Huang una discusión sobre este asunto.

Los resultados de la columna 5 en las Tablas 2, 3 y 4 permiten entender mejor la relación entre la dispersión de velocidades espaciales en una galaxia esférica y la dispersión observada espectroscópicamente.

En efecto, si la luz que recoge el espectrógrafo del centro de la imagen de una galaxia esférica proviene en su mayor parte del núcleo de la galaxia, entonces $\sigma_{\text{obs}}^2 = \frac{1}{3} \sigma_{\text{espacial}}^2$; si por el contrario la mayor parte de la luz proviene de las estrellas que están fuera del núcleo, y si éstas se mueven en órbitas rectilíneas, entonces: $\sigma_{\text{obs}}^2 = \sigma_{\text{espacial}}^2$.

Sinclair Smith⁽⁶⁾ afirma, por ejemplo que en M 32 las estrellas que se ven proyectadas sobre el núcleo comparadas con el número en el núcleo, pueden ignorarse. Sin embargo, desde el punto de vista espectrográfico, el núcleo mínimo observable está determinado por el disco del "seeing"; es poco probable que éste sea menor de unos 2" de diámetro, que para M 32 corresponde a $\alpha = 0.04$, y, según vemos por la columna 5 de las Tablas 2, 3 y 4, aproximadamente el mismo número de estrellas estará en el núcleo que fuera de él; en estas condiciones:

$$\sigma_{\text{obs}}^2 = \frac{1}{2} \left(\frac{1}{3} \sigma_{\text{esp}}^2 \right) + \frac{1}{2} \sigma_{\text{esp}}^2$$

entonces $\sigma_{\text{esp}}^2 = \frac{3}{2} \sigma_{\text{obs}}^2$. Si esta interpretación es correcta, la masa de M 32⁽²⁾ deberá ser aumentada en un 50% y lo mismo podríamos afirmar en las relaciones de M/L.

Es un placer agradecer al Dr. de Vaucouleurs la información facilitada sobre el estado actual de la fórmula de interpolación. También deseamos reconocer las grandes facilidades prestadas por el Ing. S. Beltrán, del Centro Electrónico de Cálculo de la Universidad Nacional de México.

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DENSITIES, POTENTIALS, ESCAPE VELOCITIES AND RELATED FUNCTIONS FOR A SPHERICAL REDUCED GALAXY*

Introduction.—The analysis of the stellar motions in a galaxy requires a knowledge of the density, potential, etc. as a function of distance to the center. At present, the best table available for spherical galaxies is one by de Vaucouleurs¹ which gives light densities, mass densities, potentials, etc. for 18 points, on the assumption that equipartition of energy holds for the stars in these systems. This supposition leads to a mass distribution quite different from the light distribution.

As such table is insufficient for some of the work we are carrying on, and also because we feel that the hypothesis of equipartition is incorrect,² we have decided to compute anew all the necessary variables for more points and with the assumption of a constant M/L ratio within a given galaxy.

In this work we also investigate the effect, on the computed variables, of minor variations in the coefficient b of de Vaucouleurs' interpolation formula.

The variables have been computed for a reduced galaxy and therefore are completely general, but can be transformed to any particular galaxy by means of a factor c whose index indicates the variable to which it applies. Equations (1) to (6) give these factors for the variables in question. One needs to know the mass, the mass luminosity ratio and the scale of the galaxy to be investigated. Tables 1 through 4 list the computed functions.

Finally, we discuss for an spherical galaxy the relation between the observed (spectroscopically) velocity dispersion and the dispersion of space velocities within the galaxy.

These considerations are of interest for a better application of the virial theorem to the determination of masses of galaxies.

* We understand by a reduced galaxy one whose surface brightness $B(\alpha)$ is given by de Vaucouleurs' interpolation formula:

$$\log_{10} B(\alpha) = -A(\alpha^{1/4} - 1) \quad \text{or its equivalent:} \quad B(\alpha) = e^b e^{-b\alpha^{1/4}}$$

Definitions

s	: distance of a point to the center of the reduced galaxy.
a_e	: semi major effective axis of a particular galaxy (in radians).
d	: distance from the observer to a particular galaxy (in centimeters).
$a = a_e d$	: scale of a particular galaxy (in centimeters).
$r = as$	: distance of a point to the center of a particular galaxy.
$q(r)$	: mass density to the distance r from the center of a given galaxy.
$q_R(s)$	: mass density to the distance s from the center of the reduced galaxy.
$M(r)$	: mass inside a sphere of radius r concentric with a given galaxy.
$F(r)$	: force per unit mass at the distance r from the center of a given galaxy.
$f(\alpha)$	: projected brightness contained within the isophote of radius α concentric with the image of a galaxy.
$V(r)$	: potential at the distance r from the center of a galaxy.
$v(r)$	: escape velocity at the distance r from the center of a galaxy.
$\Omega(r)$	: potential energy contained within a sphere of radius r concentric to a galaxy.
K	: mass luminosity ratio, which we take independent of position within an E galaxy.

When these variables refer to the reduced galaxy we indicate the change by adding the index R , thus $M_R(\infty) = f_R(\infty)K_R$; $K = K_R$.

The Equations.—Space densities were computed from the projected brightness by means of von Zeipel's method³; however, von Zeipel's equation has the inconvenience of being an improper integral, thus making difficult the numerical computations. We were able to transform the improper integral into another, whose integrand is defined for every point of the interval of integration. Equation (12) is the final equation which was used for the numerical integration. Equations (13) to (18) were used for the computations of the remaining variables. Here it was assumed that the validity of de Vaucouleurs' formula holds from $\alpha = 0.01$ to infinity, although in the actual computing the upper limit of the integral in (12) was taken $t = 4$, which is the same as $\alpha = 256$. For the $M(r)$ it was assumed that $q_R(s) = \text{constant} = q_R(0.01)$ for $s \leq 0.01$. In reality, for $\alpha < 0.01$ we do not know how the projected brightness goes. In the case of M32, this constant density nucleus of $\alpha = 0.01$ will have a radius of 1.2 pc. ($d = 8 \times 10^5$ pc.).

In computing these functions, it is of interest to see how they get modified when changing the parameter b . Three cases are to be considered:

Case 1. The validity of the interpolation formulae holds for any $\alpha \geq 0.01$; if this is so, it is easy to see from equation (18) and the condition $\frac{f(1)}{f(\infty)} = \frac{1}{2}$ that the parameter b becomes fixed, thus it turns out that there is not a free parameter. This "theoretical" b was found to be 7.6692494.

Case 2. The plot in the plane $\log_e B(\alpha)$, $\alpha^{1/4}$, of the empirical data by Baum, suggests that from certain point on the value of the parameter b may change; i. e. in the plane $\log_e B(\alpha)$, $\alpha^{1/4}$, the observations may be represented by two straight lines, one from $\alpha = 0.01$ to α_0 with slope $b = 7.6692494$ as in Case 1, and another from α_0 to infinity with slope $b_1 = 6.2255$. Again α_0 cannot be arbitrary for if we impose the condition that $B(\alpha)$ be continuous at α_0 , then $\alpha_0 = 1$.

Case 3. As before, we assume the log of surface brightness can be represented by 2 straight lines, one with slope $b = 7.6692494$ for $0.01 \leq \alpha \leq 1$ and the second with slope $b = 10.6291$ for $\alpha > 1$.

Most likely, cases 2 and 3 will bracket any future improvements in the photometry of E_0 galaxies.

The Computations.—To compute $M(s)$, Simpson's rule was used with 10 points. $q(s)$ was also computed with Simpson's rule but the interval was divided in subintervals with different spacing of the points, according to the behaviour of the integrand. The integrand in the potentials was computed also with Simpson's rule. All the computing was done with the IBM 650 of the National University of Mexico's, Computing Center. Some of the programming involved the use of the Livermore⁵ Routine, which we modified to include a pseudo instruction that integrates by Simpson's rule.

Description of the Tables.—Table 1 gives for the neighborhood of the origin a close net of densities with 7 significant figures. Table 2 gives for 160 points, conveniently distributed, the following:

$$s \quad \frac{q_R(s)}{K} \quad \frac{M_R(s)}{K} \quad f(\alpha) \quad \left|_{\alpha=s} \frac{M_R(s)}{K f_R(\alpha)} \right|_{\alpha=s} \quad \frac{M_R(s)}{M_R(\infty)} \quad \frac{F_R(s)}{GK} \quad \frac{V_R(s)}{GK} \quad \frac{v_R(s)}{\sqrt{GK}} \quad - \quad \frac{\Omega_R(s)}{GK^2}$$

All these functions computed with coefficient b as in case 1.

Tables 3 and 4 have the same entries as above, computed with the coefficients b as in cases 2 and 3 respectively.

The Velocity Dispersion.—The results of Tables 2, 3 and 4, column 5, allow a better determination of the relation between the velocity dispersion of space velocities and the observed dispersion. In fact, if the nuclear region contains the majority of the stars as seen in projection on the slit of the spectrograph, as Sinclair Smith⁶ claimed for the nucleus of M32, then $\sigma_{obs}^2 = \frac{1}{3} \sigma_{space}^2$; on the contrary, if the star orbits are highly eccentric and most of the light that goes through the slit corresponds to stars that are outside of the nucleus, then $\sigma_{obs}^2 = \sigma_{space}^2$. The latter was assumed by the author² in his determination of the mass of M32.

For an spectrographic analysis of M32, the minimum size of the central region investigated is given by the seeing disc. This very rarely will be less than 2" in diameter; for M32, this means $\alpha \approx 0.04$ and according to the results of Tables 2, 3 and 4, column 5, would imply that about half of the stars that appear in the seeing disc are outside of the nucleus. This fraction is quite insensitive to the assumed value for coefficient b . Then:

$$\sigma_{obs}^2 = \frac{1}{2} \left(\frac{1}{3} \sigma_{space}^2 \right) + \frac{1}{2} \sigma_{space}^2 \quad \sigma_{space}^2 = \frac{3}{2} \sigma_{obs}^2$$

If this interpretation is correct, the mass of M32 (and its M/L ratio computed before² should be increased by 50 per cent.

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TABLA 1

Densidades para s menor que 1

s	$Q(s)$	$Q(s + 0.002)$	$Q(s + 0.004)$	$Q(s + 0.006)$	$Q(s + 0.008)$	$Q(s + 0.010)$	$Q(s + 0.012)$	$Q(s + 0.014)$	$Q(s + 0.016)$	$Q(s + 0.018)$
0.011	3.888282×10^3	3.047976×10^3	2.464908×10^3	2.041149×10^3	1.722040×10^3	1.474887×10^3	1.279042×10^3	1.120887×10^3	9.911139×10^2	8.831657×10^2
0.031	7.923052×10^2	7.150341×10^2	6.487189×10^2	5.913441×10^2	5.413436×10^2	4.974849×10^2	4.587859×10^2	4.244560×10^2	3.938520	3.664468
0.051	3.418040	3.195603	2.994107	2.810979	2.644030	2.491395	2.351470	2.222872	2.104404	1.995023
0.071	1.893818	1.799992	1.712841	1.631745	1.561154	1.483581×10^2	1.419591×10^2	1.357796×10^2	1.299846×10^2	1.245431×10^2
0.091	1.194268×10^2	1.146104×10^2	1.100709×10^2	1.057877×10^2	1.017418×10^2	9.791611×10^1	9.429498×10^1	9.086416×10^1	8.761064×10^1	8.452246×10^1
0.111	8.158871×10^1	7.879929×10^1	7.611499×10^1	7.361727×10^1	7.120829×10^1	6.891079	6.671806	6.462388	6.262251	6.070857
0.131	5.887713	5.712355	5.544354	5.383309	5.228846	5.080614	4.938290	4.801566	4.670157	4.543795
0.151	4.422229	4.305223	4.192554	4.084014	3.979410	3.878552	3.781271	3.687400	3.596785	3.509279
0.171	3.424746	3.343053	3.264079	3.187705	3.113821	3.042321	2.973109	2.906086	2.841166	2.778262
0.191	2.717293	2.658183	2.600859	2.545251	2.491292	2.438920	2.388074	2.338697	2.290735	2.244135
0.211	2.198846	2.154823	2.112018	2.070390	2.029895	1.990495	1.952150	1.914826	1.878486	1.843098
0.231	1.808630	1.775050	1.742329	1.710439	1.679353	1.649045	1.619489	1.590663	1.562542	1.535104
0.251	1.508329	1.482195	1.456684	1.431775	1.407451	1.383694	1.360486	1.337813	1.315658	1.294005
0.271	1.272841	1.252150	1.231921	1.212138	1.192790	1.173865	1.155351×10^1	1.137235×10^1	1.119508×10^1	1.102158×10^1
0.291	1.085176×10^1	1.068551×10^1	1.052273×10^1	1.036334×10^1	1.020725×10^1	1.005436×10^1	9.904599×10^0	9.757874×10^0	9.614110×10^0	9.473233×10^0
0.311	9.35168×10^0	9.199843×10^0	9.067188×10^0	8.937139×10^0	8.809625×10^0	8.684586×10^0	8.561962	8.441691	8.323716	8.207982
0.331	8.094434	7.983019	7.873687	7.763388	7.661073	7.557695	7.456210	7.356573	7.258742	7.162672
0.351	7.068327	6.975665	6.884649	6.795240	6.707404	6.621104	6.536308	6.452979	6.371089	6.290603
0.371	6.211493	6.133727	6.057276	5.982112	5.908209	5.835538	5.764074	5.693792	5.624666	5.556673
0.391	5.489788	5.423989	5.359253	5.295560	5.232886	5.171211	5.110516	5.050780	4.991986	4.934111
0.411	4.877141	4.821055	4.765837	4.711470	4.657937	4.605223	4.553310	4.502183	4.451829	4.402231
0.431	4.353375	4.305247	4.257835	4.211122	4.165098	4.119749	4.075062	4.031026	3.987627	3.944856
0.451	3.902700	3.861147	3.820188	3.779809	3.740004	3.700761	3.662067	3.623916	3.586296	3.549201
0.471	3.512618	3.476539	3.440957	3.405862	3.371245	3.337098×10^0	3.303415×10^0	3.270185×10^0	3.237402×10^0	3.205039×10^0
0.491	3.173146×10^0	3.141659×10^0	3.110589×10^0	3.079928×10^0	3.049670×10^0					
s	$Q(s)$	$Q(s + 0.005)$	$Q(s + 0.010)$	$Q(s + 0.015)$	$Q(s + 0.020)$	$Q(s + 0.025)$	$Q(s + 0.030)$	$Q(s + 0.035)$	$Q(s + 0.040)$	$Q(s + 0.045)$
0.5025	2.997672×10^0	2.925427×10^0	2.855492×10^0	2.787777×10^0	2.722191×10^0	2.658651×10^0	2.597076×10^0	2.537391×10^0	2.479521×10^0	2.423400×10^0
0.5525	2.368960	2.316137	2.264873	2.215109	2.166789	2.119862	2.074277	2.029988	1.986946	1.945108
0.6025	1.904432	1.864878	1.826407	1.788982	1.752567	1.717128	1.682632	1.649049	1.616347	1.584498
0.6525	1.553475	1.523251	1.493799	1.465094	1.437115	1.409838	1.383239	1.357300	1.331998	1.307315
0.7025	1.283232	1.259730	1.236793	1.214402	1.192542×10^0	1.171198×10^0	1.150354×10^0	1.129995×10^0	1.110107×10^0	1.090678×10^0
0.7525	1.071693×10^0	1.053140×10^0	1.035007×10^0	1.017232×10^0	9.999534×10^{-1}	9.830097×10^{-1}	9.664417×10^{-1}	9.502374×10^{-1}	9.343883×10^{-1}	9.188838×10^{-1}
0.8025	9.037150×10^{-1}	8.888729×10^{-1}	8.743478×10^{-1}	8.601330×10^{-1}	8.462183	8.325975	8.192627	8.062050	7.934189	7.808965
0.8525	7.686315	7.566163	7.448463	7.333142	7.220142	7.109402	7.000870	6.894493	6.790212	6.687978
0.9025	6.587751	6.489466	6.393086	6.298558	6.205852	6.114910	6.025698	5.938174	5.852295	5.768025
0.9525	5.685330×10^{-1}	5.604170×10^{-1}	5.524508×10^{-1}	5.446311×10^{-1}	5.369553×10^{-1}	5.294190×10^{-1}	5.220196×10^{-1}	5.147539×10^{-1}	5.076194×10^{-1}	5.006126×10^{-1}

TABLA 2

s	$\frac{Q_R(s)}{K}$	$\frac{M_R(s)}{K}$	$f_R(\alpha) \Big _{a=s}$	$\frac{M_R(s)}{K f_R(\alpha)} \Big _{a=s}$	$\frac{M_R(s)}{M_R(\infty)}$	$\frac{F_R(s)}{KG}$	$\frac{V_R(s)}{GK}$	$\frac{\Omega_R(s)}{GK^2}$
1.00×10^{-2}	4.4586×10^3	1.8676×10^{-2}	8.0507×10^{-2}	2.3198×10^{-1}	8.2401×10^{-4}	1.8676×10^2	1.0346×10^1	1.3952×10^{-2}
2.00	1.5910×10^3	8.7812×10^{-2}	2.1732×10^{-1}	4.0406	3.8743×10^{-3}	2.1953	1.0134×10^1	2.0958×10^{-1}
3.00	8.584×10^2	1.7559×10^{-1}	3.7630	4.6661	7.7469×10^{-3}	1.9510	9.9272×10^0	6.2183×10^{-1}
4.00	5.1871	2.7530	5.4683	5.0344	1.2146×10^{-2}	1.7206	9.7408	1.2162×10^0
5.00	3.5380	3.8321	7.2375	5.2949	1.6908	1.5329	9.5727	1.9608
6.00	2.5660	4.9696	9.0409×10^{-1}	5.4968	2.1926	1.3805	9.4196	2.8290
7.00	1.9435	6.1494	1.0860×10^0	5.6622	2.7131	1.2590	9.2789	3.7989
8.00	1.5203	7.3597	1.2684	5.8026	3.2471	1.1500	9.1485	4.8525
9.00×10^{-2}	1.2195×10^2	8.5922	1.4503	5.9246	3.7909	1.0608 $\times 10^2$	9.0270	5.9749
1.00×10^{-1}	9.9802×10^1	9.8404×10^{-1}	1.6312	6.0327	4.3416	9.8404 $\times 10^1$	8.9131	7.1539
1.10	8.3037	1.1099×10^0	1.8107	6.1298	4.8970	9.1728	8.8059	8.3792
1.20	7.0046	1.2364	1.9885	6.2180	5.4552	8.5864	8.7046	9.6420×10^0
1.30	5.9783	1.3633	2.1644	6.2988	6.0150	8.0670	8.6084	1.0935×10^1
1.40	5.1540	1.4903	2.3383	6.3734	6.5752	7.6035	8.5170	1.2252
1.50	4.4824	1.6171	2.5101	6.4426	7.1349	7.1873	8.4298	1.3588
1.60	3.9285	1.7437	2.6796	6.5073	7.6933	6.8114	8.3464	1.4939
1.70	3.4667	1.8699	2.8470	6.5679	8.2499	6.4701	8.2664	1.6299
1.80	3.0778	1.9955	3.0120	6.6250	8.8041	6.1589	8.1897	1.7667
1.90	2.7475	2.1205	3.1748	6.6789	9.3555	5.8738	8.1160	1.9038
2.00	2.4649	2.2447	3.3354	6.7300	9.9038×10^{-2}	5.6118	8.0449	2.0412
2.10	2.2213	2.3682	3.4937	6.7786	1.0449×10^{-1}	5.3701	7.9764	2.1784
2.20	2.0101	2.4909	3.6498	6.8249	1.0990	5.1465	7.9102	2.3155
2.30	1.8258	2.6127	3.8036	6.8691	1.1527	4.9390	7.8462	2.4521
2.40	1.6641	2.7336	3.9553	6.9113	1.2061	4.7459	7.7843	2.5882
2.50	1.5216	2.8536	4.1049	6.9518	1.2590	4.5658	7.7242	2.7236
2.60	1.3955	2.9727	4.2523	6.9907	1.3115	4.3974	7.6660	2.8583
2.70	1.2834	3.0907	4.3976	7.0281	1.3636	4.2397	7.6095	2.9921
2.80	1.1833	3.2078	4.5409	7.0642	1.4153	4.0916	7.5545	3.1249
2.90	1.0936	3.3239	4.6822	7.0989	1.4665	3.9523	7.5011	3.2568
3.00	1.0130×10^1	3.4389	4.8216	7.1324	1.5173	3.8210	7.4491	3.3876
3.10	9.4039×10^0	3.5530	4.9590	7.1648	1.5676	3.6972	7.3985	3.5172
3.20	8.7468	3.6661	5.0945	7.1962	1.6175	3.5801	7.3492	3.6458
3.30	8.1509	3.7781	5.2281	7.2265	1.6669	3.4693	7.3011	3.7731
3.40	7.6091	3.8892	5.3599	7.2560	1.7159	3.3643	7.2541	3.8993
3.50	7.1153	3.9992	5.4900	7.2845	1.7645	3.2646	7.2083	4.0241
3.60	6.6641	4.1082	5.6183	7.3123	1.8126	3.1699	7.1635	4.1478
3.70	6.2509	4.2163	5.7448	7.3392	1.8602	3.0798	7.1198	4.2701
3.80	5.8717	4.3233	5.8697	7.3654	1.9075	2.9940	7.0770	4.3912
3.90	5.5231	4.4293	5.9929	7.3909	1.9542	2.9121	7.0351	4.5110
4.00	5.2019	4.5344	6.1146	7.4158	2.0006	2.8340	6.9942	4.6294
4.10	4.9055	4.6385	6.2346	7.4400	2.0465	2.7594	6.9541	4.7466
4.20	4.6315	4.7417	6.3531	7.4636	2.0920	2.6880	6.9148	4.8625
4.30	4.3777	4.8439	6.4700	7.4866	2.1371	2.6197	6.8763	4.9771
4.40	4.1423	4.9451	6.5855	7.5091	2.1818	2.5543	6.8386	5.0903
4.50	3.9237	5.0454	6.6995	7.5311	2.2261	2.4916	6.8016	5.2023
4.60	3.7203	5.1448	6.8120	7.5525	2.2699	2.4314	6.7653	5.3130
4.70	3.5308	5.2433	6.9232	7.5735	2.3134	2.3736	6.7297	5.4224
4.80	3.3541	5.3408	7.0330	7.5940	2.3564	2.3181	6.6948	5.5305
4.90	3.1890	5.4375	7.1414	7.6141	2.3990	2.2647	6.6605	5.6374
5.00	3.0347	5.5333	7.2485	7.6337	2.4413	2.2133	6.6268	5.7430
5.25	2.6902	5.7689	7.5106	7.6811	2.5453	2.0930	6.5451	5.9997
5.50	2.3960	5.9992	7.7649	7.7261	2.6469	1.9832	6.4668	6.2489
5.75	2.1432	6.2244	8.0117	7.7690	2.7462	1.8826	6.3917	6.4909
6.00×10^{-1}	1.9246×10^0	6.4445×10^0	8.2515×10^0	7.8100×10^{-1}	2.8433×10^{-1}	1.7901×10^1	6.3195×10^0	6.7257×10^1

TABLA 2 (Continúa)

s	$\frac{Q_R(s)}{K}$	$\frac{M_R(s)}{K}$	$f_R(\alpha) \Big _{\alpha=8}$	$\frac{M_R(s)}{K f_R(\alpha)} \Big _{\alpha=8}$	$\frac{M_R(s)}{M_R(\infty)}$	$\frac{F_R(s)}{KG}$	$\frac{V_R(s)}{GK}$	$\frac{\Omega_R(s)}{GK^2}$
6.25×10^{-1}	1.7347×10^0	6.6597×10^0	8.4846×10^0	7.8492×10^{-1}	2.9383×10^{-1}	1.7049×10^1	6.2500×10^0	6.9537×10^1
6.50	1.5689	6.8703	8.7111	7.8868	3.0312	1.6261	6.1830	7.1749
6.75	1.4234	7.0763	8.9315	7.9228	3.1221	1.5531	6.1184	7.3897
7.00	1.2952	7.2778	9.1459	7.9575	3.2110	1.4853	6.0560	7.5982
7.25	1.1818	7.4751	9.3546	7.9908	3.2980	1.4221	5.9957	7.8007
7.50	1.0811×10^0	7.6682	9.5579	8.0229	3.3832	1.3632	5.9374	7.9973
7.75	9.9143×10^{-1}	7.8572	9.7559	8.0539	3.4666	1.3082	5.8809	8.1882
8.00	9.1126	8.0424	9.9488×10^0	8.0837	3.5483	1.2566	5.8261	8.3736
8.25	8.3937	8.2237	1.0137×10^1	8.1126	3.6283	1.2083	5.7730	8.5538
8.50	7.7473	8.4014	1.0320	8.1405	3.7067	1.1628	5.7215	8.7288
8.75	7.1645	8.5754	1.0499	8.1675	3.7835	1.1200	5.6714	8.8988
9.00	6.6376	8.7460	1.0674	8.1937	3.8588	1.0797	5.6227	9.0641
9.25	6.1602	8.9132	1.0845	8.2190	3.9325	1.0417	5.5753	9.2249
9.50	5.7265	9.0772	1.1011	8.2437	4.0049	1.0058 $\times 10^1$	5.5293	9.3812
9.75×10^{-1}	5.3317	9.2380	1.1174	8.2676	4.0758	9.7178×10^0	5.4844	9.5332
1.00×10^0	4.9730	9.3957	1.1333	8.2908	4.1454	9.3957	5.4406	9.6810
1.05	4.6422	9.7022	1.1640	8.3354	4.2807	8.8002	5.3564	9.9632×10^1
1.10	3.8112	9.9975×10^0	1.1933	8.3777	4.4109	8.2624	5.2762	1.0231×10^2
1.15	3.3612	1.0282×10^1	1.2214	8.4178	4.5364	7.7746	5.1997	1.0484
1.20	2.9775	1.0556	1.2484	8.4560	4.6575	7.3307	5.1266	1.0725
1.25	2.6484	1.0821	1.2742	8.4923	4.7742	6.9254	5.0567	1.0954
1.30	2.3646	1.1076	1.2990	8.5270	4.8870	6.5541	4.9896	1.1171
1.35	2.1187	1.1323	1.3228	8.5602	4.9959	6.2130	4.9252	1.1378
1.40	1.9047	1.1562	1.3457	8.5919	5.1011	5.8989	4.8634	1.1574
1.45	1.7175	1.1792	1.3677	8.6223	5.2029	5.6088	4.8039	1.1762
1.50	1.5532	1.2016	1.3889	8.6514	5.3014	5.3403	4.7466	1.1940
1.55	1.4084	1.2232	1.4093	8.6795	5.3967	5.0913	4.6913	1.2111
1.60	1.2804	1.2441	1.4290	8.7064	5.4890	4.8598	4.6380	1.2273
1.65	1.1669	1.2644	1.4479	8.7323	5.5785	4.6442	4.5865	1.2429
1.70	1.0658×10^{-1}	1.2840	1.4662	8.7573	5.6652	4.4430	4.5367	1.2577
1.75	9.7561×10^{-2}	1.3031	1.4839	8.7814	5.7493	4.2550	4.4886	1.2719
1.80	8.9486	1.3216	1.5010	8.8046	5.8309	4.0790	4.4419	1.2855
1.85	8.2239	1.3395	1.5175	8.8271	5.9101	3.9139	4.3967	1.2985
1.90	7.5719	1.3570	1.5335	8.8488	5.9870	3.7589	4.3529	1.3109
1.95	6.9838	1.3739	1.5490	8.8698	6.0617	3.6131	4.3103	1.3229
2.00	6.4522	1.3903	1.5639	8.8901	6.1342	3.4759	4.2690	1.3343
2.05	5.9706	1.4063	1.5784	8.9098	6.2048	3.3464	4.2289	1.3453
2.10	5.5335	1.4219	1.5924	8.9289	6.2734	3.2242	4.1899	1.3558
2.15	5.1358	1.4370	1.6061	8.9474	6.3401	3.1087	4.1519	1.3659
2.20	4.7734	1.4517	1.6193	8.9653	6.4050	2.9994	4.1150	1.3756
2.25	4.4426	1.4660	1.6321	8.9828	6.4682	2.8959	4.0790	1.3850
2.30	4.1400	1.4800	1.6445	8.9997	6.5297	2.7977	4.0440	1.3940
2.35	3.8628	1.4936	1.6565	9.0161	6.5897	2.7045	4.0098	1.4026
2.40	3.6085	1.5068	1.6683	9.0321	6.6480	2.6180	3.9765	1.4109
2.45	3.3747	1.5197	1.6796	9.0477	6.7049	2.5318	3.9440	1.4189
2.50	3.1596	1.5323	1.6907	9.0628	6.7604	2.4516	3.9123	1.4266
2.55	2.9614	1.5445	1.7015	9.0775	6.8144	2.3753	3.8813	1.4341
2.60	2.7784	1.5565	1.7119	9.0919	6.8671	2.3024	3.8511	1.4412
2.65	2.6094	1.5681	1.7221	9.1059	6.9186	2.2330	3.8215	1.4481
2.70	2.4529	1.5795	1.7320	9.1195	6.9688	2.1666	3.7927	1.4548
2.75	2.3080	1.5906	1.7416	9.1328	7.0177	2.1033	3.7644	1.4612
2.80	2.1736	1.6014	1.7510	9.1457	7.0655	2.0426	3.7368	1.4674
2.85	2.0487	1.6120	1.7601	9.1584	7.1122	1.9846	3.7097	1.4734
2.90×10^0	1.9327×10^{-2}	1.6223×10^1	1.7690×10^1	9.1707×10^{-1}	7.1578×10^{-1}	1.9291×10^0	3.6833×10^0	1.4792×10^2

TABLA 2 (Continúa)

s	$\frac{Q_R(s)}{K}$	$\frac{M_R(s)}{K}$	$f_R(\alpha) _{\alpha=s}$	$\frac{M_R(s)}{K f_R(\alpha)} _{\alpha=s}$	$\frac{M_R(s)}{M_R(\infty)}$	$\frac{F_R(s)}{KG}$	$\frac{V_R(s)}{GK}$	$\frac{\Omega_R(s)}{GK^2}$
2.95×10^0	1.8247×10^{-2}	1.6324×10^1	1.7777×10^1	9.1827×10^{-1}	7.2024×10^{-1}	1.8758×10^0	3.6574×10^0	1.4848×10^2
3.00	1.7241	1.6423	1.7862	9.1945	7.2459	1.8248	3.6320	1.4902
3.05	1.6303	1.6519	1.7944	9.2060	7.2884	1.7758	3.6071	1.4954
3.10	1.5427	1.6614	1.8025	9.2172	7.3300	1.7288	3.5827	1.5005
3.15	1.4609	1.6706	1.8103	9.2282	7.3706	1.6836	3.5589	1.5054
3.20	1.3844	1.6796	1.8179	9.2389	7.4103	1.6402	3.5354	1.5101
3.25	1.3128	1.6884	1.8254	9.2494	7.4492	1.5985	3.5125	1.5147
3.30	1.2457	1.6970	1.8327	9.2596	7.4872	1.5583	3.4899	1.5191
3.35	1.1828	1.7054	1.8398	9.2696	7.5244	1.5197	3.4678	1.5234
3.40	1.1238	1.7137	1.8467	9.2795	7.5608	1.4824	3.4461	1.5276
3.45	1.0684	1.7218	1.8535	9.2891	7.5965	1.4466	3.4248	1.5316
3.50	1.0163×10^{-2}	1.7297	1.8602	9.2985	7.6313	1.4120	3.4039	1.5355
3.55	9.6722×10^{-3}	1.7374	1.8666	9.3077	7.6655	1.3786	3.3833	1.5393
3.60	9.2110	1.7450	1.8730	9.3167	7.6989	1.3464	3.3631	1.5430
3.65	8.7766	1.7524	1.8791	9.3256	7.7317	1.3154	3.3433	1.5466
3.70	8.3672	1.7597	1.8852	9.3342	7.7638	1.2854	3.3238	1.5500
3.75	7.9810	1.7668	1.8911	9.3427	7.7952	1.2564	3.3046	1.5534
3.80	7.6165	1.7738	1.8969	9.3510	7.8260	1.2284	3.2857	1.5567
3.85	7.2722	1.7806	1.9025	9.3592	7.8562	1.2013	3.2672	1.5598
3.90	6.9458	1.7873	1.9081	9.3672	7.8858	1.1751	3.2490	1.5629
3.95	6.6392	1.7939	1.9135	9.3750	7.9148	1.1498	3.2310	1.5659
4.00	6.3481	1.8003	1.9188	9.3827	7.9432	1.1252	3.2134	1.5688
4.10	5.8114	1.8129	1.9291	9.3977	7.9984	1.0784	3.1789	1.5743
4.20	5.3293	1.8249	1.9389	9.4121	8.0516	1.0345×10^0	3.1455	1.5796
4.30	4.8952	1.8365	1.9483	9.4260	8.1027	9.9324×10^{-1}	3.1131	1.5846
4.40	4.5036	1.8477	1.9574	9.4393	8.1520	9.5437	3.0817	1.5893
4.50	4.1497	1.8584	1.9661	9.4523	8.1994	9.1774	3.0512	1.5937
4.60	3.8291	1.8688	1.9745	9.4647	8.2452	8.8317	3.0215	1.5979
4.70	3.5382	1.8788	1.9825	9.4768	8.2893	8.5052	2.9927	1.6019
4.80	3.2739	1.8884	1.9903	9.4884	8.3319	8.1963	2.9647	1.6057
4.90	3.0332	1.8978	1.9977	9.4997	8.3730	7.9040	2.9374	1.6094
5.00	2.8137	1.9067	2.0049	9.5106	8.4126	7.6270	2.9108	1.6128
5.10	2.6132	1.9154	2.0118	9.5211	8.4510	7.3642	2.8850	1.6161
5.20	2.4298	1.9238	2.0184	9.5313	8.4880	7.1148	2.8598	1.6192
5.30	2.2618	1.9320	2.0248	9.5412	8.5239	6.8777	2.8352	1.6222
5.40	2.1077	1.9398	2.0310	9.5508	8.5585	6.6523	2.8112	1.6250
5.50	1.9662	1.9474	2.0370	9.5601	8.5920	6.4377	2.7879	1.6277
5.60	1.8360	1.9548	2.0428	9.5691	8.6245	6.2333	2.7650	1.6303
5.70	1.7161	1.9619	2.0483	9.5779	8.6559	6.0384	2.7428	1.6327
5.80	1.6056	1.9688	2.0537	9.5863	8.6863	5.8525	2.7210	1.6351
5.90	1.5035	1.9755	2.0589	9.5946	8.7158	5.6750	2.6997	1.6373
6.00	1.4092×10^{-3}	1.9819	2.0640	9.6026	8.7444	5.5054	2.6790	1.6394
7.00	7.7016×10^{-4}	2.0370	2.1062	9.6713	8.9873	4.1571	2.4940	1.6559
8.00	4.4948	2.0784	2.1374	9.7241	9.1702	3.2476	2.3420	1.6669
9.00×10^0	2.7615	2.1104	2.1610	9.7656	9.3110	2.6054	2.2143	1.6746
1.00×10^1	1.7682×10^{-4}	2.1354	2.1792	9.7988	9.4214	2.1354	2.1051	1.6801
1.20	3.9722	2.1952	2.2217	9.8809	9.6853	1.5079	1.9271	1.6868
1.40	2.1234	2.2116	2.2330	9.9042	9.7577	1.1200×10^{-1}	1.7871	1.6907
1.60	1.1973×10^{-5}	2.2232	2.2409	9.9208	9.8088	8.6391×10^{-2}	1.6734	1.6931
1.80	7.0122×10^{-6}	2.2315	2.2467	9.9326	9.8455	5.8788	1.5787	1.6945
2.00	4.2094	2.2375	2.2509	9.9408	9.8721	4.6230	1.4289	1.6955
2.20	2.5391	2.2419	2.2540	9.9462	9.8914	3.8922	1.3682	1.6961
2.40	1.4939×10^{-6}	2.2450	2.2564	9.9493	9.9050	3.3210	1.3146	1.6965
2.60	7.8375×10^{-7}	2.2470×10^1	2.2583×10^1	9.9501×10^{-1}	9.9140×10^{-1}	2.8661×10^{-2}	1.2668×10^0	1.6970×10^2

TABLA 3

s	$\frac{Q_R(s)}{K}$	$\frac{M_R(s)}{K}$	$f_R(\alpha) \Big _{\alpha=s}$	$\frac{M_R(s)}{K f_R(\alpha)} \Big _{\alpha=s}$	$\frac{M_R(s)}{M_R(\infty)}$	$\frac{F_R(s)}{GK}$	$\frac{V_R(s)}{GK}$	$\frac{v_R(s)}{\sqrt{GK}}$	$-\frac{\Omega_R(s)}{GK^2}$
1.00×10^{-2}	4.4580×10^4	1.8674×10^{-2}	8.0507×10^{-2}	2.3195×10^{-1}	5.9190×10^{-4}	1.8674×10^{-2}	1.8674×10^0	1.0381×10^1	1.3948×10^{-2}
2.00	1.5904×10^3	8.7791×10^{-2}	2.1732×10^{-1}	4.0397	2.7827×10^{-3}	2.1948	4.0406	1.0170×10^1	2.0949×10^{-1}
3.00	8.3529×10^2	1.7552×10^{-1}	3.7630	4.6644	5.5635	1.9502	6.1168	9.9637×10^0	6.2140×10^{-1}
4.00	5.1819	2.7515	5.4683	5.0317	8.7215×10^{-3}	1.7197	7.9482	9.7782	1.2150×10^0
5.00	3.5327	3.8293	7.2375	5.2910	1.2138×10^{-2}	1.5317	9.5705 $\times 10^0$	9.6109	1.9582
6.00	2.5609	4.9649	9.0409×10^{-1}	5.4915	1.5737	1.3791	1.1023×10^1	9.4585	2.8242
7.00	1.9385	6.1419	1.0860×10^0	5.6554	1.9468	1.2535	1.2338	9.3185	3.7908
8.00	1.5155	7.3489	1.2684	5.7940	2.3294	1.1483	1.3537	9.1889	4.8400
9.00×10^{-2}	1.2149×10^2	8.5772	1.4503	5.9142	2.7187	1.0589×10^{-2}	1.4640	9.0681	5.9568
1.00×10^{-1}	9.9378×10^1	9.8202×10^{-1}	1.6312	6.0204	3.1127	9.8202×10^{-1}	1.5659	8.9550	7.1288
1.10	8.2625	1.1073×10^0	1.8107	6.1155	3.5099	9.1513	1.6607	8.8485	8.3457
1.20	6.9646	1.2332	1.9885	6.2016	3.9088	8.5637	1.7492	8.7479	9.5987×10^0
1.30	5.9393	1.3593	2.1644	6.2801	4.3085	8.0430	1.8322	8.6525	1.0880×10^1
1.40	5.1160	1.4854	2.3383	6.3523	4.7082	7.5784	1.9103	8.5618	1.2184
1.50	4.4455	1.6112	2.5101	6.4191	5.1071	7.1610	1.9839	8.4753	1.3505
1.60	3.8925	1.7367	2.6796	6.4811	5.5048	6.7840	2.0536	8.3927	1.4838
1.70	3.4314	1.8616	2.8470	6.5390	5.9008	6.4416	2.1197	8.3136	1.6180
1.80	3.0434	1.9859	3.0120	6.5932	6.2947	6.1293	2.1825	8.2377	1.7527
1.90	2.7139	2.1094	3.1748	6.6441	6.6863	5.8433	2.2424	8.1647	1.8876
2.00	2.4321	2.2321	3.3354	6.6922	7.0751	5.5803	2.2995	8.0945	2.0224
2.10	2.1892	2.3539	3.4937	6.7376	7.4612	5.3376	2.3541	8.0267	2.1570
2.20	1.9786	2.4747	3.6498	6.7805	7.8442	5.1131	2.4063	7.9614	2.2911
2.30	1.7950	2.5946	3.8036	6.8213	8.2240	4.9047	2.4564	7.8983	2.4246
2.40	1.6340	2.7134	3.9553	6.8600	8.6006	4.7107	2.5044	7.8372	2.5574
2.50	1.4921	2.8311	4.1049	6.8969	8.9738	4.5297	2.5506	7.7780	2.6892
2.60	1.3666	2.9477	4.2523	6.9321	9.3435	4.3606	2.5951	7.7206	2.8201
2.70	1.2550	3.0633	4.3976	6.9657	9.7097×10^{-2}	4.2020	2.6379	7.6650	2.9499
2.80	1.1555	3.1777	4.5409	6.9978	1.0072×10^{-1}	4.0531	2.6791	7.6110	3.0786
2.90	1.0664×10^1	3.2909	4.6822	7.0285	1.0431	3.9131	2.7190	7.5585	3.2060
3.00	9.8630×10^0	3.4031	4.8216	7.0580	1.0787	3.7812	2.7574	7.5074	3.3321
3.10	9.1427	3.5140	4.9590	7.0862	1.1139	3.6567	2.7946	7.4577	3.4570
3.20	8.4906	3.6239	5.0945	7.1133	1.1487	3.5389	2.8306	7.4093	3.5804
3.30	7.8995	3.7326	5.2281	7.1394	1.1831	3.4275	2.8654	7.3622	3.7025
3.40	7.3623	3.8401	5.3599	7.1644	1.2172	3.3219	2.8992	7.3162	3.8231
3.50	6.8730	3.9465	5.4900	7.1885	1.2509	3.2216	2.9319	7.2713	3.9423
3.60	6.4262	4.0517	5.6183	7.2116	1.2843	3.1263	2.9636	7.2276	4.0600
3.70	6.0173	4.1558	5.7448	7.2339	1.3173	3.0356	2.9944	7.1848	4.1762
3.80	5.6423	4.2587	5.8697	7.2554	1.3499	2.9493	3.0243	7.1430	4.2910
3.90	5.2978	4.3605	5.9929	7.2761	1.3822	2.8669	3.0534	7.1022	4.4042
4.00	4.9807	4.4612	6.1146	7.2961	1.4141	2.7883	3.0817	7.0623	4.5160
4.10	4.6882	4.5608	6.2346	7.3154	1.4457	2.7132	3.1092	7.0232	4.6262
4.20	4.4180	4.6593	6.3531	7.3340	1.4769	2.6413	3.1360	6.9850	4.7349
4.30	4.1680	4.7567	6.4700	7.3519	1.5077	2.5726	3.1620	6.9476	4.8422
4.40	3.9363	4.8530	6.5855	7.3692	1.5383	2.5067	3.1874	6.9110	4.9480
4.50	3.7212	4.9482	6.6995	7.3860	1.5684	2.4436	3.2122	6.8751	5.0522
4.60	3.5214	5.0424	6.8120	7.4022	1.5983	2.3830	3.2363	6.8399	5.1551
4.70	3.3354	5.1355	6.9232	7.4178	1.6278	2.3248	3.2598	6.8054	5.2564
4.80	3.1621	5.2276	7.0330	7.4330	1.6570	2.2689	3.2828	6.7715	5.3563
4.90	3.0004	5.3186	7.1414	7.4476	1.6858	2.2152	3.3052	6.7384	5.4548
5.00	2.8493	5.4086	7.2485	7.4618	1.7144	2.1635	3.3271	6.7058	5.5519
5.25	2.5127	5.6293	7.5106	7.4952	1.7843	2.0424	3.3797	6.6270	5.7866
5.50	2.2262	5.8439	7.7649	7.5260	1.8523	1.9319	3.4293	6.5516	6.0130
5.75	1.9807	6.0525	8.0117	7.5545	1.9185	1.8306	3.4763	6.4794	6.2312
6.00×10^{-1}	1.7692×10^0	6.2554×10^0	8.2515×10^0	7.5809×10^{-1}	1.9828×10^{-1}	1.7376×10^1	3.5209×10^1	6.4103×10^0	6.4415×10^1

TABLA 3 (Continúa)

s	$\frac{\Omega_R(s)}{K}$	$\frac{M_R(s)}{K}$	$f_R(u) ^{a-s}$	$\frac{M_R(s)}{K f_R(u) ^{a-s}}$	$\frac{M_R(s)}{M_R(\infty)}$	$\frac{F_R(s)}{GK}$	$\frac{F_R(s)}{GK}$	$\frac{\tau_R(s)}{GK}$	$\frac{\Omega_R(s)}{GK^2}$
6.25×10^{-1}	1.5861×10^0	6.4527×10^0	8.4846×10^0	7.6053×10^{-1}	2.0453×10^{-1}	1.6519×10^1	3.5633×10^1	6.3438×10^0	6.6442×10^1
6.50	1.4269	6.6447	8.7111	7.6279	2.1062	1.5727	3.6036	6.2800	6.8395
6.75	1.2879	6.8316	8.9315	7.6489	2.1654	1.4994	3.6420	6.2186	7.0278
7.00	1.1659	7.0135	9.1459	7.6685	2.2231	1.4313	3.6786	6.1594	7.2093
7.25	1.0587×10^0	7.1906	9.3546	7.6867	2.2792	1.3680	3.7136	6.1023	7.3844
7.50	9.6401×10^{-1}	7.3632	9.5579	7.7038	2.3339	1.3090	3.7470	6.0473	7.5532
7.75	8.8021	7.5314	9.7559	7.7199	2.3872	1.2539	3.7790	5.9941	7.7162
8.00	8.0585	7.6954	9.9488×10^0	7.7350	2.4392	1.2024	3.8097	5.9426	7.8735
8.25	7.3974	7.8555	1.0137×10^1	7.7494	2.4900	1.1542	3.8392	5.8929	8.0256
8.50	6.8084	8.0119	1.0320	7.7631	2.5395	1.1089	3.8675	5.8447	8.1726
8.75	6.2832	8.1646	1.0499	7.7762	2.5879	1.0664	3.8947	5.7980	8.3148
9.00	5.8147	8.3141	1.0674	7.7891	2.6353	1.0264 $\times 10^1$	3.9208	5.7527	8.4526
9.25	5.3971	8.4606	1.0845	7.8017	2.6818	9.8882 $\times 10^0$	3.9460	5.7087	8.5864
9.50	5.0264	8.6044	1.1011	7.8143	2.7273	9.5339	3.9703	5.6661	8.7164
9.75×10^{-1}	4.7016	8.7458	1.1174	7.8271	2.7722	9.2001	3.9937	5.6246	8.8430
1.00×10^0	4.3813	8.8855	1.1333	7.8406	2.8165	8.8855	4.0163	5.5843	8.9669
1.05	3.8977	9.1582	1.1642	7.8662	2.9029	8.3068	4.0592	5.5068	9.2041
1.10	3.4808	9.4256	1.1944	7.8915	2.9876	7.7897	4.0995	5.4333	9.4325
1.15	3.1216	9.6876	1.2237	7.9164	3.0707	7.3252	4.1372	5.3633	9.6527
1.20	2.8104	9.9444×10^0	1.2523	7.9408	3.1521	6.9058	4.1728	5.2966	9.8650×10^1
1.25	2.5393	1.0196×10^1	1.2801	7.9648	3.2319	6.5255	4.2064	5.2328	1.0070×10^2
1.30	2.3020	1.0443	1.3073	7.9884	3.3101	6.1793	4.2381	5.1718	1.0268
1.35	2.0933	1.0685	1.3337	8.0115	3.3869	5.8629	4.2682	5.1133	1.0459
1.40	1.9091	1.0922	1.3595	8.0341	3.4621	5.5727	4.2968	5.0571	1.0644
1.45	1.7457	1.1155	1.3847	8.0563	3.5359	5.3057	4.3240	5.0030	1.0823
1.50	1.6004	1.1384	1.4092	8.0781	3.6083	5.0594	4.3499	4.9510	1.0996
1.55	1.4707	1.1608	1.4332	8.0994	3.6794	4.8316	4.3746	4.9008	1.1163
1.60	1.3545	1.1828	1.4566	8.1202	3.7491	4.6202	4.3982	4.8524	1.1326
1.65	1.2502	1.2044	1.4794	8.1407	3.8175	4.4238	4.4208	4.8056	1.1483
1.70	1.1561	1.2256	1.5018	8.1606	3.8847	4.2407	4.4425	4.7603	1.1636
1.75	1.0712×10^{-1}	1.2464	1.5236	8.1802	3.9506	4.0697	4.4632	4.7164	1.1784
1.80	9.9427×10^{-2}	1.2668	1.5450	8.1994	4.0153	3.9098	4.4832	4.6740	1.1927
1.85	9.2444	1.2868	1.5658	8.2182	4.0789	3.7599	4.5024	4.6328	1.2067
1.90	8.6090	1.3065	1.5863	8.2366	4.1414	3.6192	4.5208	4.5928	1.2202
1.95	8.0296	1.3259	1.6062	8.2547	4.2027	3.4869	4.5386	4.5540	1.2333
2.00	7.4963	1.3449	1.6258	8.2723	4.2630	3.3623	4.5557	4.5162	1.2461
2.05	7.0116	1.3636	1.6449	8.2896	4.3222	3.2447	4.5722	4.4795	1.2585
2.10	6.5670	1.3819	1.6637	8.3065	4.3804	3.1336	4.5881	4.4438	1.2706
2.15	6.1585	1.4000	1.6820	8.3231	4.4375	3.0286	4.6035	4.4090	1.2823
2.20	5.7861	1.4177	1.7000	8.3394	4.4938	2.9292	4.6184	4.3750	1.2938
2.25	5.4394	1.4352	1.7177	8.3554	4.5491	2.8349	4.6328	4.3420	1.3049
2.30	5.1192	1.4523	1.7349	8.3711	4.6034	2.7454	4.6468	4.3097	1.3157
2.35	4.8230	1.4692	1.7519	8.3865	4.6569	2.6604	4.6603	4.2783	1.3263
2.40	4.5487	1.4858	1.7685	8.4016	4.7095	2.5795	4.6734	4.2475	1.3365
2.45	4.2943	1.5021	1.7847	8.4165	4.7613	2.5025	4.6861	4.2175	1.3465
2.50	4.0580	1.5182	1.8007	8.4311	4.8122	2.4291	4.6984	4.1882	1.3563
2.55	3.8383	1.5340	1.8164	8.4454	4.8623	2.3591	4.7104	4.1595	1.3658
2.60	3.6337	1.5496	1.8317	8.4595	4.9116	2.2922	4.7220	4.1315	1.3751
2.65	3.4430	1.5649	1.8468	8.4733	4.9602	2.2284	4.7333	4.1040	1.3841
2.70	3.2650	1.5799	1.8616	8.4869	5.0080	2.1673	4.7443	4.0772	1.3929
2.75	3.0951	1.5948	1.8762	8.5001	5.0549	2.1088	4.7550	4.0509	1.4015
2.80	2.9432	1.6094	1.8904	8.5132	5.1012	2.0528	4.7654	4.0251	1.4099
2.85	2.7976	1.6238	1.9045	8.5261	5.1468	1.9991	4.7755	3.9999	1.4181
2.90×10^0	2.6611×10^{-2}	1.6379×10^1	1.9182×10^1	8.5387×10^{-1}	5.1917×10^{-1}	1.9476×10^0	4.7854×10^1	3.9751×10^0	1.4261×10^2

s	$\frac{Q_R(s)}{K}$	$\frac{M_R(s)}{K}$	$f_R(\alpha) _{\alpha=s}$	$\frac{M_R(s)}{K f_R(\alpha) _{\alpha=s}}$	$\frac{M_R(s)}{M_R(\infty)}$	$\frac{F_R(s)}{GK}$	$\frac{V_R(s)}{GK}$	$\frac{\Omega_R(s)}{GK^2}$
2.95×10^0	2.5331×10^{-2}	1.6519×10^1	1.9318×10^1	8.5512×10^{-1}	5.2360×10^{-1}	1.8982×10^0	3.9509×10^0	1.4339×10^2
3.00	2.4128	1.6656	1.9450	8.5634	5.2796	1.8507	3.9271	1.4416
3.05	2.2999	1.6792	1.9581	8.5755	5.3225	1.8051	3.9037	1.4490
3.10	2.1936	1.6925	1.9709	8.5874	5.3648	1.7612	3.8808	1.4563
3.15	2.0935	1.7057	1.9836	8.5990	5.4065	1.7190	3.8583	1.4634
3.20	1.9992	1.7186	1.9960	8.6105	5.4475	1.6783	3.8363	1.4704
3.25	1.9103	1.7314	2.0082	8.6218	5.4880	1.6392	3.8146	1.4772
3.30	1.8264	1.7440	2.0201	8.6329	5.5279	1.6014	3.7933	1.4838
3.35	1.7472	1.7564	2.0319	8.6439	5.5672	1.5651	3.7724	1.4904
3.40	1.6723	1.7686	2.0435	8.6547	5.6060	1.5299	3.7518	1.4967
3.45	1.6014	1.7807	2.0550	8.6653	5.6442	1.4961	3.7316	1.5029
3.50	1.5344	1.7926	2.0662	8.6758	5.6819	1.4633	3.7117	1.5090
3.55	1.4708	1.8043	2.0772	8.6861	5.7191	1.4317	3.6922	1.5150
3.60	1.4106	1.8159	2.0881	8.6962	5.7558	1.4011	3.6729	1.5208
3.65	1.3535	1.8273	2.0988	8.7062	5.7919	1.3716	3.6540	1.5265
3.70	1.2994	1.8385	2.1093	8.7161	5.8276	1.3430	3.6354	1.5321
3.75	1.2479	1.8496	2.1197	8.7258	5.8628	1.3153	3.6171	1.5376
3.80	1.1991	1.8606	2.1299	8.7354	5.8975	1.2885	3.5990	1.5430
3.85	1.1526	1.8714	2.1400	8.7449	5.9318	1.2625	3.5813	1.5482
3.90	1.1084	1.8820	2.1499	8.7542	5.9656	1.2374	3.5638	1.5534
3.95	1.0663 $\times 10^{-2}$	1.8926	2.1596	8.7634	5.9989	1.2130	3.5465	1.5584
4.00	9.5176×10^{-3}	1.9030	2.1692	8.7724	6.0318	1.1893	3.5296	1.5634
4.10	8.8396×10^{-3}	1.9233	2.1880	8.7902	6.0664	1.1442	3.4964	1.5729
4.20	8.2217	1.9432	2.2063	8.8075	6.1093	1.1016	3.4641	1.5821
4.30	7.6576	1.9625	2.2240	8.8243	6.2206	1.0614	3.4327	1.5910
4.40	7.1416	1.9814	2.2412	8.8408	6.2804	1.0234×10^0	3.4023	1.5995
4.50	6.6889	2.0177	2.2579	8.8568	6.3388	9.8755×10^{-1}	3.3726	1.6077
4.60	6.2350	2.0353	2.2742	8.8724	6.3957	9.5356	3.3437	1.6155
4.70	5.8363	2.0524	2.3054	8.8876	6.4512	9.2135	3.3155	1.6231
4.80	5.4691	2.0691	2.3204	8.9025	6.5054	8.9078	3.2881	1.6305
4.90	5.1307	2.0854	2.3349	8.9170	6.5583	8.6175	3.2613	1.6375
5.00	4.8183	2.1013	2.3491	8.9312	6.6100	8.3415	3.2352	1.6443
5.10	4.5294	2.1169	2.3630	8.9450	6.6605	8.0788	3.2098	1.6509
5.20	4.2621	2.1321	2.3764	8.9586	6.7099	7.8287	3.1849	1.6572
5.30	4.0143	2.1470	2.3896	8.9718	6.7581	7.5902	3.1606	1.6634
5.40	3.7844	2.1615	2.4024	8.9848	6.8053	7.3627	3.1368	1.6693
5.50	3.5708	2.1757	2.4148	8.9974	6.8514	7.1455	3.1136	1.6750
5.60	3.3722	2.1897	2.4270	9.0098	6.8965	6.9379	3.0909	1.6805
5.70	3.1872	2.2033	2.4389	9.0220	6.9406	6.7395	3.0687	1.6859
5.80	3.0148	2.2166	2.4505	9.0339	6.9838	6.5496	3.0470	1.6911
5.90	2.8540	2.2297	2.4618	9.0455	7.0260	6.3677	3.0257	1.6961
6.00	2.7139	2.2426	2.4732	9.0570	7.0674	6.1935	3.0049	1.7010
7.00	1.0910×10^{-3}	2.3464	2.5616	9.1599	7.4374	4.7886	2.8179	1.7406
8.00	1.0910×10^{-3}	2.4426	2.6417	9.2464	7.7425	3.8166	2.6619	1.7704
1.00×10^1	7.2715×10^{-4}	2.5232	2.7071	9.3207	7.9979	3.1151	2.5257	1.7933
1.20	5.0323	2.5917	2.7613	9.3856	8.2148	2.5917	2.4141	1.8113
1.40	2.6371	2.7015	2.8451	9.4593	8.5630	1.8760	2.2239	1.8367
1.60	1.5182×10^{-4}	2.7859	2.9061	9.5867	8.8306	1.4214	2.0719	1.8539
1.80	9.4049×10^{-5}	2.8532	2.9518	9.6660	9.0438	1.1145×10^{-1}	1.9466	1.8662
2.00	6.1725	2.9083	2.9868	9.7371	9.2185	8.9762×10^{-2}	1.8409	1.8753
2.20	4.2385	2.9546	3.0143	9.8020	9.3651	7.3864	1.7503	1.8823
2.40	3.0091	2.9941	3.0361	9.8617	9.4903	6.1861	1.6712	1.8877
2.60	2.1800	3.0281	3.0536	9.9164	9.5982	5.2571	1.6015	1.8921
2.80	1.5821	3.0573	3.0679	9.9653 $\times 10^{-1}$	9.6908	4.5226	1.5394	1.8956
2.80×10^1	1.1064×10^{-5}	3.0817×10^1	3.0797×10^1	1.0006×10^0	9.7682×10^{-1}	3.9308×10^{-2}	1.4836×10^0	1.8983×10^2

TABLA 4

s	$\frac{Q_R(s)}{K}$	$\frac{M_R(s)}{K}$	$f_R(\alpha) \Big _{\alpha=g}$	$\frac{M_R(s)}{K f_R(\alpha)} \Big _{\alpha=g}$	$\frac{M_R(s)}{M_R(\infty)}$	$\frac{F_R(s)}{GK}$	$\frac{V_R(s)}{GK}$	$\frac{v_R(s)}{\sqrt{GK}}$	$-\frac{\Omega_R(s)}{GK^2}$
1.00×10^{-2}	4.4600×10^4	1.8682×10^{-2}	8.0507×10^{-2}	2.3205×10^{-1}	1.1150×10^{-3}	1.8682×10^2	1.8682×10^0	1.0397×10^1	1.3960×10^{-2}
2.00	1.5972×10^3	8.7854×10^{-2}	2.1732×10^{-1}	4.0425	5.2433×10^{-3}	2.1963	4.0427	1.0186×10^1	2.0978×10^{-1}
3.00	8.3697×10^{-2}	1.7572×10^{-1}	3.7630	4.6697	1.0487×10^{-2}	1.9525	4.1207	9.9796×10^0	6.2273×10^{-1}
4.00	5.1978	2.7560	5.4683	5.0400	1.6449	1.7225	7.9546	9.7941	1.2187×10^0
5.00	3.5496	3.8379	7.2375	5.3028	2.2905	1.5352	9.5801 $\times 10^0$	9.6267	1.9662
6.00	2.5771	4.9797	9.0409×10^{-1}	5.5080	2.9720	1.3832	1.1037×10^1	9.4742	2.8392
7.00	1.9541	6.1652	1.0860×10^0	5.6768	3.6795	1.2582	1.2356	9.3340	3.8161
8.00	1.5305	7.3829	1.2684	5.8209	4.4063	1.1536	1.3560	9.2040	4.8791
9.00×10^{-2}	1.2294	8.6246	1.4503	5.9469	5.1473	1.0648×10^2	1.4668	9.0828	6.0138
1.00×10^{-1}	1.0068×10^2	9.8837×10^{-1}	1.6312	5.9898	5.8988	9.8837×10^1	1.5694	8.9692	7.2081
1.10	8.3887×10^1	1.1154×10^0	1.8107	6.1604	6.6572	8.6345	1.6648	8.8622	8.4508
1.20	7.0872	1.2434	1.9885	6.2528	7.4207	8.1174	1.7540	8.7609	9.7343×10^0
1.30	6.0586	1.3718	2.1644	6.3382	8.1874	7.6563	1.8377	8.6649	1.1052×10^1
1.40	5.2322	1.5006	2.3383	6.4176	8.9561	7.2423	1.9165	8.5734	1.2396
1.50	4.5586	1.6295	2.5101	6.4919	9.7253×10^{-2}	6.8686	1.9910	8.4861	1.3764
1.60	4.0028	1.7584	2.6796	6.5619	1.0494×10^{-1}	6.5295	2.0615	8.4026	1.5149
1.70	3.5391	1.8870	2.8470	6.6282	1.1262	6.2203	2.1285	8.3225	1.6549
1.80	3.1485	2.0154	3.0120	6.6911	1.2028	5.9373	2.1922	8.2456	1.7960
1.90	2.8167	2.1434	3.1748	6.7511	1.2792	5.6773	2.2530	8.1716	1.9379
2.00	2.5325	2.2709	3.3354	6.8085	1.3553	5.4375	2.3110	8.1002	2.0804
2.10	2.2874	2.3980	3.4937	6.8637	1.4311	5.2158	2.3666	8.0313	2.2233
2.20	2.0746	2.5244	3.6498	6.9167	1.5066	5.0100	2.4198	7.9647	2.3664
2.30	1.8889	2.6503	3.8036	6.9678	1.5818	4.8187	2.4710	7.9003	2.5096
2.40	1.7260	2.7756	3.9553	7.0173	1.6565	4.6402	2.5201	7.8379	2.6527
2.50	1.5822	2.9002	4.1049	7.0652	1.7309	4.4735	2.5674	7.7773	2.7955
2.60	1.4548	3.0241	4.2523	7.1117	1.8048	4.3173	2.6129	7.7185	2.9381
2.70	1.3415	3.1473	4.3976	7.1569	1.8784	4.1707	2.6569	7.6614	3.0802
2.80	1.2402	3.2699	4.5409	7.2008	1.9515	4.0329	2.6993	7.6058	3.2219
2.90	1.1494	3.3917	4.6822	7.2437	2.0242	3.9031	2.7403	7.5517	3.3631
3.00	1.0677×10^1	3.5128	4.8216	7.2856	2.0965	3.7806	2.7800	7.4989	3.5036
3.10	9.9331×10^0	3.6331	4.9590	7.3264	2.1683	3.6648	2.8184	7.4475	3.6434
3.20	9.2659	3.7527	5.0945	7.3662	2.2397	3.5552	2.8556	7.3974	3.7825
3.30	8.6601	3.8716	5.2281	7.4053	2.3106	3.4513	2.8917	7.3484	3.9209
3.40	8.1087	3.9897	5.3599	7.4436	2.3812	3.3528	2.9267	7.3006	4.0585
3.50	7.6055	4.1072	5.4900	7.4812	2.4512	3.2592	2.9608	7.2539	4.1953
3.60	7.1451	4.2239	5.6183	7.5181	2.5209	3.1701	2.9938	7.2081	4.3313
3.70	6.7230	4.3399	5.7448	7.5544	2.5901	3.0853	3.0260	7.1634	4.4664
3.80	6.3351	4.4552	5.8697	7.5902	2.6590	3.0045	3.0572	7.1196	4.6008
3.90	5.9779	4.5698	5.9929	7.6253	2.7274	2.9273	3.0877	7.0767	4.7342
4.00	5.6484	4.6837	6.1146	7.6599	2.7953	2.8536	3.1173	7.0347	4.8668
4.10	5.3439	4.7970	6.2346	7.6941	2.8629	2.8536	3.1462	6.9935	4.9985
4.20	5.0618	4.9095	6.3531	7.7277	2.9301	2.7832	3.1744	6.9531	5.1294
4.30	4.8002	5.0214	6.4700	7.7610	2.9969	2.7157	3.2019	6.9134	5.2593
4.40	4.5572	5.1326	6.5855	7.7938	3.0632	2.5892	3.2287	6.8745	5.3883
4.50	4.3311	5.2431	6.6995	7.8261	3.1292	2.5298	3.2549	6.8363	5.5165
4.60	4.1203	5.3530	6.8120	7.8581	3.1948	2.5298	3.2805	6.7988	5.6437
4.70	3.9236	5.4622	6.9232	7.8898	3.2600	2.4727	3.3056	6.7619	5.7701
4.80	3.7398	5.5708	7.0330	7.9210	3.3248	2.4179	3.3300	6.7256	5.8956
4.90	3.5677	5.6788	7.1414	7.9519	3.3892	2.3652	3.3539	6.6900	6.0201
5.00	3.4065	5.7861	7.2485	7.9825	3.4533	2.3144	3.3773	6.6549	6.1438
5.25	3.0446	6.0517	7.5106	8.0576	3.6118	2.1956	3.4337	6.5697	6.4468
5.50	2.7346	6.3134	7.7649	8.1308	3.7680	2.0871	3.4872	6.4877	6.7444
5.75	2.4666	6.5715	8.0117	8.2023	3.9220	1.9876	3.5381	6.4087	7.0367
6.00×10^{-1}	2.2334×10^0	6.8258×10^0	8.2515×10^0	8.2722×10^{-1}	4.0738×10^{-1}	1.8961×10^1	3.5866×10^1	6.3326×10^0	7.3237×10^1

TABLA 4 (Continúa)

s	$\frac{Q_R(s)}{K}$	$\frac{M_R(s)}{K}$	$f_R(a) _{a=s}$	$\frac{M_R(s)}{K f_R(a)} _{a=s}$	$\frac{M_R(s)}{M_R(\infty)}$	$\frac{F_R(s)}{GK}$	$\frac{V_R(s)}{\sqrt{GK}}$	$\frac{\Omega_R(s)}{GK^2}$
6.25×10^{-1}	2.0292×10^0	7.0766×10^0	8.4846×10^0	8.3406×10^{-1}	4.2235×10^{-1}	1.8116×10^1	6.2590×10^0	7.6054×10^1
6.50	1.8496	7.3239	8.7111	8.4070	4.3711	1.7335	6.1878	7.8820
6.75	1.6906	7.5676	8.9315	8.4730	4.5165	1.6609	6.1188	8.1534
7.00	1.5493	7.8079	9.1459	8.5370	4.6599	1.5934	6.0520	8.4196
7.25	1.4231	8.0446	9.3546	8.5997	4.8012	1.5305	5.9872	8.6807
7.50	1.3098	8.2779	9.5579	8.6608	4.9404	1.4716	5.9242	8.9366
7.75	1.2077	8.5075	9.7559	8.7204	5.0775	1.4164	5.8629	9.1874
8.00	1.1152	8.7336	9.9488×10^0	8.7785	5.2124	1.3646	5.8033	9.4329
8.25	1.0311×10^0	8.9560	1.0137×10^1	8.8350	5.3451	1.3158	5.7453	9.6731
8.50	9.5418×10^{-1}	9.1745	1.0320	8.8896	5.4755	1.2698	5.6888	9.9079×10^1
8.75	8.8355	9.3890	1.0499	8.9424	5.6035	1.2263	5.6337	1.0137×10^2
9.00	8.1827	9.5994	1.0674	8.9932	5.7291	1.1851	5.5799	1.0361
9.25	7.5748	9.8053×10^0	1.0845	9.0417	5.8520	1.1460	5.5275	1.0578
9.50	7.0021	1.0006×10^1	1.1011	9.0876	5.9721	1.1087	5.4763	1.0789
9.75×10^{-1}	6.4496	1.0202	1.1174	9.1305	6.0888	1.0732	5.4262	1.0993
1.00×10^0	6.0151	1.0391	1.1333	9.1691	6.2016	1.0391 $\times 10^1$	5.3774	1.1189
1.05	5.0656	1.0755	1.1634	9.2445	6.4190	9.7554×10^0	5.2829	1.1560
1.10	4.2919	1.1094	1.1913	9.3126	6.6209	9.1683	5.1926	1.1900
1.15	3.6582	1.1409	1.2170	9.3745	6.8089	8.6266	5.1063	1.2212
1.20	3.1353	1.1702	1.2408	9.4310	6.9842	8.1266	5.0236	1.2498
1.25	2.7010	1.1977	1.2630	9.4829	7.1479	7.6650	4.9444	1.2760
1.30	2.3378	1.2233	1.2836	9.5306	7.3010	7.2386	4.8685	1.3001
1.35	2.0324	1.2474	1.3028	9.5748	7.4445	6.8442	4.7957	1.3224
1.40	1.7743	1.2699	1.3207	9.6157	7.5791	6.4791	4.7257	1.3428
1.45	1.5549	1.2911	1.3374	9.6538	7.7055	6.1408	4.6585	1.3617
1.50	1.3675	1.3110	1.3531	9.6893	7.8245	5.8268	4.5939	1.3791
1.55	1.2069	1.3298	1.3678	9.7225	7.9365	5.5350	4.5316	1.3952
1.60	1.0685×10^{-1}	1.3475	1.3815	9.7535	8.0421	5.2636	4.4717	1.4102
1.65	9.4886×10^{-2}	1.3642	1.3945	9.7827	8.1418	5.0108	4.4139	1.4240
1.70	8.4503	1.3800	1.4067	9.8102	8.2360	4.7750	4.3581	1.4368
1.75	7.5461	1.3949	1.4182	9.8360	8.3250	4.5548	4.3043	1.4488
1.80	6.7559	1.4090	1.4290	9.8604	8.4094	4.3489	4.2523	1.4598
1.85	6.0633	1.4224	1.4392	9.8835	8.4893	4.1561	4.2020	1.4702
1.90	5.4544	1.4351	1.4488	9.9053	8.5651	3.9754	4.1533	1.4798
1.95	4.9175	1.4472	1.4580	9.9259	8.6371	3.8059	4.1062	1.4887
2.00	4.4428	1.4586	1.4666	9.9456	8.7054	3.6466	4.0606	1.4971
2.05	4.0220	1.4695	1.4748	9.9642	8.7704	3.4968	4.0164	1.5049
2.10	3.6481	1.4799	1.4826	9.9819	8.8323	3.3558	3.9735	1.5123
2.15	3.3151	1.4898	1.4899	9.9987×10^{-1}	8.8912	3.2228	3.9319	1.5191
2.20	3.0178	1.4992	1.4969	1.0015×10^0	8.9473	3.0974	3.8916	1.5255
2.25	2.7519	1.5081	1.5036	1.0030	9.0008	2.9790	3.8523	1.5316
2.30	2.5135	1.5167	1.5099	1.0045	9.0518	2.8670	3.8142	1.5372
2.35	2.2993	1.5248	1.5159	1.0059	9.1005	2.7611	3.7771	1.5425
2.40	2.1065	1.5326	1.5217	1.0072	9.1471	2.6608	3.7411	1.5475
2.45	1.9327	1.5401	1.5271	1.0085	9.1916	2.5657	3.7060	1.5522
2.50	1.7757	1.5472	1.5323	1.0097	9.2341	2.4755	3.6718	1.5566
2.55	1.6337	1.5540	1.5373	1.0109	9.2748	2.3899	3.6386	1.5608
2.60	1.5050	1.5606	1.5421	1.0120	9.3138	2.3085	3.6062	1.5647
2.65	1.3881	1.5668	1.5466	1.0131	9.3512	2.2312	3.5745	1.5685
2.70	1.2819	1.5728	1.5509	1.0141	9.3870	2.1575	3.5437	1.5720
2.75	1.1852	1.5786	1.5551	1.0151	9.4213	2.0874	3.5137	1.5753
2.80	1.0970	1.5841	1.5591	1.0161	9.4542	2.0205	3.4843	1.5784
2.85	1.0165×10^{-2}	1.5894	1.5629	1.0170	9.4858	1.9568	3.4557	1.5814
2.90×10^0	9.4294×10^{-3}	1.5945×10^1	1.5665×10^1	1.0179×10^0	9.5162×10^{-1}	1.8959×10^0	3.4277×10^0	1.5842×10^2

TABLA 4 (Continúa)

s	$\frac{Q_R(s)}{K}$	$\frac{M_R(s)}{K}$	$f_R(\alpha) ^{a-s}$	$\frac{M_R(s)}{K f_R(\alpha) ^{a-s}}$	$\frac{M_R(s)}{M_R(\infty)}$	$\frac{F_R(s)}{GK}$	$\frac{V_R(s)}{GK}$	$\frac{v_R(s)}{\sqrt{GK}}$	$-\frac{Q_R(s)}{GK^2}$
2.95×10^0	8.7558×10^{-3}	1.5994×10^1	1.5700×10^1	1.0187×10^0	9.5453×10^{-1}	1.8378×10^0	5.0136×10^1	3.4003×10^0	1.5868×10^2
3.00	8.1384	1.6041	1.5733	1.0195	9.5733	1.7823	5.0226	3.3736	1.5893
3.05	7.5718	1.6086	1.5765	1.0203	9.6003	1.7292	5.0314	3.3475	1.5917
3.10	7.0513	1.6129	1.5796	1.0211	9.6262	1.6784	5.0399	3.3220	1.5940
3.15	6.5726	1.6171	1.5826	1.0218	9.6511	1.6297	5.0482	3.2970	1.5961
3.20	6.1318	1.6211	1.5854	1.0225	9.6751	1.5831	5.0562	3.2725	1.5982
3.25	5.7255	1.6250	1.5881	1.0232	9.6982	1.5334	5.0640	3.2486	1.6001
3.30	5.3505	1.6287	1.5908	1.0239	9.7205	1.4956	5.0716	3.2252	1.6020
3.35	5.0042	1.6323	1.5933	1.0245	9.7419	1.4545	5.0790	3.2022	1.6037
3.40	4.6841	1.6358	1.5957	1.0251	9.7626	1.4150	5.0862	3.1797	1.6054
3.45	4.3878	1.6391	1.5980	1.0257	9.7825	1.3771	5.0931	3.1577	1.6070
3.50	4.1133	1.6423	1.6003	1.0263	9.8018	1.3407	5.0999	3.1361	1.6085
3.55	3.8586	1.6454	1.6025	1.0268	9.8203	1.3056	5.1065	3.1149	1.6099
3.60	3.6224	1.6484	1.6045	1.0274	9.8382	1.2719	5.1130	3.0942	1.6113
3.65	3.4031	1.6513	1.6065	1.0279	9.8555	1.2395	5.1193	3.0738	1.6126
3.70	3.1993	1.6541	1.6085	1.0284	9.8723	1.2083	5.1254	3.0539	1.6139
3.75	3.0096	1.6568	1.6104	1.0289	9.8884	1.1782	5.1314	3.0343	1.6151
3.80	2.8330	1.6595	1.6122	1.0293	9.9040	1.1492	5.1372	3.0150	1.6162
3.85	2.6685	1.6620	1.6139	1.0298	9.9191	1.1213	5.1428	2.9961	1.6173
3.90	2.5150	1.6644	1.6156	1.0302	9.9337	1.0943	5.1484	2.9776	1.6184
3.95	2.3719	1.6668	1.6172	1.0307	9.9478	1.0683	5.1538	2.9594	1.6194
4.00	2.2381	1.6691	1.6188	1.0311	9.9614	1.0432 $\times 10^0$	5.1591	2.9415	1.6203
4.10	1.9963	1.6734	1.6217	1.0319	9.9874×10^{-1}	9.9550×10^{-1}	5.1693	2.9067	1.6221
4.20	1.7845	1.6775	1.6245	1.0326	1.0012×10^0	9.5093	5.1790	2.8730	1.6238
4.30	1.5985	1.6814	1.6271	1.0333	1.0035	9.0933	5.1883	2.8404	1.6253
4.40	1.4349	1.6850	1.6295	1.0340	1.0056	8.7033	5.1972	2.8089	1.6267
4.50	1.2904	1.6883	1.6318	1.0346	1.0076	8.3375	5.2057	2.7784	1.6279
4.60	1.1627	1.6915	1.6340	1.0352	1.0095	7.9940	5.2139	2.7489	1.6291
4.70	1.0495×10^{-3}	1.6945	1.6360	1.0358	1.0113	7.6710	5.2217	2.7203	1.6302
4.80	9.4892×10^{-4}	1.6974	1.6379	1.0363	1.0130	7.3670	5.2292	2.6925	1.6312
4.90	8.5941	1.7000	1.6396	1.0368	1.0146	7.0805	5.2364	2.6655	1.6321
5.00	7.7959	1.7026	1.6413	1.0373	1.0161	6.8102	5.2434	2.6394	1.6330
5.10	7.0827	1.7049	1.6429	1.0378	1.0175	6.5549	5.2501	2.6139	1.6338
5.20	6.4443	1.7072	1.6444	1.0382	1.0189	6.3136	5.2565	2.5892	1.6345
5.30	5.8718	1.7093	1.6458	1.0386	1.0202	6.0851	5.2627	2.5651	1.6352
5.40	5.3576	1.7113	1.6471	1.0390	1.0214	5.8688	5.2687	2.5417	1.6359
5.50	4.8950	1.7132	1.6484	1.0394	1.0225	5.6636	5.2744	2.5189	1.6365
5.60	4.4781	1.7151	1.6495	1.0397	1.0236	5.4639	5.2800	2.4968	1.6370
5.70	4.1019	1.7168	1.6507	1.0400	1.0246	5.2840	5.2854	2.4751	1.6375
5.80	3.7618	1.7184	1.6517	1.0404	1.0256	5.1082	5.2906	2.4541	1.6380
5.90	3.4540	1.7200	1.6527	1.0407	1.0265	4.9410	5.2956	2.4335	1.6385
6.00	3.1750	1.7214	1.6537	1.0410	1.0274	4.7818	5.3005	2.4134	1.6389
7.00	1.4463×10^{-4}	1.7328	1.6610	1.0432	1.0342	3.5363	5.3416	2.2366	1.6418
8.00	7.1714×10^{-5}	1.7400	1.6655	1.0447	1.0385	2.7188	5.3726	2.0933	1.6434
9.00×10^0	3.8011	1.7447	1.6684	1.0458	1.0413	2.1540	5.3968	1.9743	1.6444
1.00×10^1	2.1268×10^{-5}	1.7480	1.6704	1.0465	1.0432	1.7480	5.4162	1.8735	1.6450
1.20	7.5715×10^{-6}	1.7518	1.6727	1.0473	1.0455	1.2166×10^{-1}	5.4454	1.7107	1.6456
1.40	3.0828	1.7539	1.6738	1.0478	1.0468	8.9485×10^{-2}	5.4662	1.5840	1.6458
1.60	1.3969×10^{-6}	1.7551	1.6745	1.0481	1.0475	6.8558	5.4819	1.4819	1.6460
1.80	6.9224×10^{-7}	1.7558	1.6749	1.0483	1.0479	5.4191	5.4941	1.3972	1.6460
2.00	3.7084	1.7562	1.6751	1.0485	1.0482	4.3906	5.5038	1.3255	1.6461
2.20	2.1256	1.7566	1.6752	1.0485	1.0482	3.6293	5.5118	1.2638	1.6461
2.40	1.2860×10^{-7}	1.7568	1.6753	1.0486	1.0485	3.0500	5.5185	1.2100	1.6461
2.60	8.0209×10^{-8}	1.7569	1.6754	1.0487	1.0486	2.5990	5.5241	1.1626	1.6461
2.80×10^1	4.8739×10^{-8}	1.7571×10^1	1.6754×10^1	1.0487×10^0	1.0486×10^0	2.2411×10^{-2}	5.5289×10^1	1.1203×10^0	1.6461×10^2