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DESIGN OF TELESCOPES OF THE CASSEGRAIN AND RITCHEY-CHRÈTIEN TYPES

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Sumario

En este artículo se derivan todas las ecuaciones necesarias para el diseño de telescopios del tipo Cassegrain a primer orden. Así mismo, se calculan las constantes de conicidad tanto del sistema ordinario parábola-hipérbola, como del sistema aplanático de Ritchey-Chrètien.

Introduction

The reflecting telescopes are very popular due to their low cost, good images produced, and simplicity. Among them, the most common for astronomical uses is the Cassegrain.

The ordinary type of Cassegrain produces an image almost as good as that of the Newtonian telescope, whose aperture and equivalent focal length are the same. It has been shown that in order to obtain this quality of image, great care should be taken during its construction (Porter 1954) and adjustment. (Lower 1954).

The designs of this type of telescope and its aplanatic form, known as the Ritchey-Chrètien telescope, will be considered in the present paper. An analytical expression for computing the conic constants of the mirrors in both cases will be given; avoiding, thus, the need for a computer in designing the telescope.

First Order Design of the Telescope

In order to obtain the first order design, the following quantities are defined: (See Fig. 1).

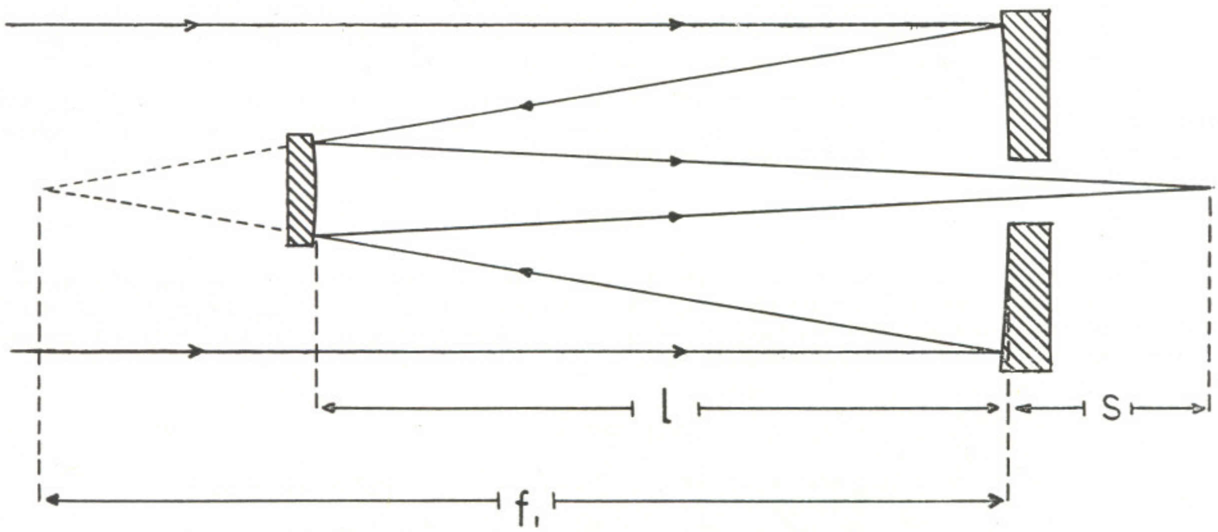

Figure 1.—*Cassegrain or Ritchey-Chrètien Telescope*

D_1 = Diameter of primary mirror
D_2 = Diameter of secondary mirror when a field with diameter I is desired in the Cassegrain plane.
F = Effective focal length of the system.
f_1 = Focal length of the primary mirror.
f_2 = Focal length of the secondary mirror
d_2 = Diameter of secondary mirror when $I = 0$
I = Diameter of field in the Cassegrain plane
l = Separation between mirrors.

S = Distance from the vertex of the primary mirror to the Cassegrain focus.
r_2 = Radius of curvature of secondary mirror.
r_1 = Radius of curvature of primary mirror.
Using the definition of effective focal length:

$$\frac{F}{D_1} = \frac{l + S}{d_2} \tag{1}$$

By similarity of triangles in Fig. 1:

$$\frac{f_1 - l}{d_2} = \frac{f_1}{D_1} \tag{2}$$

Using a well known formula of optics for paraxial rays:

$$\frac{1}{f_2} = \frac{1}{l + S} - \frac{1}{f_1 - l} \tag{3}$$

On the other hand, considering the fact that the lateral magnification given by the secondary mirror is (F/f_1), and since the diameter of the image in the Cassegrain plane is I, the diameter of the image in the prime focus would be given by $I\ (f_1/F)$. Thus, from Fig. 1 it is easy to see that the effective diameter of the secondary would be given by:

$$D_2 = d_2 + I \left(\frac{l}{F}\right) \tag{4}$$

assuming that the limiting aperture (entrance pupil) is the primary mirror.
The radii of curvature are related to the focal lengths as follows:

$$r_1 = -2\ f_1 \tag{5}$$

$$r_2 = 2\ f_2 \tag{6}$$

There are six equations and eleven quantities to be determined; thus, the system is completely defined by five parameters as follows:

$$D_1,\ F,\ f_1,\ S,\ I$$

or:

$$D_1,\ F,\ \eta,\ S,\ I$$

where $\eta = f_1/f_2$.

If the first five parameters are given, l is given from (1) and (2):

$$l = \frac{F - S}{1 + F/f_1} \tag{7}$$

d_2 can be found from (1):

$$d_2 = (f_1 - l)\ \frac{D_1}{f_1} \tag{8}$$

and f_2 can be found from (1), (2) and (3):

$$\frac{1}{f_2} = \frac{D_1}{d_2} \left[\frac{1}{F} - \frac{1}{f_1}\right] \tag{9}$$

then, D_2, r_1 and r_2 are calculated with equations (4), (5) and (6).

If the second set of parameters is given, then, from equation (9):

$$\eta = \frac{D_1}{d_2}\left[\frac{f_1}{F} - 1\right] \quad ; \tag{10}$$

from equations (7) and (8):

$$\frac{d_2}{D_1} = 1 - \frac{F - S}{F + f_1} \quad , \tag{11}$$

from (10) and (11):

$$\eta = \frac{f^2{}_1 - F^2}{F\,(f_1 - S)} \quad . \tag{12}$$

This last equation can be written in the form:

$$f^2{}_1 - \eta\, F\, f_1 - F\,(F + \eta\, S) = O \quad , \tag{13}$$

and from this:

$$f_1 = F/2\ \{\eta + [\eta^2 \ \pm [\eta^2 + 4\,(\eta\, S/F + 1)]^{1/2}\} \tag{14}$$

η is always negative, therefore, f_1 is positive only if the positive sign of the square root is taken.

$$f_1 = F/2\ \{\eta + [\eta^2 + 4\,(\eta\, S/F + 1)]^{1/2}\} \tag{15}$$

Having calculated f_1, equations (7), (8), (4), (5) and (6) are used.

Design of the Common Cassegrain

The common Cassegrain consists of a primary paraboloidal mirror and a secondary hyperboiloidal mirror. The spherical aberration is thus corrected on each mirror.

The type of hyperboloid to be used for a secondary mirror is defined by means of a conic constant K as follows: It is customary in optics to represent a conic surface by means of the equation (Spencer 1963).

$$y = \frac{C\,R^2}{1 + [1 - (K + 1)\,C^2\,R^2]^{1/2}} \tag{16}$$

where "R" is the distance from a point in the surface to the optic axis, "C" is the curvature $1/r$ near he vertex of the surface, "y" is the separation between the surface and its plane tangent to the vertex, and "K" is the conic constant.

If e is the excentricity of the conic surface, K is given by $-e^2$, and has the following values:

Sphere:	$K = 0$
Paraboloid:	$K = -1$
Hyperboloid:	$K < -1$
Elipsoid revolved about the major axis:	$-1 < K < 0$
Elipsoid revolved about the minor axis:	$K > 0$

The hyperboloid of the common Cassegrain telescope has an excentricity given by:

$$e = \frac{f_1 + S}{2l - f_1 + S} \tag{17}$$

because the distance from the center between the two branches of the hyperbola to the focus is $(f_1 + S)/2$ an the major semi-axis is $(f_1 + S)/2 - (f_1 - l)$.

Therefore, the conic constant K is given by:

$$K = -\left(\frac{f_1 + S}{2l - f_1 + S}\right)^2 \tag{18}$$

Coma of the Common Cassegrain

The common Cassegrain telescope has a large amount of Coma and is equivalent to that of the Newtonian telescope with the same aperture and effective focal length. This fact has been demonstrated by Jones (1954) showing that the quantity OSC is the same in both cases, since in the absence of spherical aberration the saggital coma is given by (Conrady 1957 A).

$$Coma_s = h\,(OSC) \tag{19}$$

where h is the height of the image, and (Conrady 1957 B):

$$OSC = \frac{Y_1 \; u'_k}{y_1 \; sen \; U'_k} - 1 \tag{20}$$

In order to demostrate that OSC is the same in both cases, it is only necessary to show that the "principal surface" has the shape of the equivalent paraboloid. This can be shown with the aid of Fig. 2 where the principal surface is clearly defined. Let us assume that the direction of the incident ray is reversed and reflected on the principal surface first, then on the back of the hyperboloid and finally comes to point P.

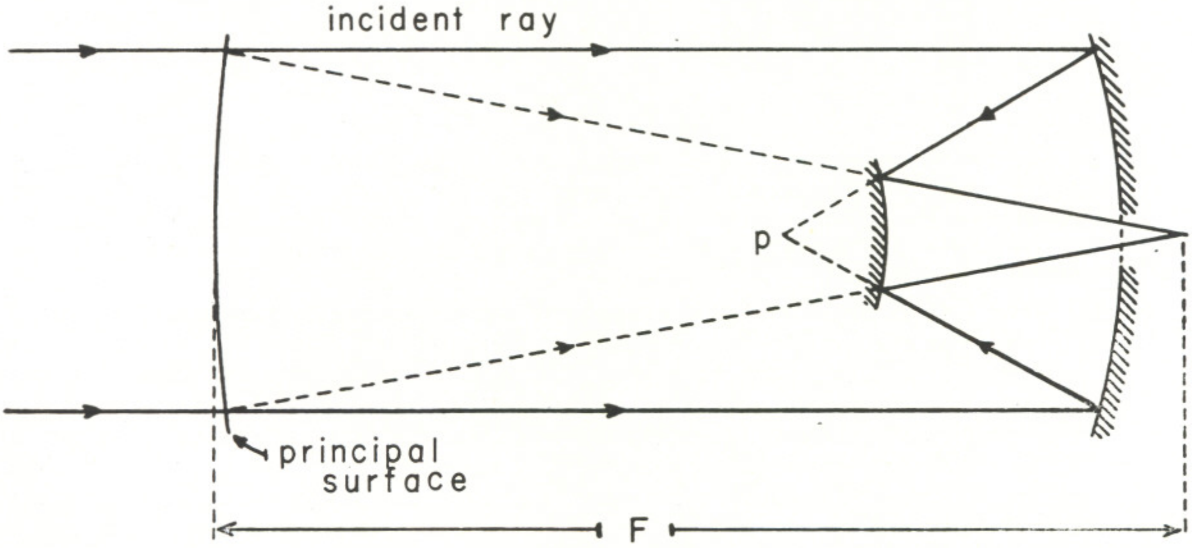

Figure 2.—*Coma of the Cassegrain Telescope*

Since point P is also the focus of the primary-paraboloid, it should be free from spherical aberration; but this is true only if the principal surface is a paraboloid with focal length F.

The magnitude of OSC is obtained by means of equation (20) and using Fig. 3:

$$OSC = \frac{D}{F} - 1 \tag{21}$$

Using the equation of the parabola, it can be shown:

$$\frac{D^2}{F^2} = 1 + \left(\frac{y^2_1}{2F^2}\right)^2 \tag{22}$$

assuming that F is much greater than y, and using (21):

$$OSC = \left(\frac{y_1}{2F}\right)^2 \tag{23}$$

therefore, from (19), the saggital Coma is given by:

$$Coma_s = h\left(\frac{y_1}{2F}\right)^2 \tag{24}$$

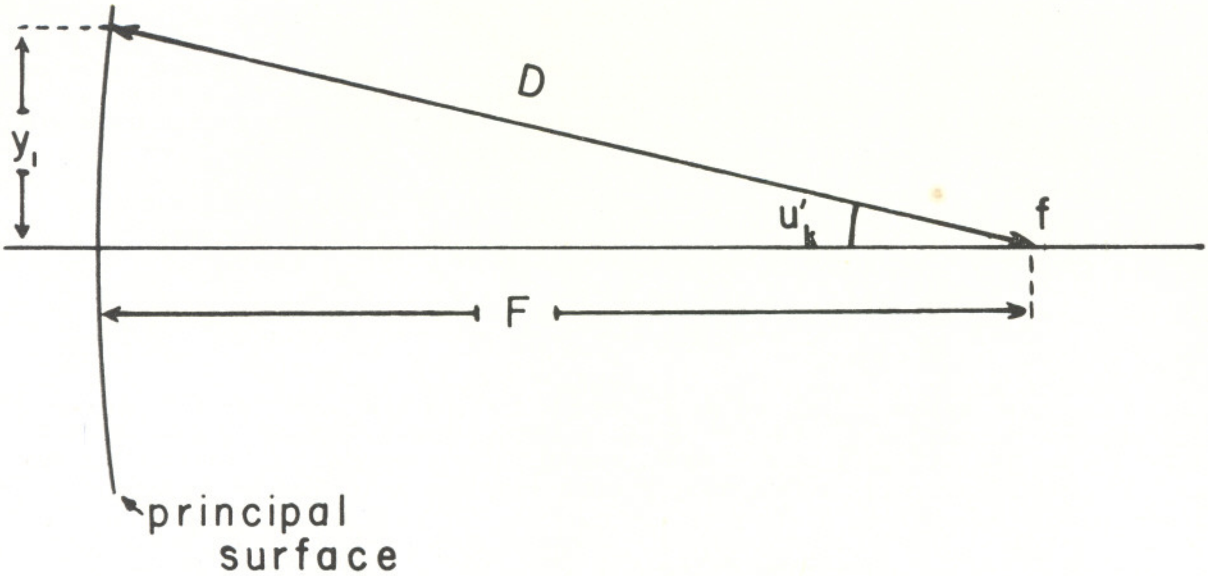

Figure 3.—*Evaluation of the Coma of a Paraboloid*

Ritchey-Chrètien Telescope

In the common Cassegrain, the spherical aberration is corrected in both mirrors; but one degree of freedom is gained if this aberration is required to be corrected only in the whole system. This degree of freedom can be used in order to correct the Coma, but then the possibility of using the primary focus is eliminated unless an optical corrector is used (Wynne 1965). However this is not a great disadvantage (Meinel 1960).

In this manner, the Coma is corrected by choosing two appropiate aspherical surfaces for the primary and secondary mirrors (Chrètien 1922), (Jones 1954), (Willey 1962), (Willey 1962 B) and since these surfaces are nearly two hyperboloids, these can be defined by means of their conic constant K.

Since there is only one constant to be determined, K can be determined for rays near the axis by means of third order equations and thus for the whole surface.

The conic constants for both mirrors will be found using the third order expressions for the spherical aberration and Coma for a conic surface, as given by G. Spencer (1963). These aberrations for a system of two surfaces, are:

$$B^T = \gamma (B_1 + B_2) \tag{25}$$

and:

$$F^T = \gamma (F_1 + F_2) , \tag{26}$$

where:

$$B = S\, i^2 + T \tag{27}$$

$$F = S\, i\bar{i} + \left(\frac{\bar{y}}{y}\right) T \tag{28}$$

$$S = y \left(\frac{N}{N'}\right) (N - N')\, (i' - u) \tag{29}$$

$$T = \frac{(N - N')(K - 1)\, y^4}{r^3} \tag{30}$$

$$\gamma = \frac{1}{2\, N'_2\, u'_2} \tag{31}$$

Quantities with bar are for the chief ray, K is the conic constant, and all others follow the usual notation in optics (Conrady 1957 C).

Let us assume that the conic constants of the common Cassegrain are modified in order to obtain the Ritchey-Chrètien; then the spherical aberrations of both mirrors are modified in such a way that the total spherical aberration reamins in zero:

$$\Delta B_1 + \Delta B_2 = 0 \tag{32}$$

If F^T is the Coma, of the common Cassegrain, the changes in the Coma, of both mirrors should satisfy the relation.

$$\gamma\,(\Delta F_1 + \Delta F_2) = -F^T \tag{33}$$

Since the entrance pupil coincides with the primary mirror $y_1 = 0$. Thus, using (27) and (28) in (32) and (33):

$$\Delta T_1 + \Delta T_2 = 0 \tag{34}$$

$$\Delta T_2 = -\frac{F^T}{\bar{\gamma}}\,\frac{y_2}{y_2} \tag{35}$$

Both are reflecting surfaces, therefore from (30):

$$\Delta T_1 = \frac{2y_1}{r^3_1}\,\Delta K_1 \tag{36}$$

and:

$$\Delta T_2 = \frac{-2\,y^4_2}{r^3_2}\,\Delta K_2 \tag{37}$$

Substituting (36) and (37) in (34) and (35):

$$\Delta K_1 = \frac{r^3_1\,y^4_2}{r^3_2\,y^4_1}\,\Delta K_2 \tag{38}$$

$$\Delta K_2 = \frac{F^T\,r^3_2}{2\,\gamma\,y^3_2\,\bar{y}_2} \tag{39}$$

In a similar way to equation (4), h is found to be:

$$h = \bar{y}_2\left(\frac{F}{l}\right) \tag{40}$$

Then, from equation (24); the saggital Coma is:

$$F^T = \frac{\bar{y}_2\,y^2_1}{4\,Fl} \tag{41}$$

Substituting this equation in (39) and using $\gamma = F/2y_1$:

$$\Delta K_2 = \frac{r^3_2\,y^3_1}{4\,F^2\,l\,y^3_2}\ ; \tag{42}$$

on the other hand, from (8), it can be shown that

$$\frac{y_2}{y_1} = \frac{f_1 - l}{f_1} \tag{43}$$

Using (5), (6) and (43) in (38):

$$\Delta K_1 = -\frac{(f_1 - l)^4}{f_1 f^3_2} \Delta K_2 \ , \tag{44}$$

and using (6) and (43) in (42):

$$\Delta K_2 = \frac{2 f^3_2 f^3_1}{F^2 l (f_1 - l)^3} \tag{45}$$

Substituting (45) in (44):

$$\Delta K_1 = -\frac{2 (f_1 - l) f^2_1}{F^2 l} \tag{46}$$

The effective focal length of the telescope is given by:

$$F = \frac{f_1 f_2}{f_1 + f_2 - l} \tag{47}$$

therefore, substituting in (45):

$$\Delta K_2 = \frac{2F (f_1 + f_2 - l)^3}{l (f_1 - l)^3} \tag{48}$$

The conic constant for a paraboloid is -1, therefore, adding -1 to equation (46) the conic constant for the primary mirror of a Ritchey-Chrètien telescope is given by:

$$K_1 = -\frac{2 (f_1 - l) f^2_1}{F^2 l} - 1 \tag{49}$$

Adding equations (18) and (48), the conic constant for the secondary mirror is:

$$K_2 = -\left(\frac{f_1 + S}{2l - f_1 + S}\right)^2 + \frac{2F (f_1 + f_2 - l)^3}{l (f_1 - l)^3} \tag{50}$$

Field Curvature

The curvature of the Petzval surface is independent of the conic constants, and using the Petzval theorem (Conrady 1957 D) is given by:

$$\frac{1}{\varrho} = -2 \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{51}$$

where ϱ is the radius of curvature of this surface.

In the absence of astigmatism the surface of best definition coincides with the Petzval surface, but not when astigmatism is present.

However, the astigmatism is small. Meinel (1960) states that the astigmatsim of the common Cassegrain is equal to that of the equivalent paraboloid, multiplied by the magnification of the secondary mirror.

The Petzval surface is flat when $\eta = -1$.

Analysis of Results

These equations were used in order to design and Ordinary Cassegrain and a Ritchey-Chrètien with the same effective focal length of 324 cm, a primary mirror F/3, and with an aperture of 24 cm. Both designs are shown in Fig. 4.

DESIGN OF A CASSEGRAIN AND A RITCHEY-CHRETIEN TELESCOPE

EFFECTIVE FOCAL LENGTH =	324.0 cm.
FOCAL RATIO =	F/13.5
PRIMARY MIRROR =	
DIAMETER =	24.0 cm.
FOCAL LENGTH =	72.0 cm.
K_1 FOR THE CASSEGRAIN =	-1.0
K_1 FOR THE RITCHEY-CHRÈTIEN =	-1.029678
SECONDARY MIRROR =	
MINIMUM DIAMETER (I=0.) =	5.545455 cm.
FOCAL LENGTH =	-21.389610 cm.
K_2 FOR THE CASSEGRAIN =	-2.469388
K_2 FOR THE RITCHEY-CHRÈTIEN =	-2.742377
DISTANCE BETWEEN MIRRORS =	55.363636 cm.
PETZVAL CURVATURE ($1/\rho$) =	-0.032863 1/cm.

Figure 4.—*Designs of two Telescopes*

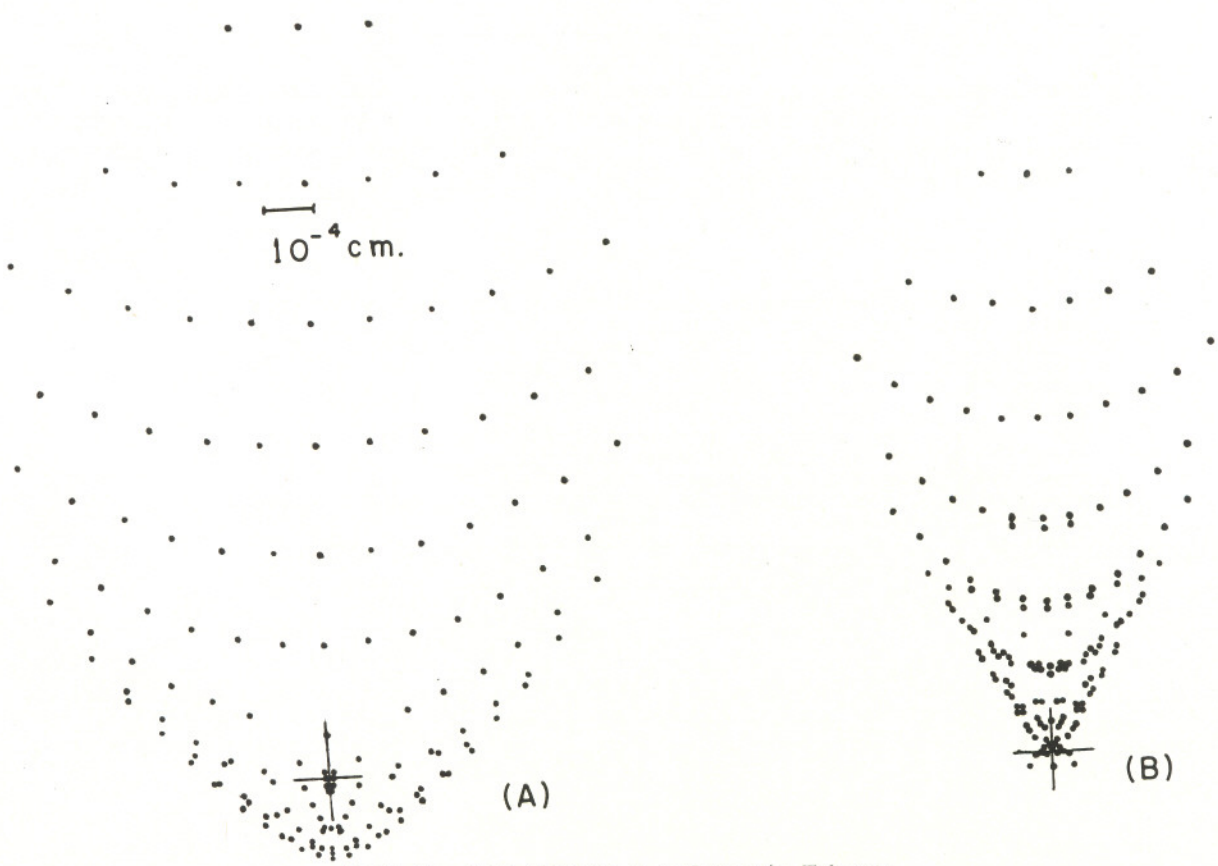

Figure 5.—*Spot Diagrams of a Cassegrain Telescope*

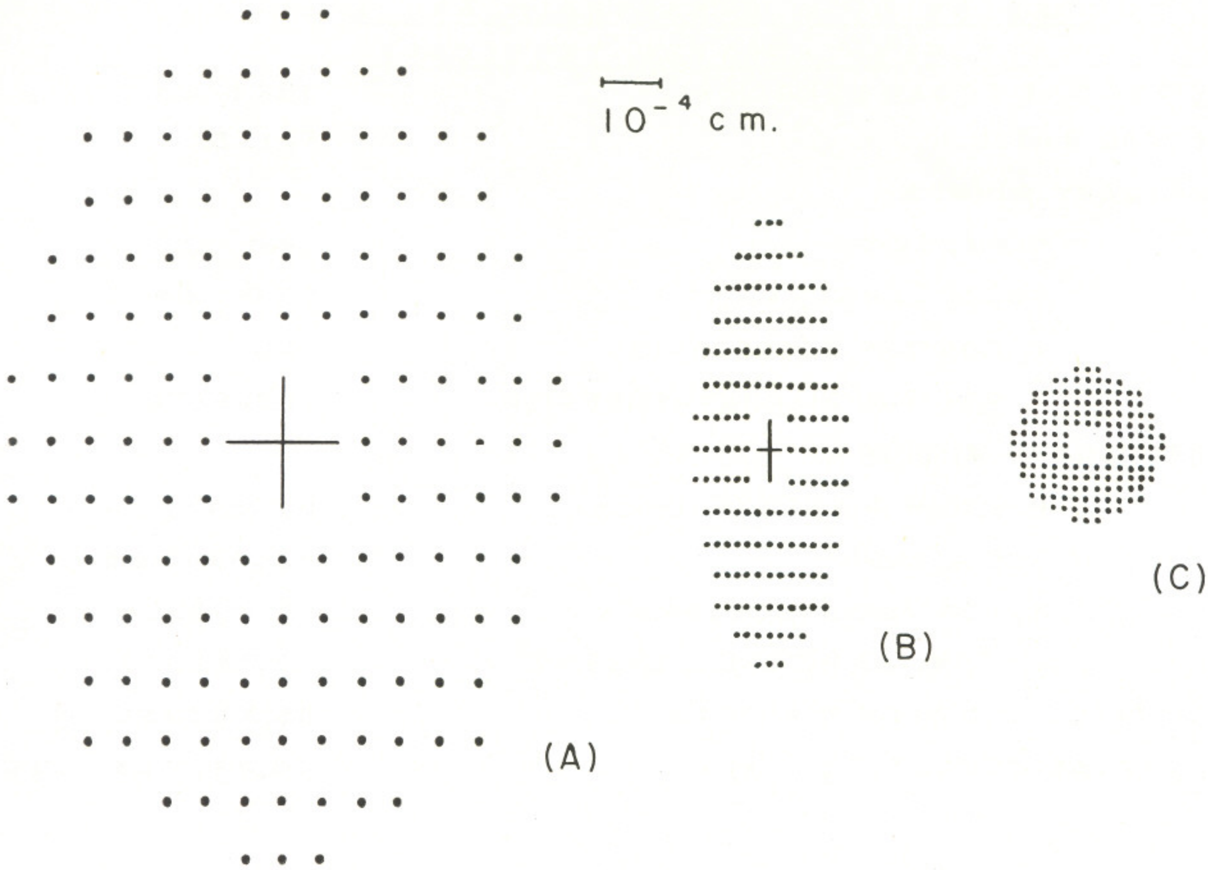

Figure 6.—*Spot Diagrams of a Ritchey-Chrètien Telescope.*

The Spot Diagrams for an image 0.75 cm. off axis are given in the following figures: Fig. 5 (a) shows it on the focal plane of the Cassegrain, and Fig. 5 (b) on its Petzval surface. Also, Fig. 6 (a) shows the spot diagram on the focal plane of the Ritchey-Chrètien, and Fig. 6 (b) on its Petzval surface, while Fig. 6 (c) is taken on its surface of best definition.

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