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SPECTROPHOTOMETRY OF THE ORION NEBULA. I. ON THE NEBULAR STRUCTURE.

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Abstract

Results of spectrophotometry observations of the Orion Nebula in the wavelength range $\lambda 3600$ — 11000 Å are presented. Apparent emission line intensities in various areas of the Huyghenian region have been corrected for interstellar and internebular extinction using a reddening law obtained from the intensity ratio between Paschen and Balmer lines, arising from the same upper level. Self-absorption in the Balmer lines has been shown to be entirely negligible and the intrinsic Balmer decrement found is very close to the value predicted by pure recombination theory. The value of the total absorption is determined from the comparison between the surface brightness obtained with the hydrogen lines with that derived through observations in the radio frequency region. The resulting ratio between total absorption and reddening, R_v, is close to 4.8 in agreement with the results of recent multicolor photometry. In the above derivation, effect of density fluctuations in the nebula have been considered. The same effect on the ratio of the nebular to the auroral lines of the OII ion is given in detail.

Sumario

Algunos resultados de observaciones espectrofotométricas de la Nebulosa de Orión, en el rango de $\lambda 3600$-11000 Å, se presentan. Las intensidades aparentes de líneas en emisión, producidas en ciertas áreas de la región Huygheniana, han sido corregidas para tomar en cuenta la absorción producida por los medios internebular e interestelar. Tal corrección se obtiene utilizando las relaciones de intensidad de líneas, de las series de Balmer y Paschen, que se originan del mismo nivel superior. Se demuestra que la autoabsorción en las líneas de Balmer es completamente despreciable. El decremento de Balmer encontrado concuerda con el calculado teóricamente para recombinación radiativa. El valor absoluto de la absorción, en las regiones más densas, se determina comparando el brillo superficial observado, en las líneas de hidrógeno, con aquel derivado a través de observaciones en la región de radiofrecuencia. La razón resultante entre la absorción total, en el visual, y el enrojecimeinto es alrededor de 4.8, en perfecto acuerdo con resultados recientes de fotometría multicolor. Los efectos producidos por fluctuaciones en densidad han sido considerados en el análisis teórico. Los mismos efectos son también estudiados usando la relación de intensidades de las líneas aurorales y nebulares producidas por el ión O II.

I. Introduction

The spectrum of the Orion Nebula has been the subject of numerous photometric surveys, from which a great part of the existing knowledge regarding the physical conditions, structure and composition of the H II regions has been derived. The recent photoelectric studies of Aller and Liller (1959), Mathis (1962), and O'Dell *et al.* (1964), because of their high precision, have considerably helped in our understanding of the unsolved problems related to emission nebulae. However, one of the fundamental difficulties encountered in the study of the Orion Nebula, namely the effect of the dust particles existing within the nebula on the line intensities, deserves further attention. The ratio between visual absorption and reddening —the so-called R— has shown (Sharpless, 1952; Johnson and Borgman, 1963) to differ markedly, for the stars embedded in the Orion Nebula, from its average value in the galactic plane. Since it appears quite certain that most of the extinction, suffered by in the Trapezium stars, actually occurs within the Nebula (Münch and Wilson, 1962; Wurm, 1961,) the questions related to the nature of the dust particles suspended within the nebular material naturally arises. An observable bearing some relation to this question is the dependence on wavelength of the extinction produced by solid grains.

Because of the uncertain location of the Trapezium stars within the nebula, it is clear that the use of the reddening curve, determined from their direct photometry, to correct the nebular line intensities is open to question. It is then proposed to determine the reddening law in any region of the Orion Nebula from observations of the lines themselves. For this purpose we shall compare the intensity of pairs of H-lines, in the Paschen and Balmer series, having the same upper level, to determine the differential extinction between the corresponding wavelengths. Although the above method yields important results (O'Dell, 1963), the absolute value of the absorption can not be determined safely by extrapolating the differential correction (Aller, 1965). The absolute absorption should be derived on different grounds. It is then proposed that, the extent and effect of density fluctuations within the nebula are studied; then using the work done by Osterbrock and Flather (1959) on the O II doublet ratio in the Orion Nebula, which includes the analysis of radio observations, one can readily obtain the total absorption.

II. The Observations

The observations of the Orion Nebula were carried out during the winter of 1962-63 with the Cassegrain scanner of the 60-inch reflector at Mt. Wilson. The spectral range $\lambda 3500$-5500 Å, in the sec-

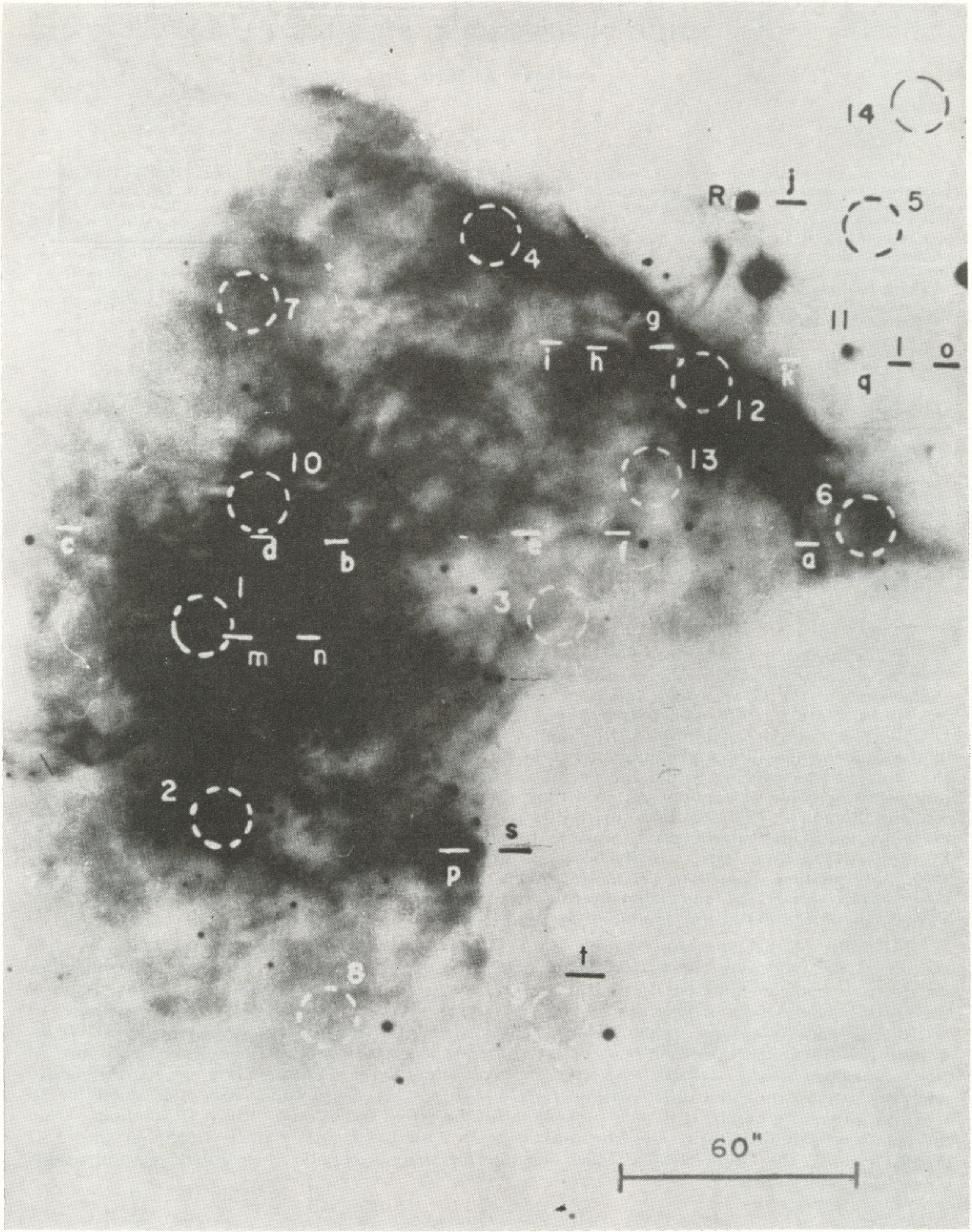

Fig. 1.—*The location of the points studied in the present investigation as seen on a short exposure plate. (Münch and Wilson, 1962).*

ond order with a dispersion of 10 A/mm, was recorded with a 1P21 tube. The range λ4800-11000 Å was observed, in the first order, with a RCA 7102 multiplier. Fourteen areas, designated with numbers in Fig. 1, were observed with circular diaphragms of 14″ in diameter, which project, on the exit slit plane of the spectrometer, on 34 Å in the first order. In order to measure the intensity of H_α, separately from that of the [NII] lines, an entrance aperture one half as large was used for this case.

The areas observed were so selected as to be conspicous enough to enable one to set on them without offset procedures. The photomultiplier responses were obtained at the present wavelengths of the lines and the nearby continuum as well, by letting the D.C. amplified deflections be recorded for a sufficiently long time. In reducing the observations, a precision planimeter was used to integrate the signals; elimination of noise fluctuations was attained in this way.

The fundamental calibration of the observations was based on the standard stars γ Gem, α Leo and ε Ori (Oke, 1964), which as pointed out by Code (1959) might contain errors as large as 0.1 mag. in absolute value. The internal consistencies of the nebular measures, however, indicate an accuracy of the order of five per cent for the relative intensities. Regarding the absolute values of the flux received outside the atmosphere that is not the case. The mean error of the energy fluxes in absolute units is fourteen per cent, for points numbers 3, 5 and 7 probably as a result of errors in the settings. Absolute fluxes, for other more easily identifiable points, have a mean error around 7 per cent, on the average.

III. The reddening correction by the intensity ratio of the hydrogen lines

As it is well known, self-absorption of the Balmer lines can take place if the population of the 2p level is highly increased by resonance scattering of Ly-α radiation, if the medium has a sufficiently large optical depth τ_o. As pointed out first by Greenstein (1954) and mathematically proved by Pottasch (1960) self-absorption could alter significantly the hydrogen recombination spectrum. Consequently, if one desires to use the recombination lines for obtaining the reddening correction, a critical analysis of the self-absorption process, as a function of the number of scatterings, Q, suffered by a Ly-α photon in the nebula, must be considered.

The probability of absorption of a H_α photon, per H atom, is:

$$\lambda_\alpha = \frac{I_{H\alpha}\ B_{23}}{A_{21}} \tag{1}$$

where $I_{H\alpha}$ is the intensity of H_α line radiation produced in the nebula, given by:

$$I_{H\alpha} = N_e N_{H+}\ c\alpha_{32} \frac{h\nu_\alpha}{4\pi\Delta\nu_\alpha} \cdot \tag{2}$$

in the last equation α_{32} is the effective recombination coefficient and $\Delta\nu_\alpha$ is the frequency width of the line. According to Seaton (1959), at $T_e = 10^4$ °K, $\alpha_{32} = 8.86 \times 10^{-14}$ cm^3/sec.; when microturbulent motions of about 10 km/sec are considered, the Dopper width is, for an electron temperature of 10^4 °K, 1.35×10^{11} sec^{-1}. $\Delta\nu_\alpha$ is taken equal to the latter number one has:

$$\lambda_\alpha = 4.31 \times 10^{-23}\ N_e^2 \tag{3}$$

The number of H_α quanta absorbed in a unit volume viz, a cylinder of unity area and height L, is thus:

$$P = Q\lambda_\alpha\ (I - X)\ N_e^2\ \alpha_B L \tag{4}$$

Since for a nebula which is optically thick for Lyman radiation, (the so-called case B), the number of quanta in the Balmer lines is equal to the total number of Ly-α produced. However, one must distinguish between 2p → 1s and 2s → 1s transitions; the X gives the fraction of Balmer photons which eventually lead to the two-photon emission, produced by the 2s → 1s transition. For 10^4 °K, X is equal to 0.34; α_B is the total recombination coefficient, which has been computed by Seaton (1959). On the other hand, the energy absorbed in H_α, per the unit volume is, with the usual notation

$$Ph\nu_\alpha = 4\pi I_{H\alpha}\ N_{2p}\ L \frac{\pi e^2}{mc} f_{2p} \tag{5}$$

Introducing the proper numerical constants and using equations 4 and 5, the optical thickness at the center of H_α is given by:

$$\tau_{H\alpha} = 3.65 \times 10^{-22} N_e^2 \, k_{H\alpha} \, L \, Q \qquad (6)$$

If the resonance scattering is strictly coherent, the *Ly-α* quantum path inside the nebula, is merely described as a random walk. The number of scatterings suffered by a *Ly-α* photon before leaving the nebula is τ_o^2. Adopting the value of 10^5, given by Pottasch (1960), for the optical depth at the center of the line, one finds that an average *Ly-α* photon bounces back and forth between the 1s and 2p levels 10^{10} times before it can escape from the nebula. This number would indicate a large population of the 2p level, as one can directly notice from equation (6). More specifically, if one adopts 1500 cm^{-3} as an underestimate of the average central electron density for the Orion Nebula, with $L = 1/3$ pc., the optical depth at H_α is about 25. According to Capriotti's calculations (1964), in which the processes of recombination and self- absorption are considered, the Balmer decrement obtained with that optical depth is significantly different from the decrement predicted on grounds of pure recombination, as shown in Table 1. In this table the unreddened relative intensities of some hydrogen lines observed in this work are also given. The latter values were derived on the assumption that the Paschen to Balmer intensity ratio is not altered by self-absorption of the Balmer lines. This tacit assumption is proved in what follows.

Table 1

Line	*Pure Recombination (Te = 10 000°K)*	*Self-Absorption* $\tau_H = 25$	*Point (1) Observed Ratios*
10938	9.0	11.6	8.9
10049	5.8	7.4	5.4
9229	3.3	4.1	3.1
6562	(272)262	347	257
4861	100	100	100
4340	(50.6)48.9	55.1	48.7
4101	(29.8)27.6	32.6	27.4
3970	(19.2)17.2	20.7	24.0
3889	(13.2)11.2	13.8	15.5
3835	(9.5)	11.6	9.3
3798	(7.2)	8.7	6.1
3770	(5.4)	6.6	4.8

Although the direct comparison of the Balmer decrement, observed in the present investigation, with that predicted by theory would provide a sound test for the existence of self-absorption, a more direct approach to the problem can be obtained from the fact that H_α in absorption has never been observed in the spectra of the stars embedded in diffuse nebulae (Struve *et al.,* 1939). From this evidence, one can directly derive the population of the 2p-level and compare it with the values derived above. If one assumes that the minimum detectable line in absorption has an equivalent width of 0.01 Å, using the linear part of a suitable curve of growth, (Unsöld, 1955), one obtains for the abscissa.

$$\log \frac{N_{2p} \, L \, f_{2p}}{\Delta\omega_o} = 0.5$$

with $f_{2p} = 0.71$, one obtains the condition

$$N_{2p} \, L \leqq 7 \times 10^{10} \text{ or } \tau_{H\alpha} \leqq 0.22 \qquad (7)$$

The upper limit for the optical depth, $\tau_{H\alpha}$ derived in this way is two orders of magnitude lower than the value derived above. This result clearly indicates the non-existence of coherent *Ly-α* scattering within the nebula. Furthermore, the possibility of significant self-absorption of the Balmer lines can be ruled out for the case of the Orion Nebula.

As it will be shown in a later paper, the electron temperature in the densest regions of the nebula is never higher than 10^4 °K; therefore, one can also neglect the collisional excitation mechanisms. In a conclusion, one can safely state that the hydrogen nebular spectrum is predominantly produced by pure radiative recombination processes. Consequently, the Paschen to Balmer line in-

Table 2

	Point 1		Point 2		Point 3		Point 4		Point 5		Point 6		Point 7		Point 8		Point 9		Point 10	
Hydrogen Lines																				
10938	17.8	8.9	21.0	10.0	33.4	9.4	20.4	9.8	15.0	10.0	20.5	8.4	19.2	8.2	22.4	9.4	22.0	9.4	—	—
10049	10.1	5.4	10.5	5.6	16.1	5.2	10.5	5.6	—	—	11.1	5.8	13.2	6.4	11.3	5.4	11.9	5.7	—	—
9229	4.7	3.1	5.0	3.2	6.8	2.9	4.5	2.6	Too weak		6.2	3.5	6.7	3.5	5.4	3.1	5.5	3.3	—	—
6562	346.6	257	375.3	270	464.4	265	421.2	300*	394.2	320*	432.0	288*	401.3	268	441.7	300*	437.4	295*	—	—
4861	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
4340	40.8	48.7	41.5	49.8	32.6	45.6	40.1	48.5	41.1	46.0	36.1	45.0	36.1	46.0	36.8	47.0	36.8	46.5	36.9	48.7
4101	20.9	27.4	22.4	29.6	17.2	27.6	18.9	25.4	23.9	27.6	19.9	28.0	20.2	28.0	20.4	27.0	20.6	28.2	20.3	29.8
3970	17.4	14.0	15.0	21.0	—	—	14.0	19.8	—	—	13.0	19.7	15.6	23.1	14.6	21.9	14.5	21.0	15.0	24.8
3888	10.9	15.5	10.2	14.7	9.0	17.0	9.3	13.5	—	—	10.1	16.0	11.4	17.4	11.5	17.5	9.4	14.2	10.3	17.6
3835	6.3	9.3	6.6	9.6	5.0	9.5	5.3	8.0	—	—	5.9	9.2	5.7	8.8	5.9	9.0	—	—	5.5	9.4
3798	4.1	6.1	5.1	7.4	—	—	4.4	6.5	—	—	3.1	5.1	3.4	6.0	3.6	5.7	—	—	2.9	5.1
3770	3.2	4.8	3.4	5.2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
3750	2.4	3.6	3.2	5.0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Helium Lines																				
10830	80	40.5	115	54	175	56	78	39	45	30	70.0	29.0	150.0	65.0	120	54	104	51.0	—	—
7065	11.3	8.6	—	—	11.9	9.1	12.7	8.5	—	—	—	—	—	—	—	—	12.7	8.3	—	—
5876	12.0	9.8	11.1	9.1	17.7	12.1	15.4	12.6	10.4	7.8	13.1	10.1	14.3	11.0	12.5	9.9	15.4	11.8	—	—
4471	3.5	4.1	3.6	4.2	3.5	4.5	3.8	4.3	3.5	3.9	3.2	3.8	3.2	3.8	3.8	4.4	3.3	3.9	3.4	4.2
4026	1.8	2.4	1.7	2.3	1.4	2.4	1.6	2.1	—	—	1.5	2.1	1.6	2.2	1.9	2.6	—	—	1.6	2.4
3889	—	4.3	Ext	3.5	Ext	5.8	Ext	2.3	—	—	—	4.8	Ext	5.2	Ext	6.3	Ext	3.0	Ext	6.4
O III Lines																				
5008	360.0	357	324.3	315	329.0	309	148.5	227	148.5	147	159.8	152.0	289.5	277.0	291.4	280.0	246.3	238.0	290.5	266.0
4959	117.1	115	103.7	105	105.6	104	76.8	78	46.1	45	49.9	49.0	97.9	97.0	89.1	87.5	80.5	79.0	97.0	94.0
O II Lines																				
7325	10.3	7.5	14.9	9.7	17.6	8.8	16.0	10.5	—	—	15.9	9.5	15.0	9.1	12.9	8.1	15.7	9.9	—	—
3727	66.3	109	139.4	220.0	45.9	102	120.7	194.0	164.9	212	150.5	263.0	68.0	117.0	112	190.0	13.9	240.0	90.1	170.0
NE III Lines																				
3967	Ext	6.8	Ext	3.8	Ext	—	Ext	2.6	—	—	Ext	2.5	Ext	5.9	Ext	4.7	Ext	3.8	Ext.	5.3
3868	15.5	22	7.5	11.1	7.4	20	6.4	9.2	—	—	4.1	6.5	11.6	17.7	8.7	13.3	6.0	9.2	10.5	17.1
S III Lines																				
9532	125.0	77.0	190.0	137.0	82.2	34.5	99.5	60.5	78.0	59.0	133.3	72.0	236.0	127.0	190.3	108.0	251.3	142.0	—	—
9069	49.5	31.2	48.1	30.0	74.2	32.0	47.9	30.3	16.5	13.0	58.8	33.0	97.4	54.0	70.6	41.0	95.2	56.3	—	—
S II Lines																				
6724	4.9	3.6	10.1	7.0	0.8	0.4	19.0	13.3	—	—	21.6	14.0	21.6	14.0	15.4	10.2	20.2	13.4	—	—
4072	2.2	3.1	1.4	1.9	—	—	1.9	2.6	—	—	1.7	2.4	1.9	2.6	—	—	—	—	—	—
A III Lines																				
7135	11.4	8.0	16.0	11.0	16.8	8.6	15.0	10.3	6.6	5.3	13.3	8.4	17.8	11.2	12.3	8.0	15.7	10.2	—	—

* Non corrected for [NII] emmision.

tensity ratio could provide a sound way to determine the local reddening corrections, at any point on the nebula. The actual procedure has already been outlined by Aller and Liller (1959). The theoretical values for the intensity rations have been computed by Burgess (1958), who took in account the orbital degeneracy of the hydrogen atom.

The lines used in the present work are: the two lines arising from the level n = 6; namely λ10938 and λ4101; and the pair originating from level n = 9; that is, λ9229 and λ3835. Those four lines are fairly strong and free of blending lines, a difficulty that prevents the use of other members of the series. Moreover, the atmospheric molecular absorption does not effect the intensity of the relevant lines significantly. The observed ratios are larger than the theoretical values by a factor, which represents the differential absorption, in the spectral interval bounded by the corresponding lines. If, for pure numerical convenience, the absorption at λ4861 is set equal to zero, one can fit by standard numerical means a reddening curve to the values of differential selective absorption obtained by direct comparison. The reddening corrections at other wavelengths can then be derived graphically. Such a curve, derived from the observations of point 1. is shown in Figure 2. The so-called classical reddening curve has also been plotted. At this stage it must be pointed out that if there were significant photometric errors present in this investigation, these errors would be reflected in the P/B curve, effecting its shape.

As it can be seen, the two curves look alike in the red region of the spectrum, however if one employed the Whitford curve to correct for extinction, the reddening in the ultraviolet part would be underestimated. In figure 2, the arrows on the P/B curve indicate the wavelengths of the corresponding pair of H lines used. The parts of the curve outside the region limited by arrows represent a reasonable extrapolation only.

In Table 1, the observations of point 1 are compared with the theoretical recombination values, as given by Burges (1958). The numbers in brackets, represent the intensity values computed

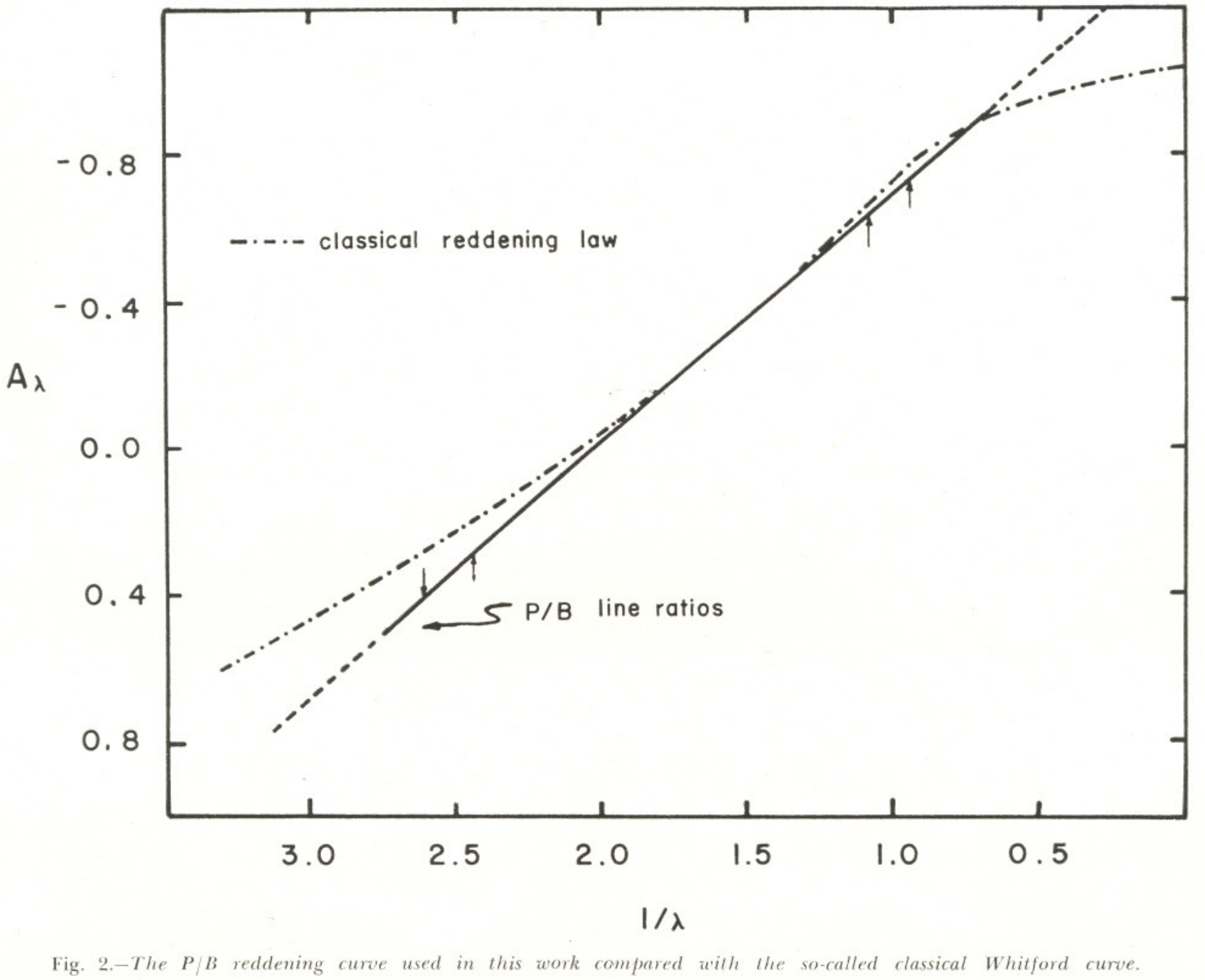

Fig. 2.—*The P/B reddening curve used in this work compared with the so-called classical Whitford curve.*

under the assumption that levels are populated according to their statistical weights. This latter assumption might not be well grounded for the case of the Orion Nebula, where the electron density is not high enough. In fact the observations do not support it. As seen in the table, there are systematic discrepancies in the ultraviolet lines; however, due to the crowding of the lines in that spectral region the accuracy, there, is lower than in the zone of longer wavelengths.

For other points the reddening correction was determined in the same way, by constructing a reddening curve for each region. For points not observed with the two cells, the shape of the curve for point 1 was used, and the scale fixed by the relative intensity of H_γ and H_δ in the blue and with the intensity of $\lambda 10049$ in the red. Some results of the observations are given for 10 points, in table 3. Both unreddened and reddened values of the observed intensities are presented. The analysis of this table will be carried out elsewhere.

Table 3

Point	Flux 10^{10} ergs $cm^{-2} sec^{-1}$	$S \times 10^3$ ergs $cm^{-2} sec^{-1} steradian^{-1}$	$Ne^2 L \times 10^{-6}$ cm^{-6} parsec	Distance seconds of arc
1	13.0	36.0	1.18	45
2	9.7	10.1	0.68	68
3	12.0	10.0	0.34	32
4	9.2	22.4	0.76	104
5	0.5	3.0	0.10	151
6	4.0	14.4	0.49	105
7	6.3	11.2		87
8	3.4	9.5	0.32	100
9	2.1	6.2	0.21	115
10	11.5	26.1	0.88	55
11	4.8	10.7	0.36	120
12	8.1	22.8	0.77	89
13	3.2	9.0	0.30	70
14	0.1	0.6	0.02	185
rim	2.2	6.4	0.22	125

IV. The absolute value of the absorption.

With the curve obtained above, one only derives relative intensities; that is, the ratio of the intensity of a given line to that of H_β. However, for the determination of absolute fluxes, one must determine the amount of light absorbed by dust at any of the wavelengths observed. In other words, one must obtain the intersection of the P/B reddening curve with the line, where $1/\lambda = 0$. Since a regional curve has been adopted, none of the standard techniques can be used for this problem. Consequently, a new method needs to be developed. The radio observations, which are not affected by interstellar absorption, provide the starting point.

As Stromgren (1951) has shown, the knowledge of the emission measure can lead to the determination of the electron density, provided that the absorption in absolute units is given. Once the electron density is obtained, the radio flux can be easily computed and compared with the observed flux. By trial an error one can in principle obtain the value of the absorption, which best fits the observations.

An extra parameter must be included in the problem. As a direct inspection of Figure 1 clearly indicates, the Orion Nebula is far from having a homogeneous distribution in density. Clumpiness of gas is definitely observed. The effects of such density fluctuations are of utmost importance in the interpretation of the observations, both in optical and in radio regions.

The determination of surface brightness becomes an easy problem when the scanner observations are used. Due to the intrinsic linearity of the photoelectric methods, the deflections obtained on the paper recorder can be immediately converted into absolute energy units, since the deflections obtained when scanning of the standard stars provide the relation flux-deflection necessary to calibrate the instrumental scale. Intensities in absolute units can now be readily obtained for the lines of the nebular spectrum. Table 3 gives the flux coming from all points observed, at the wavelength of H_β. This energy flux is as received at the Earth; then it must still be corrected for absorption.

The surface brightness, being independent of distance, is a more useful quantity. It is obtained by dividing the flux by the solid angle subtended, by the area observed, at the telescope. The values thus derived appear in the third column of Table 3. The same quantity has been obtained by other authors in the past. Their values refer to a larger area than the ones covered with the scanner apertures, yet they always cover the brightest region in the nebula; therefore, those determinations can be compared with the observations of point 1. Ambartsumian (1933) obtained a value of 0.022, later Strömgren (1951) found a much higher value of 0.16 and, more recently, Wurm (1961) derived a surface brightness of 0.032, in perfect agreement with the present determination of point. 1.

The surface brightness at H_β is given by the equation

$$S = \frac{N_e^2 L}{4\pi} \alpha_{4,2} \; (T) \; h\nu_\beta \tag{8}$$

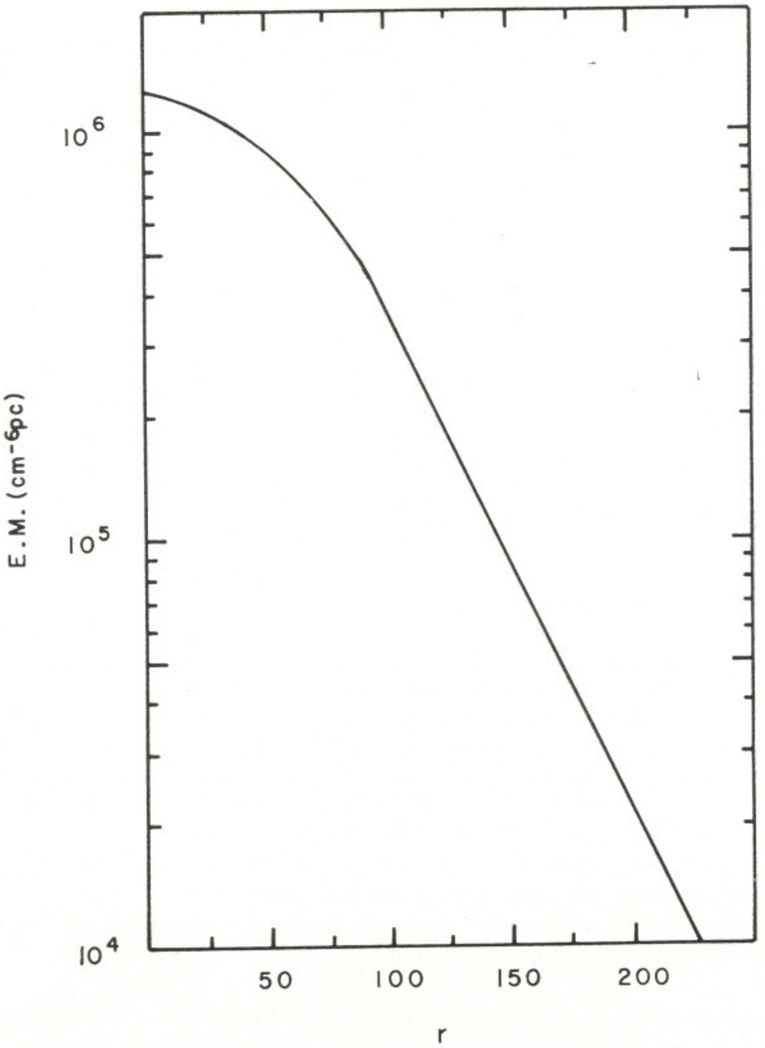

Fig. 3.—*The reddened emission measure as a function of the projected distance on the Nebula.*

$\alpha_{4,2}$ is the effective recombination coefficient, (Seaton, 1959); L is again the path along which the line is formed. The product N_e^2L is the so-called emission measure. Here, the electron temperature will be taken equal to 10^4 °K for all the computations. The emission measure is also given in Table 3, and plotted in Figure 3 as a function of distances.

Osterbrock and Flather, utilising the intensity ratio of the [OII] lines, λ3726 and λ3729, determined the electron density distribution of the nebula as a function of the distance from the center. Assuming homogeneity in the nebula, they then computed the radio flux at six different wavelenghts. The computed fluxes were several times the observed; therefore, the idea of density fluctuations was introduced. The fluxes were recomputed using now an effective radius αL. In this way, α is defined as the fraction of the length of the average line of sight occupied by condensations; the optical depth is now decreased relatively to the homogeneous model. Using a constant value α of $1/30.3$, the agreement with the observations was better, although not complete. The origin of the remaining discrepancy could lie in the assumption of the constancy of α. As Pariskii (1962) found, from the radio brightness, α increases toward the center. Peimbert (1962), by trial and error, derived an empirical formula for α, which for regions close to the center can be written as:

$$\alpha(r) = \frac{1}{1 + 5.0\, r^{3/2}} \tag{9}$$

Here r is the projected distance to the center of the nebula in minutes of arc. With a similar formula, Peimbert obtained complete agreement with the radio observations.

The square of the density can be obtained through the emissions measure observed; then a comparison with the projected density, as given by the [OII] lines, can provide the real amount of absorption. In order to do so, let the nebula be considered as a spherical cloud with radius R, and assume that the density can be expressed as a function of r, the distance from the center.

Then the reddened emission measure E, at point P as derived through equation 8, located at r minutes of arc from the center is:

$$E(r) = \int_0^{(R^2 - r^2)^{1/2}} N_e^2(s)\, e^{-\tau(s)}\, ds \tag{10}$$

where $e^{-\tau(s)}$ represents the extinction due to the dust component of nebula particles. The solution of this integral equation, which is a Volterra equation of the first kind, can be solved by numerical methods, leading to the knowledge of the function $N_e^2(r)$. The solution would require $\tau(s)$ to be given, which in turn would imply that the density distribution of the solid component is known. Although it is clear that the dust is heavily concentrated toward the center, its distribution as a function of the density cannot be determined. As a consecuence, one approximation to simplify the problem must be introduced; namely that $\tau(s)$ is a constant. The exponential is then transferred to the left side of equation (10), and the unreddened emission measure is obtained, through the factor $e^{\tau(s)}$, to be determined. Let this factor be called K.

$$K\; E(r) = \int_0^{\sqrt{R^2 - r^2}} N_e^2(l)\, dl \tag{11}$$

The integration is carried over half the sphere only, since that is what is actually observed. For otherwise, the far side of the nebula could be observed. However, when theoretical radio fluxes are to be computed, the whole sphere must be considered. Using 1 as a variable, one has the classical Abell equation, the solution of which is well known; Then, $N_e^2(l)$ is given by:

$$N_e^2\ (l) = \frac{2K}{\pi} \int_l^R \left| \frac{dE(r)}{dr} \right| \frac{dr}{(r^2 - l^2)^{1/2}} \tag{12}$$

Following the same arguments given by Osterbrock, the density squared given by (12) must be equal to the proyected density $N_e^2(r)$, obtained through the [OII] doublet, multiplied by the condensation parameter α.

The values of $\dfrac{dE\ (r)}{\mathrm{dr}}$. can be determined directly from Figure 3. The integration was carried out using the scale $1' = 0.131$ parsec. The values for $r > 4'$ were obtained by a rough extrapolation ($E(r)$ decays almost exponentially).

The parameter $\alpha(l)$ introduced by Osterbrock is then merely the ratio:

$$\alpha(l) = \frac{2\mathrm{K} \displaystyle\int_l^R \left| \frac{dE(r)}{dr} \right| \frac{dr}{(r^2 - l^2)^{1/2}}}{\pi [N_e^2(l)]_{[\mathrm{OII}]}} \tag{13}$$

Table 4

Distance (min. of arc.)	$Ne^2(1) \times 10^{-6}$ (eq. 8)	$\alpha^{-1}(1)$ (eq. 9)	$Ne^2(1) \times 10^{-6}$ (OII) lines	K	Total Absorption at H_β (mag.)
1	3.68	6	108	4.9	1.70
2	0.70	14.2	34	3.3	1.30
3	—	27	4.2	2.1	0.82
4		41	3.6	—	—

In this equation the unknown quantity is K, since α is given by equation (9). Integrating numerically, one can determine K, which is the value of the absorption at various distances from the center of the nebula. Table 4 gives the results.

As it can be noticed, the absorption decreases with distance, a very well known observational fact. This change in K supplies additional support to the variation of the parameter α, namely, that it decreases outward. One can then conclude that the nebula is more homogeneous in the center than in the outer regions. It is clear that the density distribution depends largely on the initial condition (before the 06 star was born). However, this increasing inhomogeneity in the outer layers could be explained in terms of the interaction of the expanding gas with the highly disordered medium around the nebula. This kind of interaction would certainly corrugate the ionization front, leading to fragmentation. Consequently, the closer to the ionization front the larger the inhomogeneity in the electron density.

From the numbers shown in Table 4, one can find, by extrapolating the value of the total absorption at the center of the nebula, that is, at the position of the Trapezium stars. If this quantity is multiplied by the factor $4861/\lambda_v$, where λ_v is the effective wavelength of the V filter, one obtains for the total visual absorption.

$$A_v = 1.8$$

The mean color excess (for B-V) of the four Trapezium stars is close to 0.37, if one adopts the photometry carried out by Sharpless (1962). The ratio of those two numbers is 4.9 —which is the value of R— and can be compared with other determinations. In fact, this quantitly R has been the subject of several investigations, some of which clearly indicate a deviation from the so-called classical absorption curve (R as derived from this curve is 3.0). Sharpless (1952) studied this ratio of total to selective absorption, for several stars inmersed in the Orion Nebula, and found a value of nearly 6.0. More recently, Walker (1964) using H-γ narrow band photometry data redetermined R for some early type stars in the nebula. Taking his results for just the Trapezium stars, one obtains an average value of 5.0 By the techniques of multicolor photometry Johnson (1965) finds the same number. Therefore, the two latter determinations are in excellent agreement with the present investigation, account taken of all the simplifying mathematical assumptions made above. Moreover, one can notice that for θ^2 Ori, and O9 star placed at about $2'$ from the center, the absorption at λ_v is 1.18. The color excess for this star is about 0.25 (Sharpless, 1952); one then obtains for R a value of 4.7 indicating that this quantity might apply to the entire Orion Nebula.

These results can be also compared with those obtained by direct radio observation. Thompson and Krishnan (1965), with a beam width of $1'.0$, have observed the Orion Nebula at a wavelength of 9.1 cm. Their observations permit the calculation of the emission measure over a region of radius $0.'65$, obtaining a mean measure of 7.6×10^6. Using the data from tables 3 and 4, one can compute the corresponding quantity, deriving a mean unreddened emission measure of 6.5×10^6 cm^{-6} pc. which is very close to the value observed.

The agreement of the results derived in this work with those obtained by such a wide variety of techniques, strongly supports the existence of a law of absorption for the Orion Nebula which is different from the one predicted by the classical reddening curve.

However, there have been some attempts in the past tending to show that the same classical law of extinction holds even for the stars embedded in emission nebulae (Divan, 1954, Underhill, 1964). This interpretation is difficult to accept, because uniqueness of the extinction law would require also the same nature of dust particles everywhere. The larger radiation pressure present in HII regions would certainly affect the mechanism of growth and destruction of solid grains; therefore, a different mechanism of reddening must hold for the Orion Nebula. Although, according to the present results, this seems to be the case, the possibility of unseen red companions remains an open question. Nevertheless the latter effect might not be sufficient to account for the high value of R.

V. Density fluctuation effects on the [OII] lines.

The computations carried out above have established the character of the density fluctuations, namely that the nebula is more homogeneous close to the center than in the outer regions. However, it was assumed that the density outside the condensations was zero; this hypothesis was adopted in order to show that the apparent desagreement of the optical and radio observations can be satisfactorily explained in terms of density fluctuations. One can make a more realistic estimate of the extent of those fluctuations, if it is now considered that the density outside the condensations does not vanish. Let N_e be the electron density of those condensations, as given by the [OII] lines and let N_e' be the density of the outside. This difference in densities would directly affect the intensity ratio of the nebular and the auroral lines of [OII]. Since the unreddened ratios are available from Table 2 for some points in the Nebula, and the electron density within the condensations is known, (Osterbrock and Flather, 1959), N_e' can be computed.

Using an optical path $\alpha' L$ for the condensations, and consequently $(1 - \alpha')\, L$ for the path in between, the intensity ratio of the nebular to the auroral lines of [OII] becomes:

$$\frac{I_n}{I_a} = \frac{\boldsymbol{A}_2 (N_e)\alpha' b_2 (N_e) + (1-\alpha')b_2 (N_e')\ \boldsymbol{A}_2 (N_e')}{\boldsymbol{A}_3[\alpha' b_3 (N_e) + 1 - \alpha')\ b_3 (N_e')]}\ e^{\frac{1.44}{\lambda_a}} \tag{14}$$

Here the quantities $\boldsymbol{A}_2$ and $\boldsymbol{A}_3$ involve the optical transition probabilities, the wavelengths, the statiscal weight of the upper level and the factor $p_n\ (N_e)$, (Méndez, 1965), which takes into account the collisional deexcitation; that is,

$$\boldsymbol{A}_i\ (N_e) = \sum_n \frac{A_{ni}\ \omega_n}{\lambda_n}\ p_n\ (N_e) \tag{15}$$

The summation is carried over all possible lines arising from the term i; λ_a is the mean wavelength of the auroral lines. All calcualtions have been done using an electron temperature of 10^4 °K. The atomic constants of the [OII] ion, given by Seaton and Osterbrock (1957), were used. Since the densities involved are small, the de-activation coefficient, can be taken as unity for all the auroral lines. One thus obtains:

$$\boldsymbol{A}_3\ (\mathrm{OII}) = 1.41$$

The nebular lines have transition probabilities of the same order of magnitude as the collisional de-activation rate; then the factor p is less than unity. If one neglects the interaction between levels, one has

$$\boldsymbol{A}\ (N_e) = (193 p_{3726} + 68.5 p_{3729})\ 10^{-5}$$

Equation (14) can now be rewritten as;

$$\frac{I_n}{I_a} = 5.08\ \frac{\alpha'(r) b_2 (N_e) \boldsymbol{A}_2 (N_e) + [1-\alpha'(r)]\boldsymbol{A}_2(N_e') b_2(N_e')}{\alpha'(r) b_3 (N_e) + [1-\alpha'(r)] b_3 (N_e')} \tag{17}$$

The quantities $b_i\ (N_e)$ are the well known factors, which take into consideration the departures from thermodynamic equilibrium. In equation 17, there are three variables involved: $\alpha'(r)$, N_e' and N_e. The value of α' must be smaller than that given in the last section because now the electron density in the space between the condensations is not zero; thus α' was taken, as starting point, for the computations like

$$\alpha'\ (r) = \alpha\ \ (r)\ \frac{N_e'}{N_e}$$

and then slightly changed. The value of N_e is given by the observations of the λ3726-29 lines. The electron density outside the condensations was fixed by trial and error until the variation in the ratio, as determined by the observations, was closely matched. Figure 4 illustrates the variation in the ratio obtained by trial and error method. Table 5 gives explicitly the results. The ratios corresponding to a nebula with a smooth density distribution (varying as $\frac{1}{r^2}$) are also given, as well as the ratios for the electron densities of the condensations. The latter are given to stress the fact that the Orion Nebula does indeed contain density fluctuations. The comparison is plotted in Figure 4.

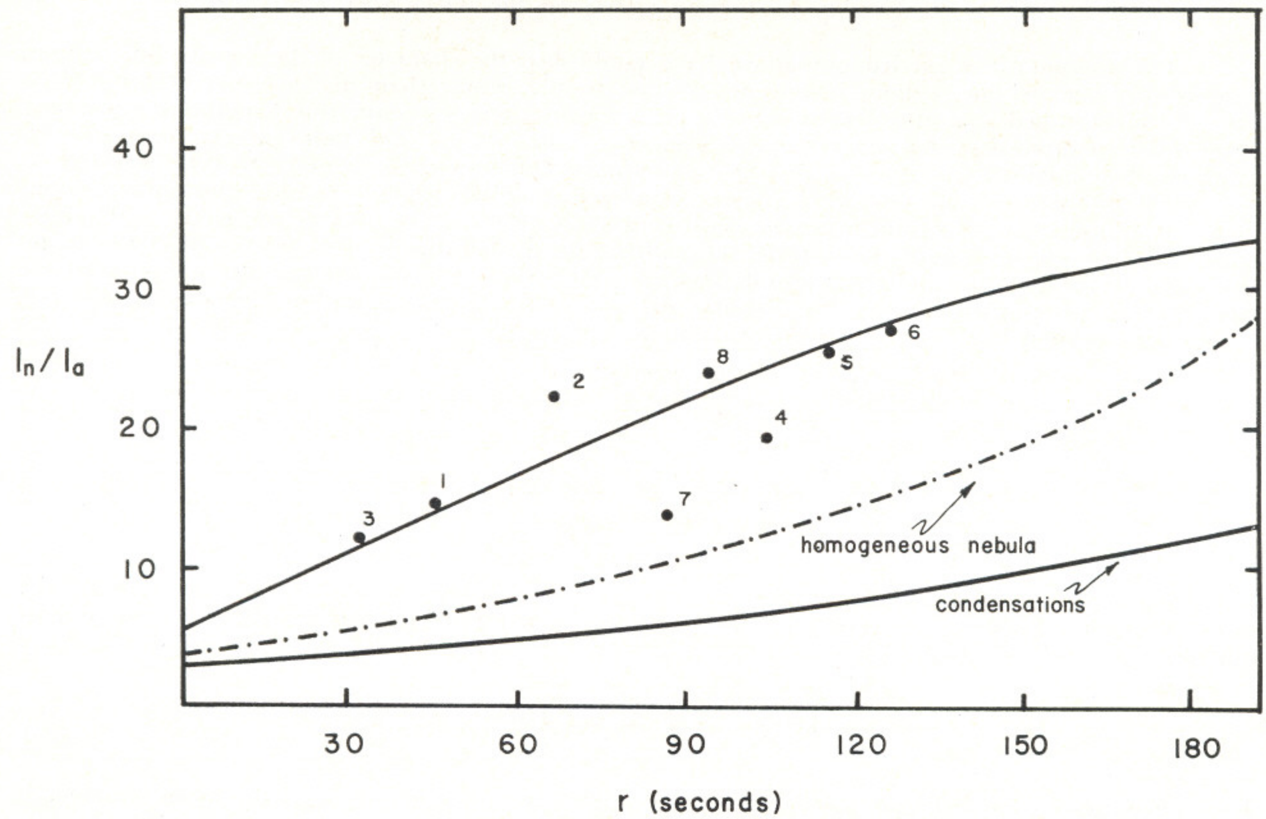

Fig. 4.—*The variation of the intensity ratio of the nebular to the auroral lines of OII is given for three cases: a) for the density distribution of condensations; b) for an homogeneous nebula; c) the distribution which is a better fit to the observed ratios (circles).*

Table 5

Distance	$N_e \times 10^{-4}$	$N_e \times 10^{-4}$	α^{-1}	$\frac{I\ (neb)}{I\ (aur)}$
0	1.8	0.35	6	6.9
1′	1.05	0.25	17	16.6
2′	0.58	0.08	98	28.1
3′	0.31	240	0.03	32.5

VI. Concluding remarks.

In the discussion presented above the electron temperature was always taken as to be 10^4 °K; within the Huygenean region the variation in electron temperature is not very large. As it will be shown in a later paper it does not vary by more than 1 000 °K from the assumed value of 10^4 °K. Therefore, the results derived would not change significantly when variations in temperature are introduced in the computation scheme. Although the conclusions and results obtained correspond to the central parts of the Orion Nebula, where the concentration of dust is higher, there are certain results which must hold for the entire nebula. The predominance of the radiative recombinations over all other processes producing the hydrogen spectrum is certainly valid. Here one must remark the implicit fact used earlier that the exciting star is in the center of the emitting gas cloud and not in front of it. If this hypothesis is accepted one can use the results obtained here to estimate the reddening correction for other nebulae as well. That is, if the spectra of the exciting stars do not show H_α in absorption, the self-absorption of the Balmer lines is not important; therefore, the Paschen to Balmer intensity ratio provides the best method to derive the differential absorption.

The estimations made above could also serve to test the validity of an alternative theory proposed for the Ly-α resonance scattering mechanisms; Osterbrock (1962), using an approximate diffusion model, has considered the case when the scattering of the Ly-α photons is not a coherent process. He has considered the general case when there is a correlation between the frequencies of the Ly-α

quantum, before and after the scattering. The correlation function used in this work was derived by Unno (1952). Those results indicate that for a nebula with $\tau_0 = 10^5$, in the center of Ly-α, one average photon is scattered about 8×10^5 times before it cat leave the nebula. Using this number for Q in equation (6) one finds that for the Orion Nebula the optical depth, at H_α, is close to 0.02 in good agreement with condition 7. Here, the effects of dust on the Ly-α radiation have not been taken into account. Evidently, the solid grains must be very effective in reducing the value of Q, although in this case the Ly-α photon does not leave the nebula but it is eliminated. For $\tau = 10^6$, value to be found in certain planetary nebula where the electron density is of the order of 4×10^4 cm^{-3}, Q becomes close to 10^7. The optical depth for H_α comes near to 10; as a consequence, for those planetary nebulae, the possibility of self-absorption cannot be ignored. As a final point, the results clearly indicate that the radiative processes, corresponding to the so-called base B, hold in the Orion Nebula.

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