BOLETÍN DE LOS OBSERVATORIOS DE TONANTZINTLA Y TACUBAYA, VOL. 4, N°29, 1967.

INTENSITY AND CONTRAST OF A HOLOGRAPHIC IMAGE

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ABSTRACT

In this paper the intensities of holographic images are described by means of graphs obtained using a digital computer. The film characteristic curve is either assumed to be linear of that the maximum and minimum exposures are both on the straight portion of this curve. The contrast of a hologram from a continuous tone object is also found. All of these results are found assuming that the photographic emulsion is thin enough to be considered as two dimensional.

Introduction

The holographic formation of images was first proposed by Gabor (1949) and later advanced by H. M. A. El-Sum (1952), but the extensive research in this field really began when E. Leith and J. Upatnieks (1963) published the procedure in order to make holograms from continuous tone objects using a gas laser. Leith and Upatnieks (1964) also made the first three-dimensional holograms and now even three dimensional color holograms have been made by several authors, like A. A. Friesem and R. J. Fedorowicks (1966) as first proposed by Leith and Upatnieks (1964).

Many papers have been published about the applications of holography, for instance to microscopy by Leith, Upatnieks and Haines (1965); to correction of lens aberrations by Upatnieks, Vander Lugt and Leith (1966); to surface deformation measurement by Haines and Hildebrand (1966); to interferometry by Hildebrand and Haines (1966), etc.

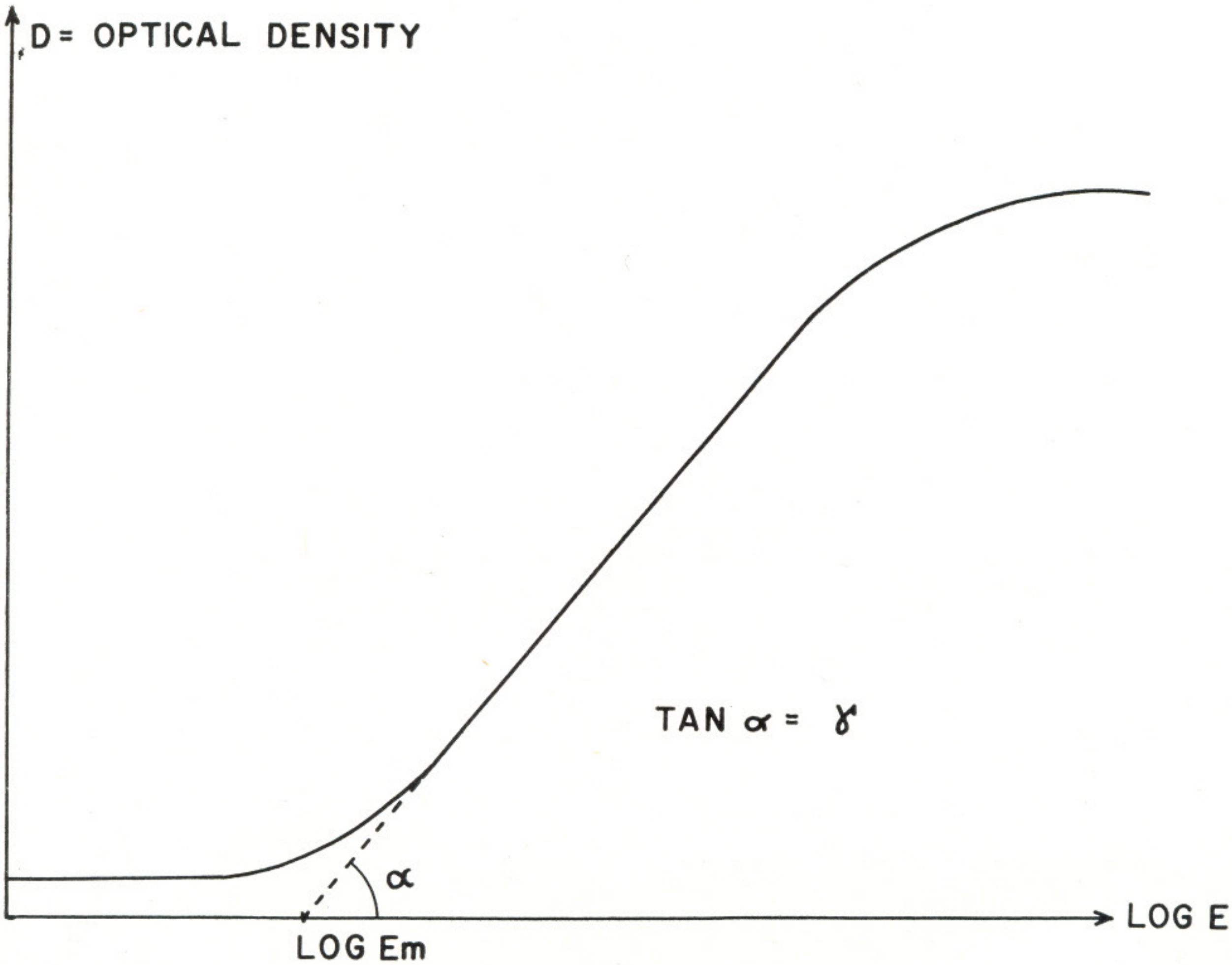

Fig. 1.—*Characteristic Curve of the Film.*

The purpose of this paper is to describe the intensities of the holographic images my means of graphs obtained using a digital computer. The contrast of the image from a continuous tone object is also found.

All of these results are found assuming that the photographic emulsion is thin enough to be considered as two-dimensional. Storage of information in thick emulsions has been considered in detail by P. J. Heerden, (1963) Y. N. Denisyuk (1963) and later by Leith *et al.* (1966).

Mathematical Treatment

The characteristic curve of the film is in Fig. 1 where the slope of the straight portion is γ and log E_m is the inertia of the film. Now, let us assume that the maximum and minimum exposures given to any part of the hologram are such that the straight portion of the curve is always used. Under these conditions the resulting amplitude film transmission is given by (J. C. Wyant, 1966).

$$T = \left[\frac{E(x)}{E_m}\right]^{-\gamma/2}, \tag{1}$$

where $E\ (x)$ is the exposure as defined by:

$$E(x) = I\ (x)\ t; \tag{2}$$

here, $I(x)$ is the intensity of the light and t is the exposure time.

If the exposure is to remain within the desired limits, if must be satisfied that:

$$E(x) > E_m. \tag{3}$$

If the amplitude of the reference beam is A_o and the amplitude of the beam to be reconstructed is A_x, it can be easily shown that the intensity $I(x)$ is given by: (see Stroke, 1966)

$$I(x) = A_o^2 + A_x^2 + A_oA_x \cos\ (\alpha\, x + \phi\ (x)) \tag{4}$$

where α gives the relative inclination of one wavefront with respect to the other and $\phi(x)$ gives the deformations of the wavefront to reconstruct with respect to the reference wavefront. The reference wavefront is not necesarily flat.

Sustituting (2) and (4) in (1), the amplitude transmission of the processed film is:

$$T(x) = \frac{A_o^2 + A_x^2 + 2A_oA_x \cos\ (\alpha\, x + \phi\ (x))}{E_m/t} \tag{5}$$

If γ is equal to -2 the analysis of this equation is very simple, but it becomes more complicated otherwise. This equation can be analized by expanding it in a series, and has been done by Stroke (1966) and M. Françon (1966) but not in the correct manner, since it was a Taylor series and not a Fourier series and even the first order term is different for both series.

Expanding in a Fourier series, the coeficients represent the amplitude of each diffraction order. As it has bee said (Wayant, 1966) there is one image for each diffraction order, but the most important ones are the two first order images.

Intensities of the Images

In order to find the intensities of the image we must first find the amplitude by expanding in a Fourier series as explained before. Let us define an average exposure as follows:

$$E_{av} = (A_o^2 + A_x^2); \tag{6}$$

which is the exposure that would be given the hologram if the light intensity were measured with a exposure meter having a large sensitive area. It is assumed to be the same for all holograms and it is therefore considered as a constant. With this definition Eq. 5 can be written as:

$$T(x) = \left[\frac{E_{av}}{E_m}\right]^{-\gamma/2} \left[1 + 2\,\frac{(E_oE_x)^{1/2}}{E_{av}} \cos\ (\alpha\, x + \phi\ (x))\right]^{-\gamma/2} \tag{7}$$

From Eq. 3 it can be shown that:

$$0 \leqslant 2 \frac{(E_o \; E_x)^{1/2}}{E_{av}} < 1; \tag{8}$$

thus, definin:

$$A = \frac{2 \, (E_o E_x)^{1/2}}{E_{av}} , \tag{9}$$

and

$$\psi = \alpha \; x + \phi \; (x), \tag{10}$$

$T(x)$ is given by:

$$T(x) = \left[\frac{E_{av}}{E_m}\right]^{-\gamma/2} \left[1 + A \; cos \; \psi\right] . \tag{11}$$

It will be assumed that ψ is such that the fringe spacing is so small that A can be considered constant in a small region containing several fringes. The Fourier series representing $T(x)$ is given by:

$$T(x) = \frac{B_o}{2} + \sum_{n=1}^{\infty} B_n \; cos \; n\psi \tag{12}$$

where Bn represents the amplitude of the orden n. For simplicity it will be assumed that there is a single well defined wavefront to reconstruct and not many as in the case where an extended object is to be reconstructed. The coefficients B_n will be given by:

$$B_n = \frac{1}{\pi} \left[\frac{E_{av}}{E_m}\right]^{-\gamma/2} \int_{-\pi}^{\pi} [1 + cos \; \psi]^{-\gamma/2} \; cos \; n \; \psi \; d\psi. \tag{13}$$

In the case of an extended object, we can consider that there are several wavefronts with different amplitudes Ex and directions ψ each interfering with the reference beam; therefore for any point on the image, the ratio of amplitudes B_n/B_o will be much smaller than predicted if there were a single point object to reconstruct. The image size is directly proportional to the order n therefore if the object is extended there should be considered an extra factor $\frac{1}{n^2}$ on the ratio of intensities $(B_n/B_o)^2$. This explains why the high order images are not easily observed.

Computation of Amplitude Ratios

Considering the object as a point, the ratio of the amplitude of its image to the amplitude of the zero order is:

$$\frac{B_1}{B_o} = \frac{\int_{-\pi}^{\pi} \left[1 + A \; cos \; \psi\right]^{-\gamma/2} cos \; \psi \; d\psi}{\int_{-\pi}^{\pi} \left[1 + A \; cos \; \psi\right]^{-\gamma/2} d\psi} ; \tag{14}$$

considering this equation and integrating numerically on a digital computer for several values of A in the range defined by Eq. 8, the curves in Fig. 2 were found. It can be seen that the ratio of intensities $(B1/B_o)^2$ is almost independent of the sign of the γ of the film, and that in increases with A. The quantity A is related to the ratio R of the intensities of the two interfering beams by means of the equation:

$$A = \frac{2 \; R^{1/2}}{1 + R}, \tag{15}$$

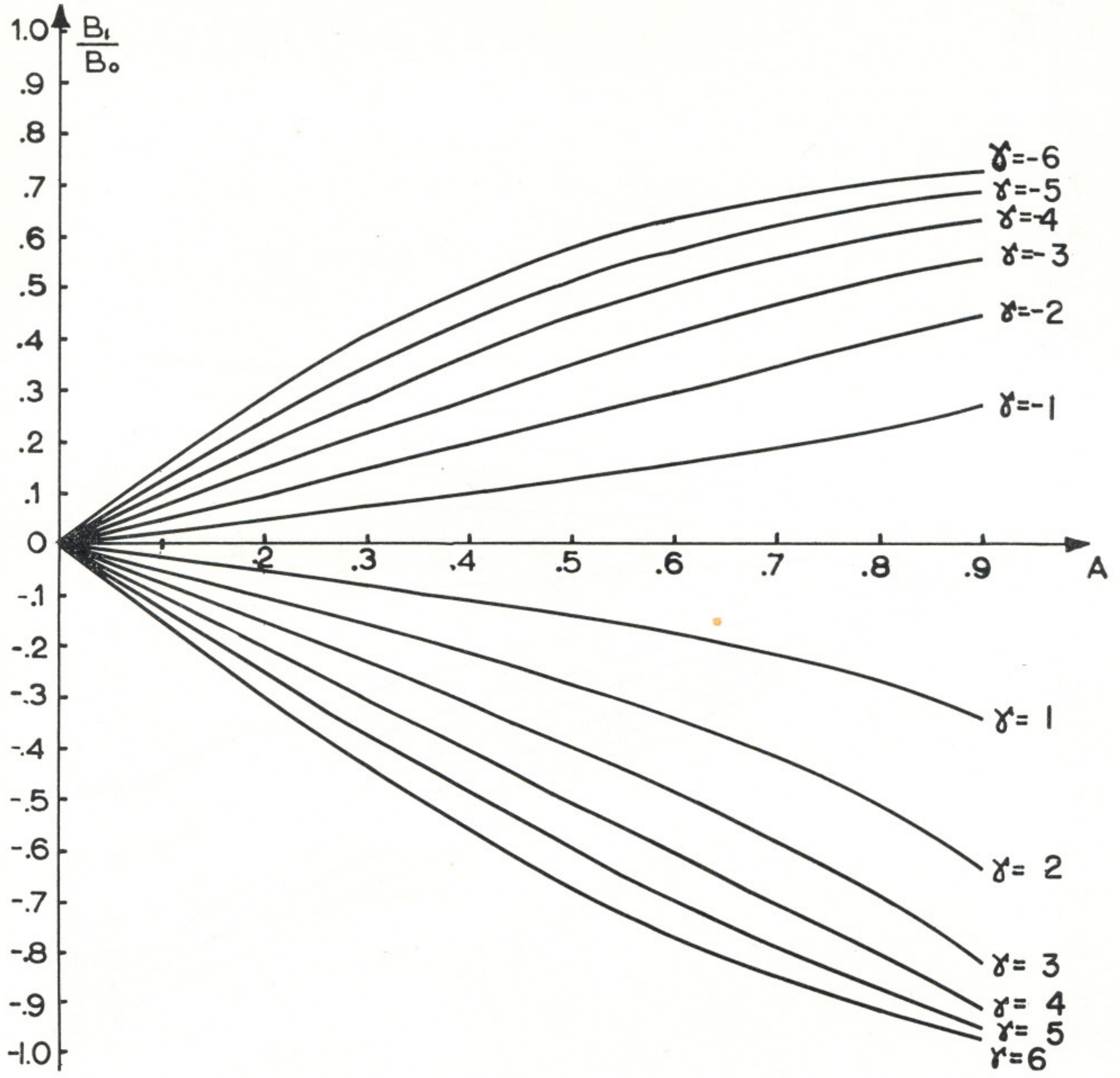

Fig. 2.—*Amplitude of the First Order Relative to the Zro Order vrs A.*

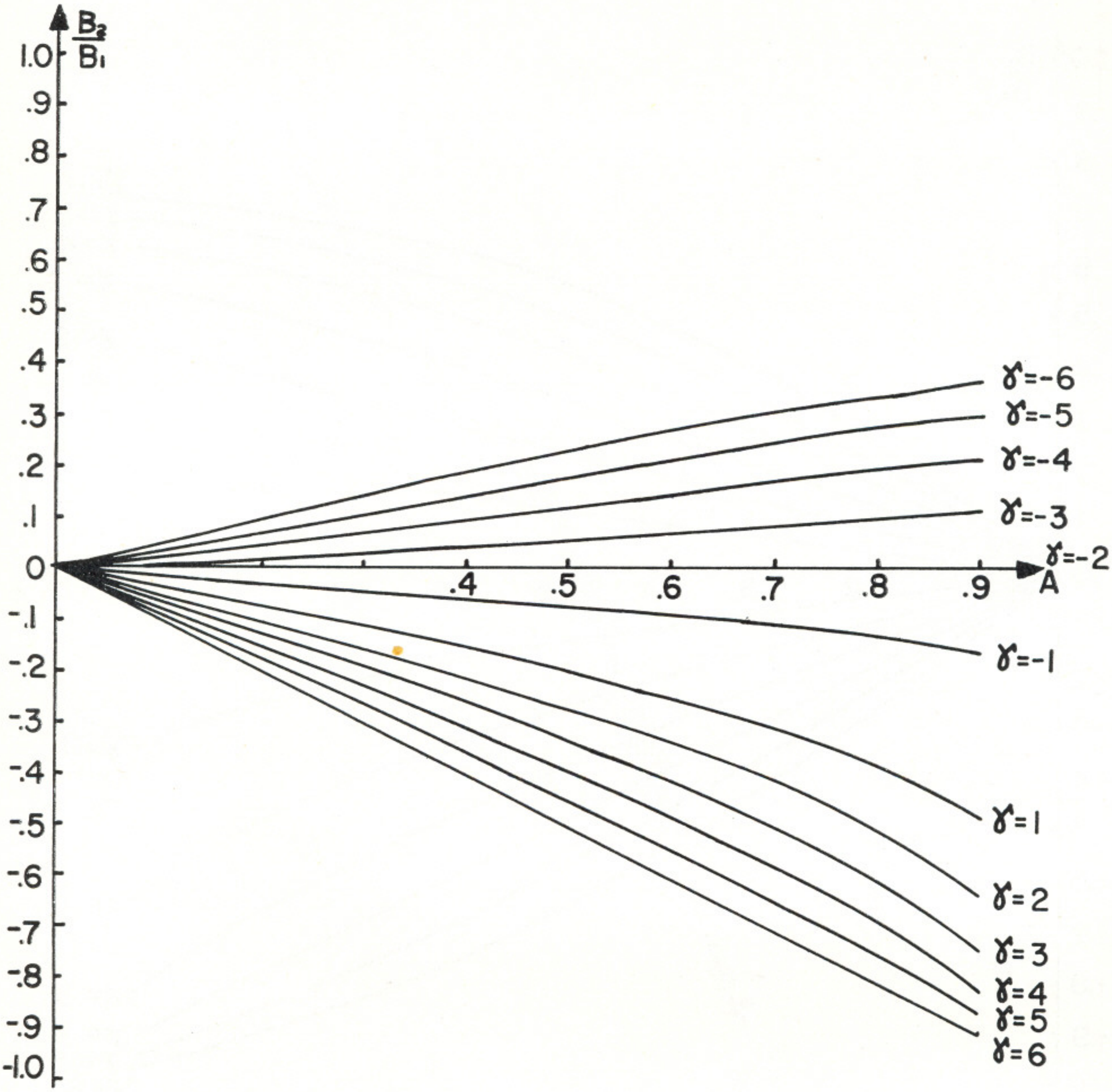

Fig. 3.—*Amplitude of the Second Order Relative to the First Order vrs A.*

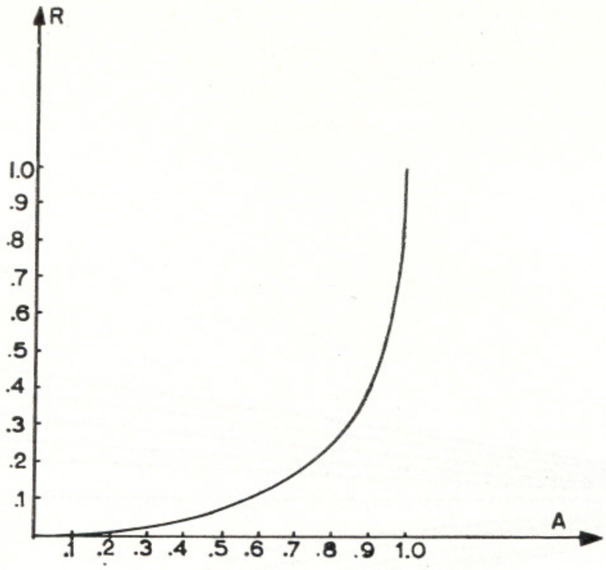

Fig. 4.—*Ratio of Intensities R vrs A.*

which was obtained from Eqs. (9) and (6). This relation is plotted in Fig. 4.

Not taking into account the mentioned factor $\frac{1}{n^2}$, the ratio of the amplitude of the second order image to the amplitude of the first order image is given by:

$$\frac{B_2}{B_1} = \frac{\int_{-\pi}^{\pi} \left[1 + A \cos \psi\right]^{-\gamma/2} \cos 2\psi \, d\psi}{\int_{-\pi}^{\pi} \left[1 + A \cos \psi\right]^{-\gamma/2} \cos 2\psi \, d\psi}, \tag{16}$$

The results obtained for several values of A and are plotted in Fig. 3 where it can be seen that the second order image does not exist when $\gamma = -2$ as it was to be expected. The photographic negative (positive γ) is seen to give more intensity to the second order image.

The absolute amplitude of the first order image, that is, without refering it to the zero order amplitude is much more complicated to obtain because of the factor in front of the integral dependes on γ and it is not exactly known. This amplitude is given by:

$$B_1 = \frac{1}{\pi} \left[\frac{E_{av}}{E_m}\right]^{-\gamma/2} \int_{-\pi}^{\pi} [1 + A \cos \psi]^{-\gamma/2} \, d\psi \tag{14}$$

The difficulty in evaluating exactly the integral arises because the ratio $\left(\frac{E_{av}}{E_m}\right)$ is not exactly known but at least a rough estimation of B_1 can be obtained setting this approximate value:

$$\log E_{av} - \log E_m = \pm \frac{1}{2} \tag{15}$$

where the positive sign is taken when γ is positive and viceversa. Therefore:

$$B_1 = \frac{1}{\pi} (10)^{-|\gamma|/2} \int_{-\pi}^{\pi} [1 + A \cos \psi]^{-\gamma/2} \cos \psi \, d\psi \tag{16}$$

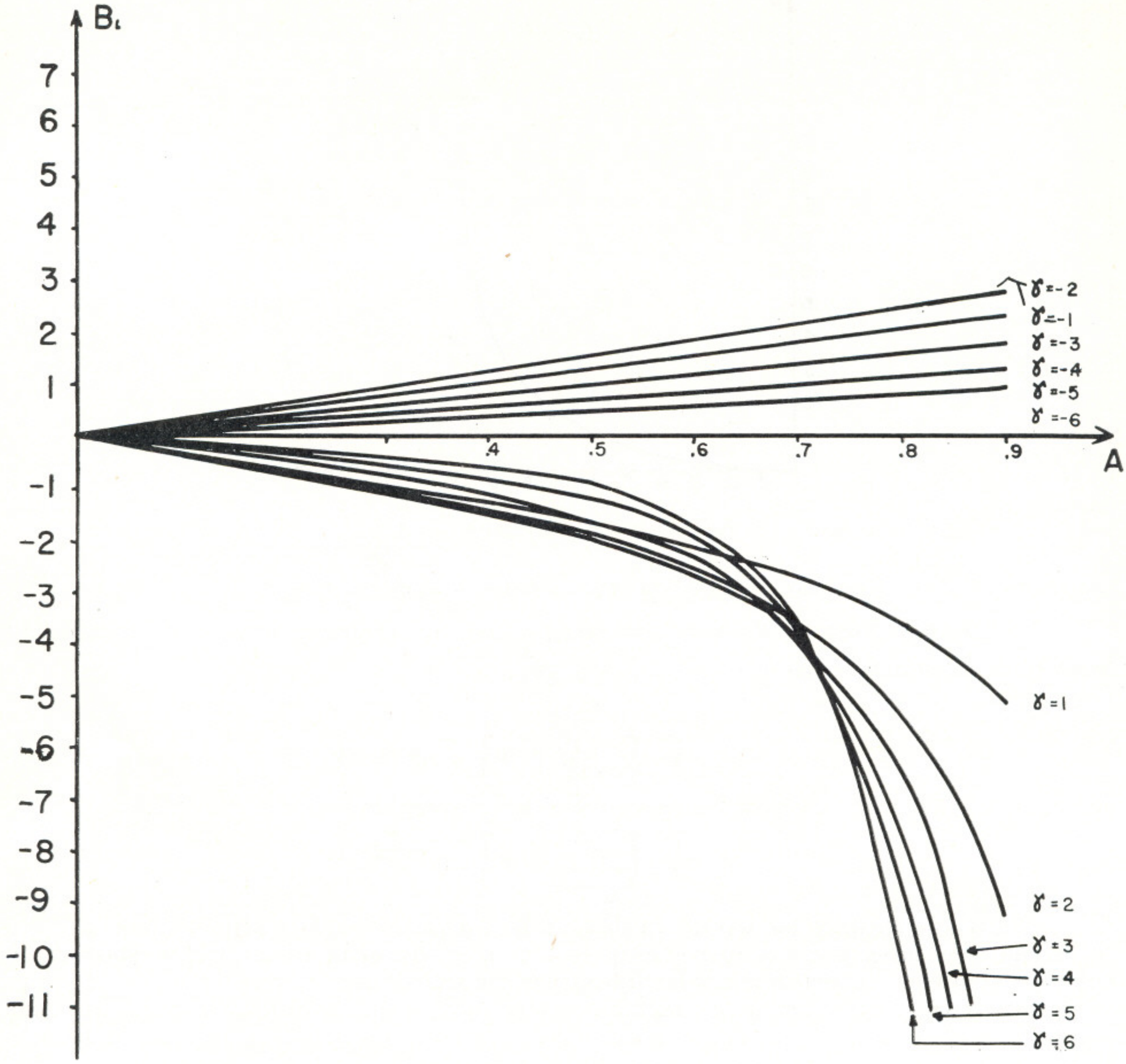

Fig. 5.—*Amplitude of First Order vrs A.*

This amplitude B_1 vrs A is plotted in Fig. 5 where it can be noticed that the relation is olmost linear when it is negative. Any uncertaintly in Eq. 15 changes the relative position of the curves but not its shape.

The amplitude B_1 can also be plotted aga inst γ, but any error in Eq. (15) affects the shape of the curves.

Contrast of Continuous Tone Images

In order to find the contrast of a continuous tone image, it is necessary to compute the chages in the intensity B^2 of the first order image when the intensity of the object is changed, keeping the reference beam constant. Under these conditions the average intensity can not be considered constant, and therefore it can be shown that B_1 has to be wri tten as:

$$B_1{}^1 = \frac{1}{\pi} \left[\frac{E_o}{E_m} \right]^{-|\gamma|/2} \int_{-\pi}^{\pi} \Big[(1+R)(1+\cos\psi) \Big]^{-\gamma/2} \cos\psi \, d\psi \qquad (17)$$

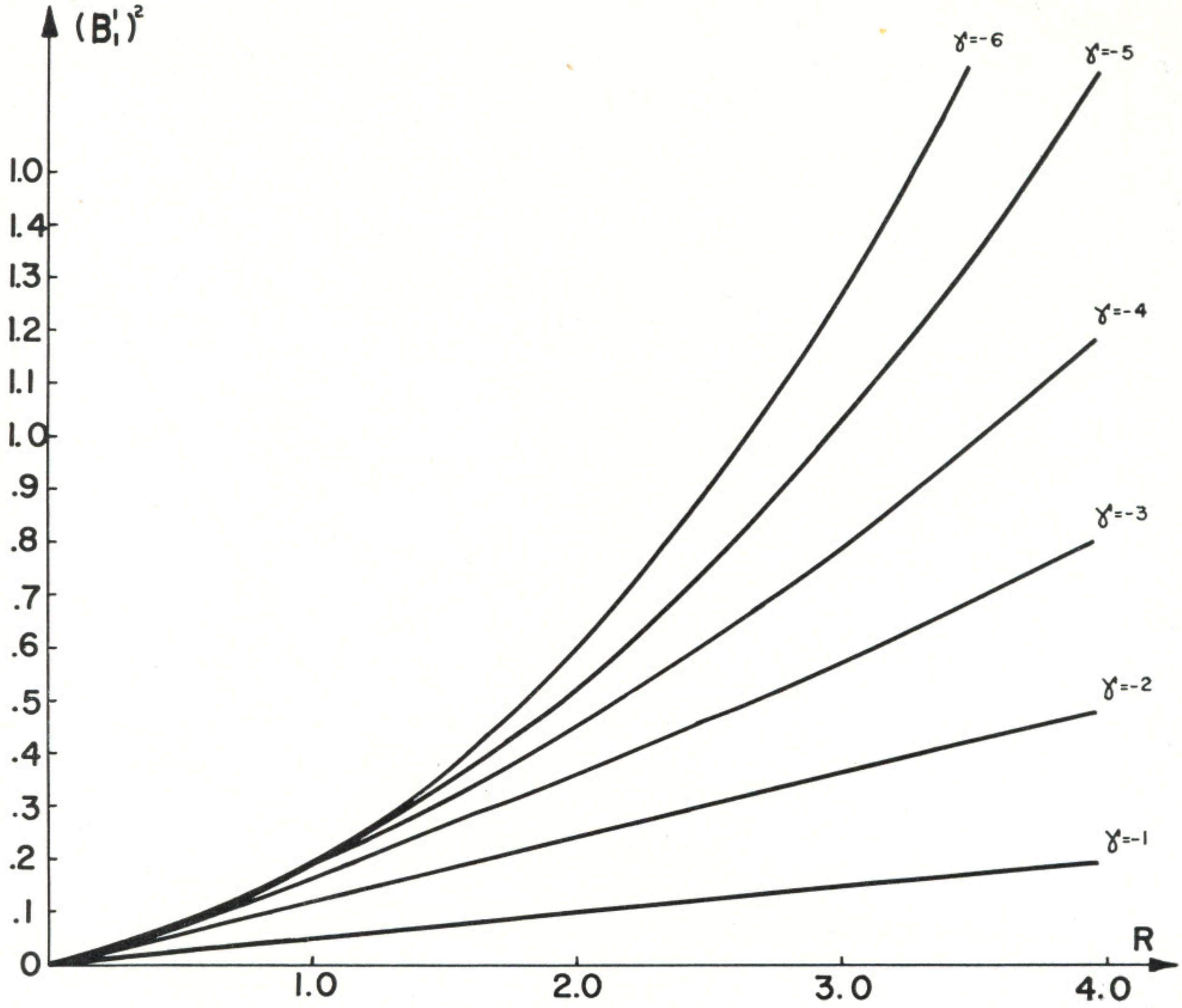

Fig. 6.—*Contrast of Continuous Tone Image for Negative* γ.

where E is the intensity of the reference beam multiplied by the exposure time. R is the ratio of intensities defined before $B_1{}^1$ was written, to distinguish it from the B_2 defined in Eq. (16) in which E_{av} is constant.

Here we take following approximate value for E_o/E_m:

$$log\ E_o - log\ E_m \pm \frac{1}{4}, \tag{18}$$

where the positive sign is taken when γ is positive, and viceversa.

The graphs in Figs. 6 and 7 where obtained using these relations. Noticing that the scale is not the same in both curves, it can be concluded from them that the contrast is much higher for positive γ, that is, for the negative picture. The contrast of the reconstructed image is seen to depend on the value of γ, mainly for positive γ. Also, from Eq. 17, the contrast depends on the ratio (E_o/E_m).

Aknowledgements

The author wishes to express his thaks to Mr. Ignacio Rizzo for helpful discussions and to Mr. Arquimedes Morales for making the drawings.

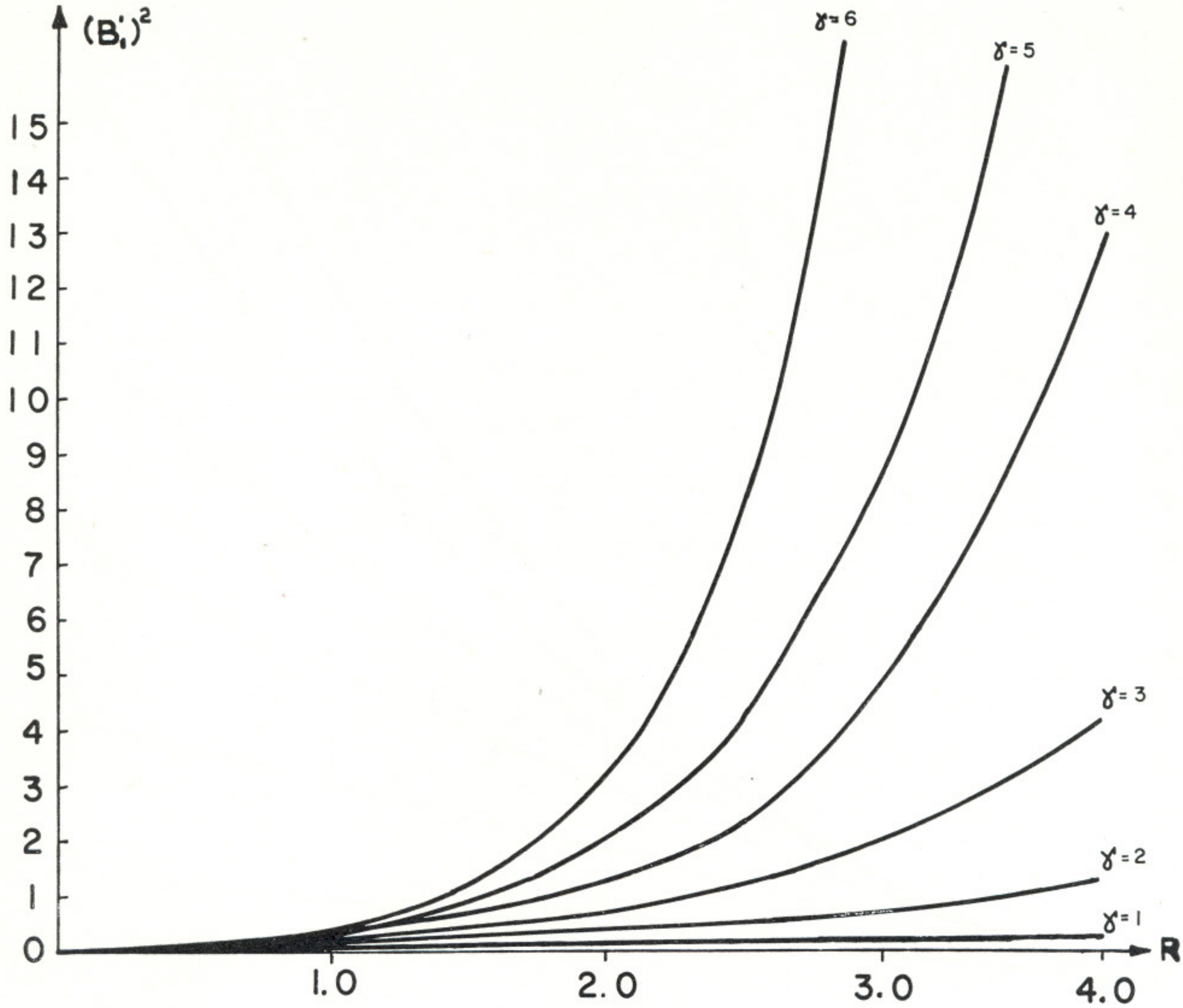

Fig. 7.—*Contrast of Continuous Tone Image for Positive* γ.

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