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THE SELF-ABSORPTION SPECTRUM OF THE TRIPLET SYSTEM OF HE I

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ABSTRACT

The relative line intensities for the triplet system in He I, have been re-computed for various nebular optical depths at $\lambda 3888$. The profile of the line has been considered due to its importance. The electron collisions have not been taken into account since a low density nebula has been assumed. A direct comparison with observational results supports the idea of a depopulating mechanism for the 2^3S level, such as the Ly-α ionization radiation. A three dimensional spherical symmetric model has been used.

SUMARIO

Las intensidades relativas de las líneas para las transiciones en el sistema de los tripletes en el He I han sido recalculadas para varias profundidades ópticas nebulares en $\lambda 3888$. Se ha tomado en cuenta el perfil de la línea debido a su importancia y las colisiones entre electrones no se han considerado ya que se ha supuesto una nebulosa con baja densidad. Una comparación directa con los resultados observacionales apoya la idea de la existencia de un mecanismo de despoblación del nivel 2^3S, tal como el efecto de la ionización de la radiación Ly-α. Se ha considerado un modelo en tres dimensiones con simetría esférica.

I. Introduction

Observational results obtained in recent spectrophotometric studies of nebulae (O'Dell 1963, Méndez 1967), lead one to conclude that the recombination process alone does not explain the helium nebular spectra. The relative intensities of some He I lines differ significantly from those computed considering that the various levels are populated only by downward cascades after the electron has been recombined. The reason for such discrepancies is the well known metastability of the 2^3S level; as an illustration one can note that, for the physical conditions prevailing in the Orion Nebula, the mean life time of the 2^3S level is about 10^4 sec., this is five orders of magnitude longer than the corresponding value for the 2^3S level of hydrogen.

In fact, the optical depth at λ 3888 is $2.25 \times 10^{-5} N_e^2 L$ for the center of the line. This value is computed ignoring the depopulation effects of the resonance *Ly-α* radiation. Since the unreddened measure for the Orion Nebula, in the central regions, is close to 6.5×10^6 (Méndez, 1967), one can expect an optical depth for the $\lambda 3888$ line of about 140. This value could produce effects on the He I spectrum, which effect should be easy to detect observationally.

Pottasch (1962) solved the problem of self-absorption in the helium spectrum using a simplified model: he considered a plane-parallel atmospehere and neglected the Doppler widening in frequency produced by thermal motion. However when the line profile is ignored, the effects of optical depth on the line intensities are exagerated, as shown by Capriotti (1964). That is, under the Pottasch treatment of the problem, for a given line intensity distribution, the optical depth is underestimated by a certain factor which in turn depends on optical depth. Since the knowledge of electron population in the 2^3S level is extremely useful in the study of *Ly-α* radiation transfer in nebulae, one must include the line profile in the formulation of the problem.

In the present work a 14 level helium atom has been treated. This 14 level atom is the same as the one used by Pottasch (1962). Therefore, the effects of higher levels on the population of the lower ones are the same as those found by Pottasch.

In order to simplify the problem further the electron density will be taken as 1 elec./cm.^{3}, which permits us to neglect all the possible effects arising from electron collisions. In fact, at the electron temperature of 10^4 °K adopted in the calculations, the electron collisions would increase the population of the 2^3P level, increasing thus the number of 2^3P-2^3S transitions. The net effect obtained would be the emission of a larger number of $\lambda 10830$ photons. However, the other lines will not show appreciable changes. For the physical conditions prevailing in most diffuse nebulae, the above assumption is valid.

II. Computations

a) *Atomic data for He I.*

The mathematical procedure for solving the self-absorption problem has already been developed by Capriotti (1964), for the case of a spherical nebula. He studied the hydrogen radiation spectrum produced by self-absorption; this method could be easily applied to the radiative spectrum of He I; the changes needed are only concerned with the conversion probabilities. Moreover, the absorbed photons are just those contained in the Principal series, that is, those produced by transitions occurring from the metastable 2^3S level to the n^3P levels, where n = 2,3,4 and 5. To be specific, the conversion probability, B (n,n'), that the absorption of an n photon of the Principal series, will induce the emission of an n' photon, should be considered in a different formulation depending on the emission n' photon:

The emission of lines in the Principal series has the following probability:

$B(n,n'\text{-}2^3S) = C(n,1;n',1)\ P(n',1;2,0) \qquad n>n'$
$B(n,n) = P(n,1;2,0)$

For the lines in the Sharp series, those produced in the transition $n^3S\text{-}2^3P$, the probability is:

$B(n,n'\text{-}2^3P) = C(n,1;n',0)\ P(n',0;2,1) \qquad n\geq n'$

In the Diffuse series, the lines produced by the transitions $n^3D\text{-}2^3P$, the probability is:

$B(n,n'\text{-}2^3P) = C(n,1;n',2)\ P(n',2;2,1) \qquad n>n'$

And in particular for the transitions $4^3S\text{-}3^3P$ and $4^3P\text{-}3^3D$:

$B(n,4^3S\text{-}3^3P) = C(n,1;4,0)\ P(4,0;3,1) \qquad n\geq 4$
$B(n,4^3P\text{-}3^3D) = C(n,1;4,1)\ P(4,1;3,2) \qquad n>4$
$B(n,4^3P\text{-}3^3D) = P(4,1;3,2) \qquad n=4$

Where $P(n,l;n',l')$ is the probability that an atom in an n,l level will make a direct transition to an n',l' level; $C(n,l;n',l')$ is the probability that an atom in an n,l level will cascade down to an n',l' level, but through all possible routes, that is:

$$C(n,l;n',l') = P(n,l;n',l') + \sum_{m=n'+1}^{k(-)} (C(n,l;m,\ l'-1)\ P(m,l'-1;n',l') + \sum_{m=n'+1}^{k(+)} C(n,l;m,l'-1)\ P(m,\ l'-1;n',l')$$

Where:

$k(-) = n-|l-l'|+1 \qquad \text{if} \qquad l'>l+1$
$k(-) = n-|l-l'|-1 \qquad \text{if} \qquad l'\leq l+1$
$k(+) = n-|l-l'|-1 \qquad \text{if} \qquad l'\geq l-1$
$k(+) = n-|l-l'|+1 \qquad \text{if} \qquad l'<l-1$

TABLE 1

Conversión Probabilities

$B(2^3S-5^3P,\ 5^3P-2^3S)$	= 0.83636
$B(2^3S-5^3P,\ 4^3P-2^3S)$	= 0.0000033854932
$B(2^3S-5^3P,\ 3^3P-2^3S)$	= 0.0266853909791
$B(2^3S-5^3P,\ 2^3P-2^3S)$	= 0.1369294198711
$B(2^3S-4^3P,\ 4^3P-2^3S)$	= 0.83372
$B(2^3S-4^3P,\ 3^3P-2^3S)$	= 0.132390572866
$B(2^3S-4^3P,\ 2^3P-2^3S)$	= 0.153035323686
$B(2^3S-3^3P,\ 3^3P-2^3S)$	= 0.91708
$B(2^3S-3^3P,\ 2^3P-2^3S)$	= 0.082917
$B(2^3S-2^3P,\ 2^3P-2^3S)$	= 1.0
$B(2^3S-5^3P,\ 4^3S-2^3P)$	= 0.0185556958336
$B(2^3S-5^3P,\ 3^3S-2^3P)$	= 0.0220490330162
$B(2^3S-4^3P,\ 4^3S-2^3P)$	= 0.0167048946
$B(2^3S-4^3P,\ 3^3S-2^3P)$	= 0.074170997986
$B(2^3S-3^3P,\ 3^3S-2^3P)$	= 0.082917
$B(2^3S-5^3P,\ 4^3D-2^3P)$	= 0.04914346074
$B(2^3S-5^3P,\ 3^3D-2^3P)$	= 0.0471812302812
$B(2^3S-4^3P,\ 3^3D-2^3P)$	= 0.0621594310999
$B(2^3S-5^3P,\ 4^3S-3^3P)$	= 0.016024085779
$B(2^3S-4^3P,\ 4^3S-3^3P)$	= 0.01442579409
$B(2^3S-5^3P,\ 4^3P-3^3D)$	= 0.000000252454341
$B(2^3S-4^3P,\ 4^3P-3^3D)$	= 0.06217

The Einstein spontaneous emission coefficients were obtained from the oscillator strength tabulated by Wellman (1952) except for the transitions $n^3S\text{-}2^3P$ which were taken from Allen (1955). The numerical results of the computations for the conversion probabilities are given in Table 1.

The efective recombination coefficients for the helium atom have been computed from the recombination coefficients given by Burgess and Seaton (1960) and Seaton (1960). Although the intensities of the lines produced by the n^3D-2^3P transitions are insensitive to whether we take case A or case B, all the effective recombination coefficients were calculated for case B.

b) *Computation of line intensities.*

The energy escaping from the nebula in the various He I lines, can be computed by counting the number of conversions of absorbed photons into photons of equal or lower energy. In this way, the number of n photons escaping from the nebula, can be expressed for a nebula in steady state as follows:

$$\mathbf{E}_n = R_n - A_n + \sum_{m=n}^{N} B(m,n)A_m$$

Where N represents the highest ^3P level, R_n is the number of n photons emitted per second by direct recombination and A_m is the number of m photons absorbed per second.

The solution of the transfer equation involved, has been obtained following the same method developed by Capriotti (1964). So that the number of absorbed n protons in the frequency range ν and $\nu+d\nu$ in the volume element d^3r' about $\bar{r}'$, *is* expressed by the integral equation:

$$A_n(r',\nu)d^3r'd\nu = \int_r (S_n(r) + B(n,n)A_n(r))\, \pi^{-1/2}e^{-\nu^2}\, G_n(r,r',\mu,\nu)d^3rd^3r'd\nu$$

Hhere S_n is the total number of n photons emitted per second through pure recombination process and through the conversion of higher energy self-absorbed photons.

In this work shell models fere not considered and therefore the probability $G_n(r,r''\mu,\nu)d^3r'$ that an n photon emitted with frequency ν at the point $\bar{r}$, will be absorbed in a volume d^3r' about the point $\bar{r}'$, is given by:

$$G_n(r,r',\mu,\nu) = \frac{K_n(\nu)}{4\pi(r^2+r'^2-2rr'\mu)}\, e^{-K_n(\nu)\sqrt{r^2+r'^2-2rr'\mu}}$$

Here μ is the cosine of the angle between $\bar{r}$ and $\bar{r}'$

The Doppler widening in frequency for agiven line can be taken into account by introducing the probability, $p(\nu)$, that a photon absorbed can be re-emitted with a frequency between ν and $\nu+d\nu$ (where $\Delta\nu_o$ is the Doppler with):

$$p(\nu) = \frac{1}{\sqrt{\pi}} \exp - \left(\frac{\nu-\nu_o}{\Delta\nu_o}\right)^2$$

The numerical solution has also been obtained following essentially Capriotti's method (Block 1967), so that further details need not be given.

III. *Results and conclusion.*

Several models with various optical depths, were constructed by dividing the nebula in 16 concentrics shells. All computations were carried out using the G-20 computer of the University of Mexico. The number of operations involved in the solution of the integral equation is such that 20 minutes of computer time are needed for each model. The assumption of a nebula with constant density and temperature was considered. The resultant line intensities for the triplet system are given as a function of the nebular optical depth in $\lambda 3888$ ($2^3S - 3^3P$), in Table 2. The line intensities are normalized such that the intensity of $\lambda 5876$ is always equal to 100. Two infrared lines not included in Pottasch's analysis are also given, since one of them shows significant changes with varying optical depth. However, the low intensity of these lines will make their observation a difficult task.

Although the intensity of $\lambda 5876$ is normalized, the number of photons in this line increases steadily with optical depth. As this line is frequently used in abundance determinations the variations of its intensity are given in Table 3. The changes should be taken into account to avoid an overestimate of helium in nebulae.

TABLE 2

Wavelength	*0*	*1*	*10*	*30*	*50*	*100*	*150*	*300*	*Transition*
10830.224	158	158.85	166.34	176.04	180.62	185.46	187.42	189.58	2P – 2S
7065.276	18.3	19.73	32.38	49.29	57.68	66.96	70.89	75.38	3S – 2P
3888.646	108	105.48	83.21	53.71	39.29	23.56	16.99	9.56	3P – 2S
5875.651	100	100	100	100	100	100	100	100	3D – 2P
4713.201	3.2	3.24	3.61	4.29	4.78	5.5	5.9	6.46	4S – 2P
3187.743	45.1	44.52	39.34	30.22	14.22	16.11	12.11	7.02	4P – 2S
4471.507	39.4	39.38	39.19	38.9	38.8	38.82	38.93	39.2	4D – 2P
4120.857	1.05	1.05	1.04	1.02	1.00	0.98	0.97	0.95	5S – 2P
2945.000	29.3	29.17	27.55	24.29	21.58	16.74	13.64	8.81	5P – 2S
4026.218	21.0	20.97	20.74	20.31	20.0	19.57	19.34	19.02	5D – 2P
21120.000	0.65	0.65	0.66	0.69	0.71	0.75	0.78	0.84	4S – 3P
19543.000	0.46	0.49	0.77	1.26	1.58	2.01	2.22	2.48	4P – 3S

From Table 2 one can see that the lines most sensitive to the overpopulation of the 2^3S level are λ3187, λ3888 and λ7065. Their variation in intensity is shown in Figure 1. These three lines are very conspicous in nebula spectra, and therefore easy to observe.

A direct comparison of the theoretical calculations with the observational results obtained in a spectrophotometric study of the Orion Nebula (Méndez, 1967) can now be carried out. Table 4 gives

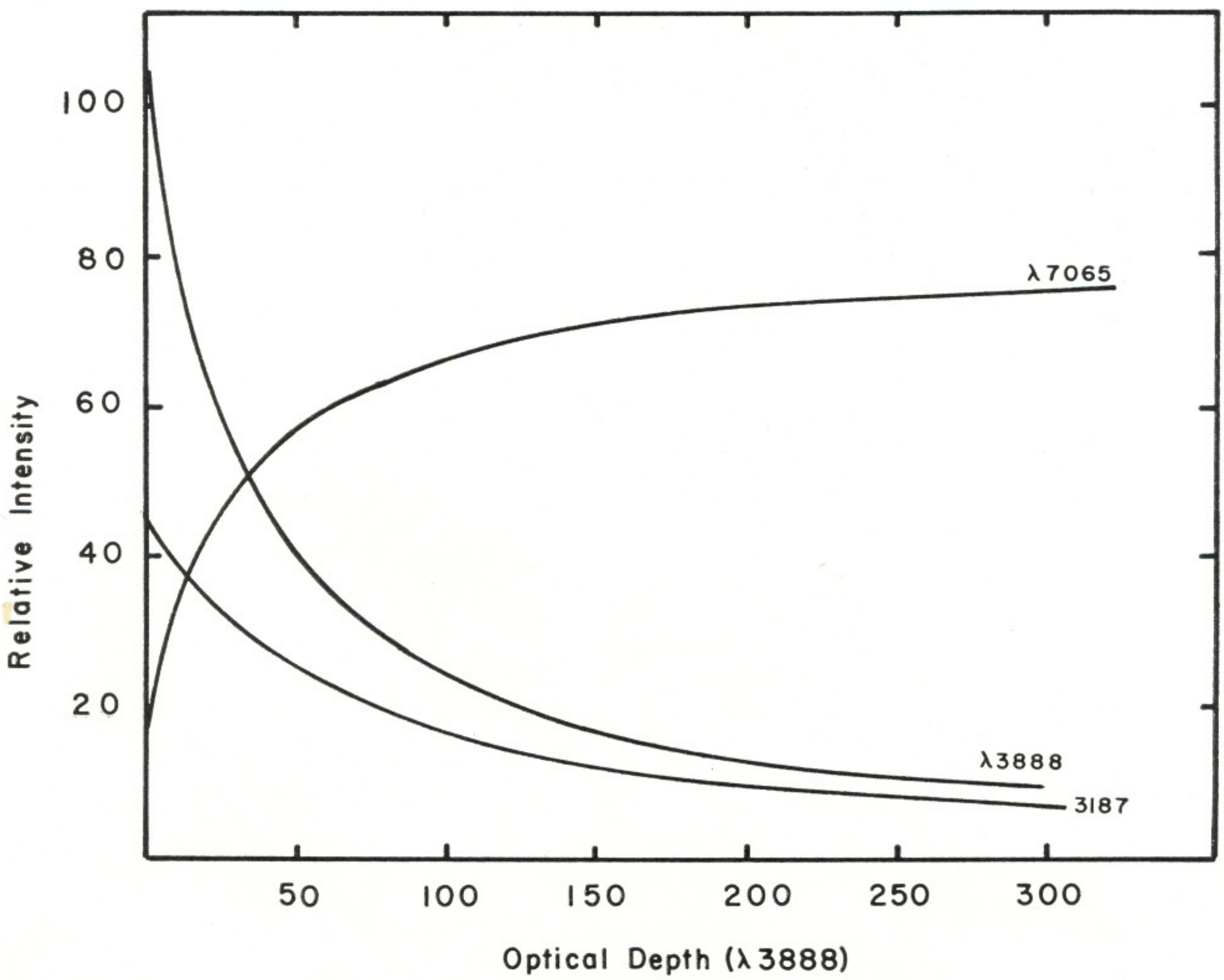

Figure 1. *Variation of line intensity with optical depth.*

TABLE 3

Intensity of the λ5876 line

τ	Intensity
0	100
1	100.12
10	101.27
30	103.42
50	104.96
100	107.28
150	108.58
300	110.41

TABLE 4

Observed helium line strengths for NGG 1976.

λ	Line strengths
10830.224	405.0
7065.276	80.0
5875.651	100.0
4471.507	41.0
3888.646*	43.0 (23.0)
3187.743	14.9

* The number in brackets is obtained considering that the hydrogen levels are populated in accordance with their statistical weights (The intensity of the hydrogen line λ3888 is 13.2).

the intensities of the relevant lines obtained from the central regions of the nebula. The intensity of λ3187 comes form the work of Kaler et al. (1965), with the proper reddening correction. One can conclude, from the strengths of λ3187 and λ3888, that the optical depth lies marginally between 50 and 100. Since the computed value is close to 150, one must expect the existence of an additional mechanism for depopulating the 2^3S level, such as the ionizing effect of the *Ly-α* radiation (Münch, 1963), to be discussed in a later paper. The large discrepancy for the λ10830 line is produced by electron collisions. However, the collisional cross sections are not well known, therefore they can not be included in the computations.

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