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ON THE AGES OF THE GALACTIC CLUSTERS NGC 188, M67 AND NGC 6791

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SUMARIO

Se presentan curvas de igual edad para tres diferentes composiciones químicas y se comparan con los diagramas color-magnitud de NGC 188, M67 y NGC 6791.

En el caso de NGC 188 se encuentra que las abundancias metálicas están comprendidas entre $0.05 \leq Z \leq 0.10$; este resultado confirma la "supermetalicidad" del cúmulo. La edad que se deriva para NGC 188 es de 3.6×10^9 años bajo la hipótesis de $E(B-V) = 0.15$ mag. Para M67 la edad es de 2 a 3×10^9 años aunque esta determinación es muy incierta ya que las curvas teóricas no corresponden al diagrama H—R que se observa. Para NGC 6791 se estima una edad de 4×10^9 años.

ABSTRACT

Theoretical isochrones for three chemical compositions are presented and compared to the color-magnitude diagrams of NGC 188, M67 and NGC 6791.

The metal abundances that provide the best agreement between theory and observations for NGC 188 are in the range $0.05 \leq Z \leq 0.10$; this result confirms the "supermetallicity" derived from the spectral features. For NGC 188 an age of 3.6×10^9 years was determined assuming $E(B-V) = 0.15$ mag. In the case of M67 the age is in the range of 2 to 3×10^9 years; this determination is however very uncertain since the shape of the predicted curves does not correspond to the observed H—R diagram. For NGC 6791 an age of 4×10^9 years was estimated.

I. Introduction

The ages of the oldest galactic clusters provide an important clue for the history of the galaxy and the problem of assigning ages to M67 and NGC 188 has been undertaken by several workers.

Demarque and Schlesinger (1969) have determined the age and chemical composition of NGC 188 from the effective temperature at which the gap, or lack of stars, occurs. Aizenman, Demarque and Miller (1969) have derived the age and helium abundance of M67 and NGC 188 based on the size of the gap; more recently, Demarque and Heasley (1971) have proposed that from the size of the gap and the luminosity of the yellow subgiants it is possible to determine not only the age and helium abundance but also the metal content of a cluster. Sandage and Eggen (1969) have constructed curves of equal age from models by Iben (1967) and by Aizenman, Demarque and Miller (1969) and by fitting these to the C-M diagram of M67 and NGC 188 have derived their ages and distances.

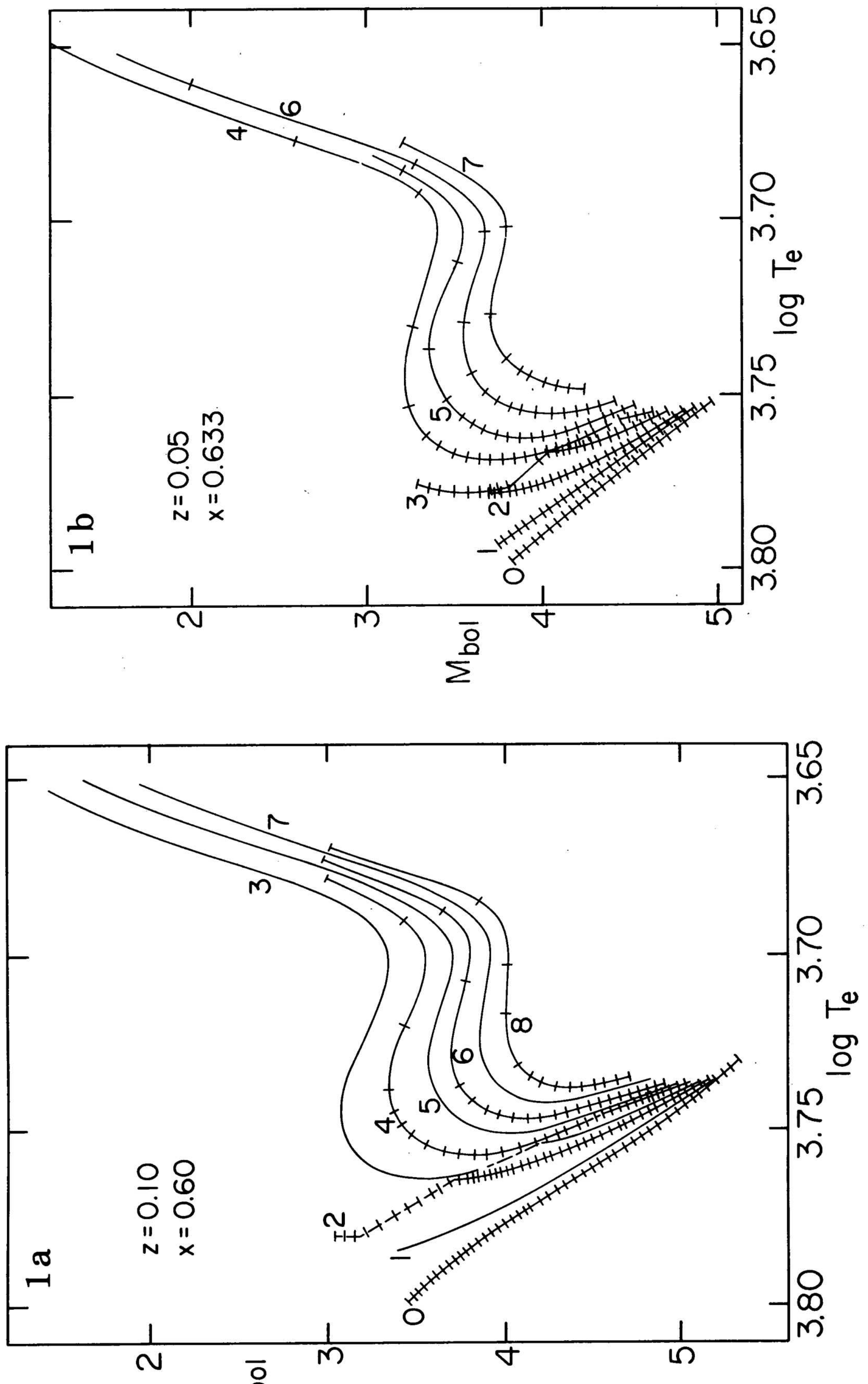
Since it has been found that individual stars of M67, NGC 188 and NGC 6791 have photospheric metal to hydrogen ratios higher than the solar one (Spinrad and Taylor 1967, 1969, 1971; Spinrad, Greenstein, Taylor and King 1970) it was felt that it was possible to carry out a comparison of the models with the observations in more detail than previously from the evolutionary models (Torres-Peimbert 1969, 1971) of different chemical compositions, that include cases of very high Z.

In this paper we present isochrones derived from the model sequences by Torres-Peimbert for different chemical compositions. We compare the C-M diagrams of M67, NGC 188 and NGC 6791 to the theoretical curves and derive the ages of the clusters, as well as their distance moduli.

II. Theoretical H—R Diagrams

Curves of equal age have been derived from the individual evolutionary tracks computed by Torres-Peimbert (1971) for those compositions for which there were two or more evolutionary sequences available. The isochrones are presented in Figure 1 for ($Z = 0.10$, $X = 0.60$), ($Z = 0.05$, $X = 0.633$), and ($Z = 0.023$, $X = 0.651$). The position of the models for each 0.01 $M_\odot$ interval has been marked for most of the isochrones. These diagrams thus not only contain the expected locus of a cluster of stars of equal age but also the expected stellar density for a uniform mass function. The ages marked on the isochrones have been obtained using the origin of time as the beginning of the main sequence.

The curves of equal age reflect the behavior of the individual tracks and thus at the time of fast evolution on the H-R diagram the isochrones follow very closely the evolutionary path. In partic-



1b
z=0.05
x=0.633
1a
z=0.10
x=0.60
M bol
log Te

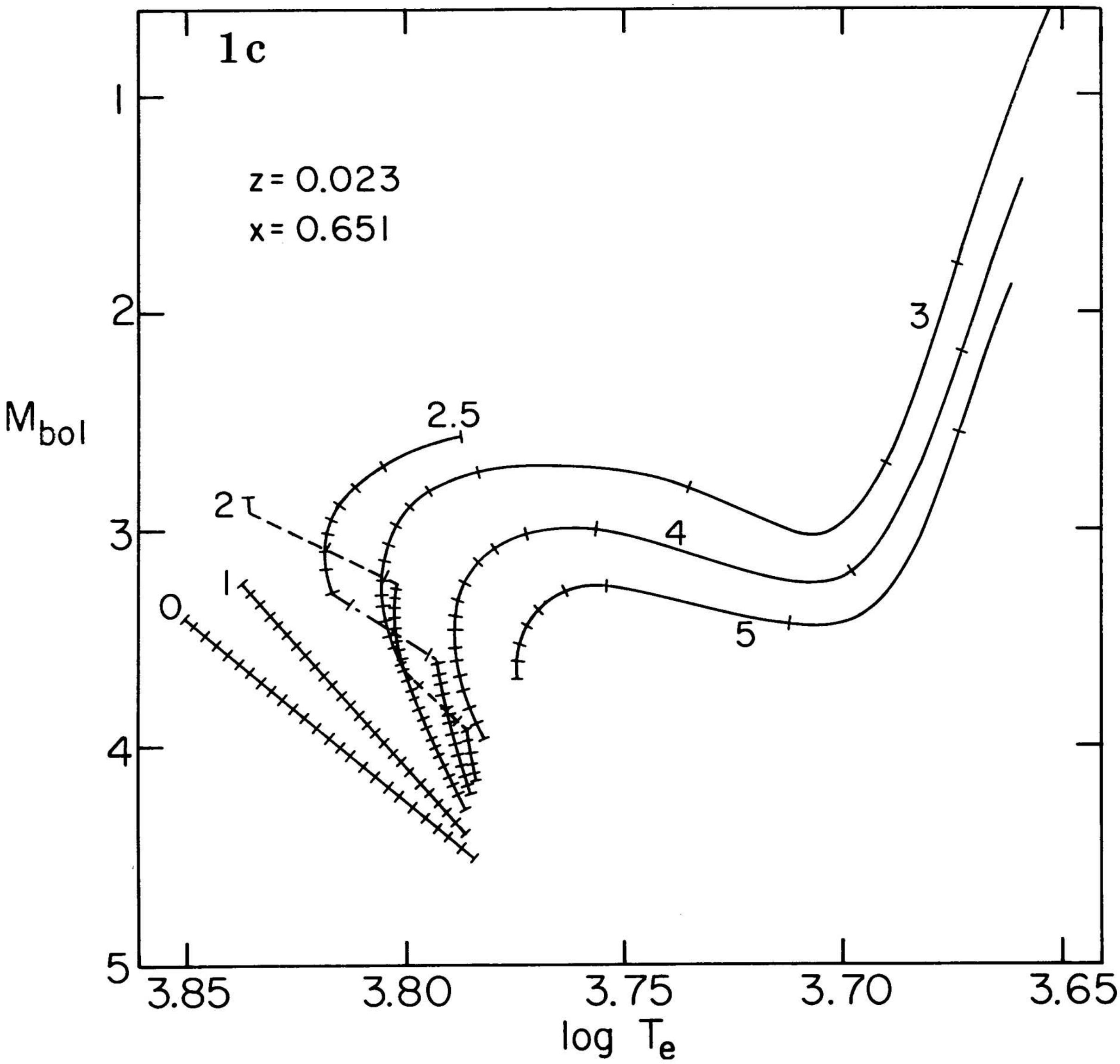


Fig. 1.—*Isochrones for the three compositions available. The labels on each curve are the corresponding ages in units of 10^9 years. The short slashes indicate the expected stellar density under the assumption of a uniform mass function. These isochrones were computed with* l/H = *1.5. To compare them with observations a uniform shift of the observations of* Δ *log* T_e = *—0.012 is necessary, as has been explained in* § *II.*

ular, the tracks show a sharp change in direction toward higher luminosities and temperatures for the phase of hydrogen exhaustion of objects with a main sequence convective core. This effect appears in the "younger" isochrones almost as a discontinuity, or gap, since the time necessary to traverse it is relatively short compared to the immediately preceding and following phases. The upper boundary of the gap is at higher temperatures and luminosities than the lower one. For "older" isochrones the gap becomes less marked until it disappears completely. For a given composition the isochrones with a gap are necessarily younger than the age of the least massive star that has a main sequence convective core.

For the comparison between the computed lines of equal age and the observed color magnitude diagram for any given cluster, the hypothesis is made that magnetic fields, stellar rotation, stellar instability and mass loss are not present in the stars of the observed clusters; furthermore, it is assumed that all the stars within the cluster are coeval and have the same primordial composition.

An adjustment is necessary before carrying out the comparison between observations and predictions. Since the isochrones are based on individual models computed with a ratio of mixing length to pressure scale height equal to l/H = 1.5, and Torres-Peimbert, Simpson, and Ulrich (1969) have calibrated this quantity for the sun at $l/H = 2.0$, the computations need to be corrected to $l/H = 2.0$.

Torres-Peimbert *et al.* (1969) have computed that for an evolutionary model on the main sequence of mass 1 $M_{\odot}$ the influence of l/H on the H-R diagram can be expressed as

$$\frac{\Delta \log T_e}{\Delta (l/H)} = 0.023 . \quad (1)$$

Also, from a study of Bodenheimer (1966) on the influence of initial conditions on the pre-main sequence convective phase, the following relation can be derived

$$\frac{\Delta \log T_e}{\Delta (l/H)} = 0.04 . \quad (2)$$

Based on relations (1) and (2) a uniform shift of the predicted isochrones of $\Delta \log T_e = 0.012$ compensates approximately for an increase of 0.5 in l/H.

For the transformation of the observed quantities to the ($\log T_e$, M_{bol}) plane, the calibration of bolometric correction and effective temperature for luminosity classes III and IV by Johnson (1966) was used. The temperatures are intermediate and thus apply to the region where the calibration is best determined. The problem of the applicability of a transformation based on normal population I stars to super metal rich stars needs further study.

From the narrow band measurements by Spinrad and Taylor (1969) for luminosity class III it appears that for the same effective temperature the SMR stars have a heavier blanketing in B, and thus have a larger $(B-V)$; on the average the change in color is $\Delta (B-V) \sim +0.03$. Conversely, for a given $B-V$, the SMR star has a higher effective temperature than a normal population I star ($\Delta \log T_e \sim +0.005$ for $0.9 \leq B-V \leq 1.5$) as well as a smaller bolometric correction ($0.025 \leq \Delta$ B. C. ≤ 0.075 for the range considered). This effect is also present in SMR dwarfs (Taylor 1969) from which the derived values were approximately the same. The differential blanketing was not included in the results; should a blanketing of $\Delta (B-V) \sim 0.03$ be considered, the only significant change would be a reduction in the ages by $\sim 20\%$.

III. Comparison with NGC 188

Photoelectric *UBV* photometry for this cluster has been obtained by Eggen and Sandage (1969). On the rising branch of the evolving main sequence the color-magnitude diagram of this cluster shows a moderate lack of stars; Aizenman *et al.* (1969) made a statistical study of the number of stars near the turnoff point and concluded that there is a gap of stars of width $\Delta V = 0.085 \pm 0.023$ mag at $V \sim 15.63$ mag. The absolute luminosity of the gap depends on the comparison with theoretical models and will be discussed below.

Eggen and Sandage (1969) from a conventional two-color diagram determined the reddening to be $E(B-V) = 0.09$ mag. However, Spinrad, Greenstein, Taylor, and King (1970) from the strength of Hα of two K0 subgiants find $E(B-V) > 0.10$ mag for NGC 188; Abt and Goldson (1962) obtained a reddening of 0.09 to 0.12 mag from distant stars in the direction of the cluster; and Greenstein and Keenan (1964) determined $E(B-V)$ to be 0.18 mag by comparing the spectral types of giants and subgiants with approximately normal MK standards. In view of the different possible values of the interstellar reddening for NGC 188, the comparison with the isochrones was carried out for several values of the reddening correction, and is given in Table 1 for each of the three available compositions.

In Table 1 the ages and distances to NGC 188 listed are those derived from the best fit of the color of the turnoff point as well as the general shape; and although not in all cases the fit is adequate, it gives an indication of the variations of the age determination assuming different chemical composition and reddening. Given a chemical composition and reddening, the mean errors in the age determined by this method are approximately 10% for the age and 0.20 mag for the distance modulus. The age determined by fitting the shape and color of a cluster is smaller for higher metal abundance, since for the same age an increase in metals corresponds to lower temperatures and hence to lower luminosities; the corresponding change in the distance modulus is not well defined although in general, for higher metal abundances the determined distance increases. For similar reasons, if larger reddening corrections are applied the determined age decreases, and the distance increases.

TABLE 1

Age and Distance of NGC 188

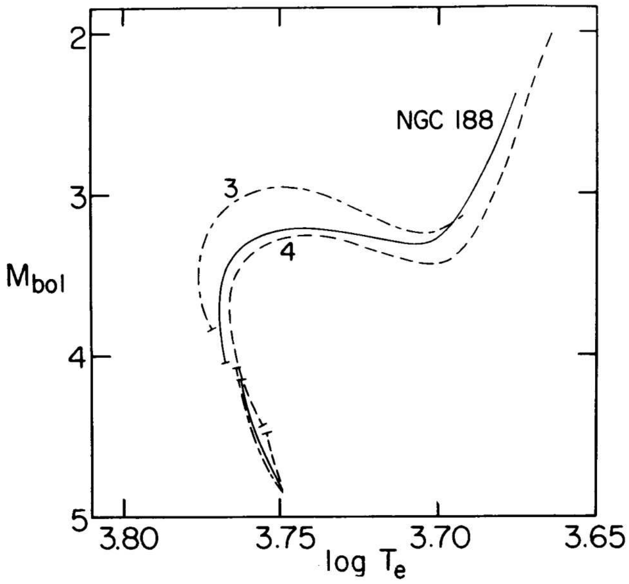
	$E(B-V)=0.05$			$E(B-V)=0.10$			$E(B-V)=0.15$		
	Age (y)	$(m\text{-}M)_o$	*Comments*	*Age (y)*	$(m\text{-}M)_o$	*Comments*	*Age (y)*	$(m\text{-}M)_o$	*Comments*
Z = 0.10 X = 0.60	5×10^9	11.0	No gap predicted	3.7×10^9	11.2	Predicted gap 0.20 fainter	2.5×10^9	11.4	Predicted gap 0.15 brighter
Z = 0.05 X = 0.633	7×10^9	10.9	No gap predicted	5.6×10^9	11.0	No gap predicted	4×10^9	11.1	Predicted gap 0.25 fainter
Z = 0.023 X = 0.651	$>7 \times 10^9$	—	No gap predicted	$\geqslant 6 \times 10^9$	—	No gap predicted	5×10^9	11.1	No gap predicted

From columns 4, 7 and 10 of Table 1, it is apparent that not all chemical compositions and values of the reddening yield a good fit for both the overall shape of the cluster and the position of the gap. In general a family of solutions of different Z, Y, and reddening, fit both characteristics. In the present study, among the compositions available, the most likely value lies between $0.05 \leq Z \leq 0.10$, for $E = 0.15$ mag.

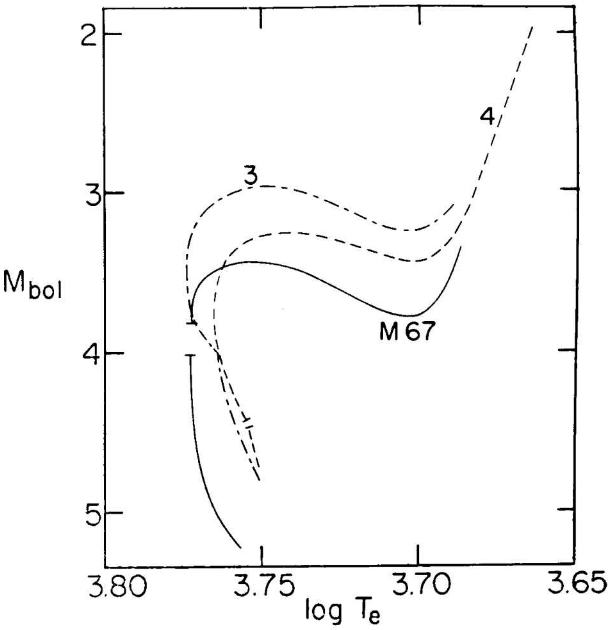
Eggen and Sandage (1969) from the two color diagram of NGC 188 concluded that there is a small ultraviolet excess relative to the Hyades which they interpreted to be due to a slightly lower metal abundance in the stars of NGC 188 than in those of the Hyades. Alternatively, Spinrad and Taylor (1969) obtained photoelectric spectrum scans of seven individual giant stars of NGC 188; in all the objects studied they found relatively strong Na, Mg, N, Ca, and Fe lines and concluded that the anomalous intensities were produced by a photospheric overabundance of these elements. Moreover, Spinrad *et al.* (1970) have classified the spectrum of four members of NGC 188, estimating spectral types from both the hydrogen absorption lines and the metallic lines. The classification with the metallic lines gave later types than did Hδ and Hγ. The interpretation advanced by Spinrad *et al.* is that indeed there is an overabundance of all metals with observable lines in the blue region of the spectra (Fe, Ca, Mn and C).

As has been reported (Torres-Peimbert and Peimbert 1971) the only change of chemical composition that occurs in the surface of the giant stars at stages prior to helium burning is an increase of nitrogen at the expense of carbon, due to the internal CNO burning and partial mixing to the surface. It results in an increase of approximately two per cent in the helium abundance while the abundances of O, Na, Mg, Ca, and Fe remain the same. The variation of helium is so small that it cannot produce appreciable changes in the metallic line strengths; furthermore, it is almost independent of Z, which means that it would affect similarly both SMR and normal stars. Thus in case of a photospheric metallic anomaly it is indeed likely that such anomaly was already present when the stars were formed. The observations point to an overall metal-to-hydrogen abundance ratio of about a factor of three larger than the solar one. Since the solar metal abundance has been estimated to be between 0.015 and 0.025 by mass (Torres-Peimbert *et al.* 1969; Bahcall, Bahcall, and Shaviv 1968; Lambert 1968; Lambert and Warner 1968*a, b;* and Warner 1968), a metal abundance of $Z \sim 0.05$ is indicated by the photospheric evidence. From Table 1 the best agreement is reached for high metal content; therefore, our comparison with theoretical models confirms the interpretation of Spinrad and Taylor (1969) and Spinrad, Greenstein, Taylor and King (1970). Furthermore, it implies that the metal richness is not confined to the atmosphere of these objects but applies to the interior as well. Based on the comparison of the $H-R$ diagram of the cluster with the computed isochrones, and the photospheric chemical abundance determinations, an age can be derived for NGC 188 provided that the color excess is well determined. The best agreement is obtained for $Z = 0.06$ and $X = 0.627$ assuming $E(B-V) = 0.15$ mag; this fit yields an age of 3.6×10^9 years and a distance modulus of $(m\text{-}M)_0 = 11.1 \pm 0.2$ mag. The gap is then located at $M_{bol} = 4.1$ mag. The distance modulus yields a distance of 1660 pc, and a height of 650 pc above the galactic plane. The value of the distance modulus by Eggen and Sandage, $(m\text{-}M)_0 = 10.85 \pm 0.15$ mag, does not differ substantially from the distance obtained here.

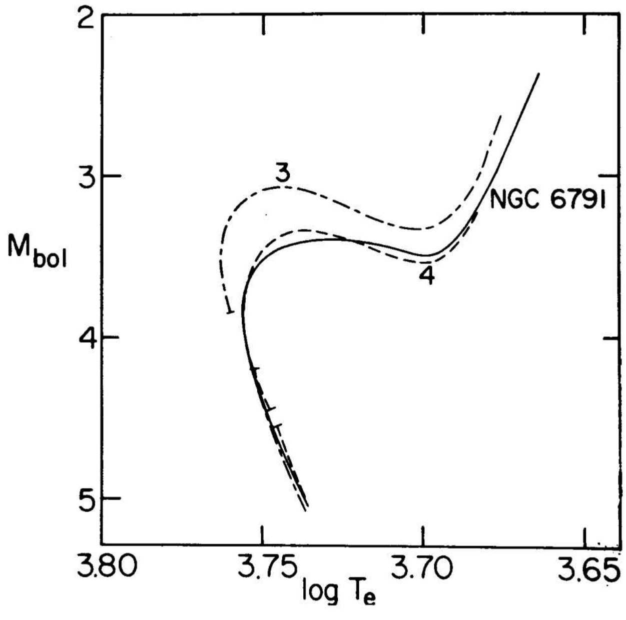
Figure 2*a* shows the observed diagram of NGC 188 transformed to the theoretical plane after applying a reddening correction of $E(B-V) = 0.15$ mag. The observations were shifted by -0.012 in $\log T_e$ to compensate for l/H (see §II). Superimposed are the isochrones for $Z = 0.06$, $X = 0.627$; the isochrones for this composition have not been computed directly, but are obtained by interpolation between Figures 1 and 2.



(a)



(b)



(c)

Fig. 2.—*C-M diagrams superimposed to the isochrones. The ages are marked in units of* 10^9 *years. The observations were shifted by* -0.012 *in* $\log T_e$ *to compensate for* l/H *(see§ II). a) NGC 188 with* $E(B-V) = 0.15$ *and* $Z = 0.06$, $X = 0.627$; *b) M67 with* $E(B-V) = 0.06$ *and* $Z = 0.06$, $X = 0.627$; *c) NGC 6791 with* $E(B-V) = 0.22$ *and* $Z = 0.10$, $X = 0.60$.

The age here derived is smaller than the values found by other workers. Sandage and Eggen (1969) have derived an age of 7.7×10^9 years from isochrones based on the models of $Z = 0.03$, $X = 0.67$ by Aizenman *et al.* (1969), or that of 9.5×10^9 years based on computation of $Z = 0.02$, $X = 0.71$ by Iben (1967). It is interesting to note that neither of the fits predicts a gap. The conclusions derived by Aizenman *et al.* by examining the width of the observed gap were that for $Z = 0.03$ the helium abundance is $Y = 0.30 - 0.35$ and the age is of 3.5 to 4.6×10^9 years, and for $Z = 0.06$, the helium abundance is $Y = 0.35$ and the age is of 5.5×10^9 years. Demarque and Schlesinger (1969) obtained an age of 7×10^9 years for $Z = 0.07$ and $Y = 0.26$.

The difference in the age determinations is due in part to the high Z value adopted here and also to the circumstance that for the original models no configuration with $\tau > 4.2 \times 10^9$ years possessed a main sequence convective core where τ is the time spent on the central hydrogen burning phase (Torres-Peimbert 1971).

A determination of the age of the cluster based only on the size of the gap at a given magnitude (Aizenman *et al.* 1969) is uncertain due to the assumption made a priori that the bolometric magnitude of the observed gap is known. This is not a valid assumption since the comparison with the theoretical curves gives the distance to the cluster and, therefore, these are not independent determinations. Moreover, such a comparison does not take into account other regions of the diagram, which might show other effects, as is the case of M67.

The possibility of increasing the age of the theoretical models by increasing considerably the helium abundance should be examined, since an increase in Y of 0.30 lowers the minimum luminosity of the model with a main sequence convective core by $\sim$ 1.2 mag (Demarque 1968). The change in the main sequence lifetime for a configuration with a convective core of 4% of the total mass can be derived by comparing the main sequences computed by Demarque and Schlesinger (1969) for the same metal abundance. Thus for a given metal abundance of $Z = 0.03$ the mass of the object with a convective core of 4% of the total mass is different according to the helium abundance, and is of $M_1 = 1.10\ M_\odot$ for $Y_1 = 0.30$ and $M_2 = 0.58\ M_\odot$ for $Y_2 = 0.60$. The lifetimes are

$$\frac{\tau_1}{\tau_2} = \frac{X_1 M_1/L_1}{X_2 M_2/L_2} = 1.14 \qquad (3)$$

which means that even if the luminosity is decreased by a factor of three, the lifetime of the model with a convective core of 4% with very high helium is smaller than the lifetime of the model with a 4% convective core with normal helium. Therefore, from the evolutionary configurations used here, no age over 4.2×10^9 years is compatible with a gap in the H-R diagram of a cluster.

IV. Comparison with M67

The photometry for this cluster was obtained by Eggen and Sandage (1964). The C-M diagram of M67 shows some special characteristics. The most striking feature is a lack of stars on a small area along the turnoff from the main sequence at $V = 12.90$. Due to the completeness of the coverage of the central zone of the cluster, the absence of stars in this zone of the C-M diagram can be considered to be real. Aizenman *et al.* (1969) in a statistical study found that indeed there is a gap present on the diagram of M67; of width $\Delta V = 0.167 \pm 0.010$ mag, the width of the gap is 0.24 ± 0.02 mag according to Racine (1971). Another interesting feature to notice is that the density of stars within $0.55 \leq B-V \leq 0.64$ projected along a line of constant $B-V$ is higher for the region above the gap than below the gap, being of 53 and 40 per magnitude, respectively. This is a peculiarity of M67 since the number density of stars for the case of NGC 188 is 44 stars per magnitude for both regions. Furthermore, the upper boundary of the gap lies directly above the lower boundary of the gap, i. e., they are at the same effective temperature. Finally, the sub-giant branch of M67 is very well defined and the brighter part of the sub-giant branch is only 0.35 mag above the upper boundary of the gap.

From the two-color diagram for the individual stars Eggen and Sandage (1964) determined the reddening correction to be $E(B-V) = 0.06$ mag. However, Spinrad *et al.* (1970) measured the line strength of Hα of five late F main sequence stars in M67, and from a relationship between $B-V$ and Hα line strength for F and G dwarfs derived an intrinsic color and estimated the reddening to be of $E(B-V) = 0.11 \pm 0.01$ mag. On the other hand, Racine (1971) has reported a reddening of $E(B-V) = 0.09 \pm 0.02$ mag.

A composite diagram on the theoretical plane has been made for NGC 188, M67, and NGC 6791, after correcting for interstellar reddening by 0.15, 0.13, and 0.22 mag, respectively and is shown in Figure 3. This diagram was prepared assuming that these clusters have the same main sequence.

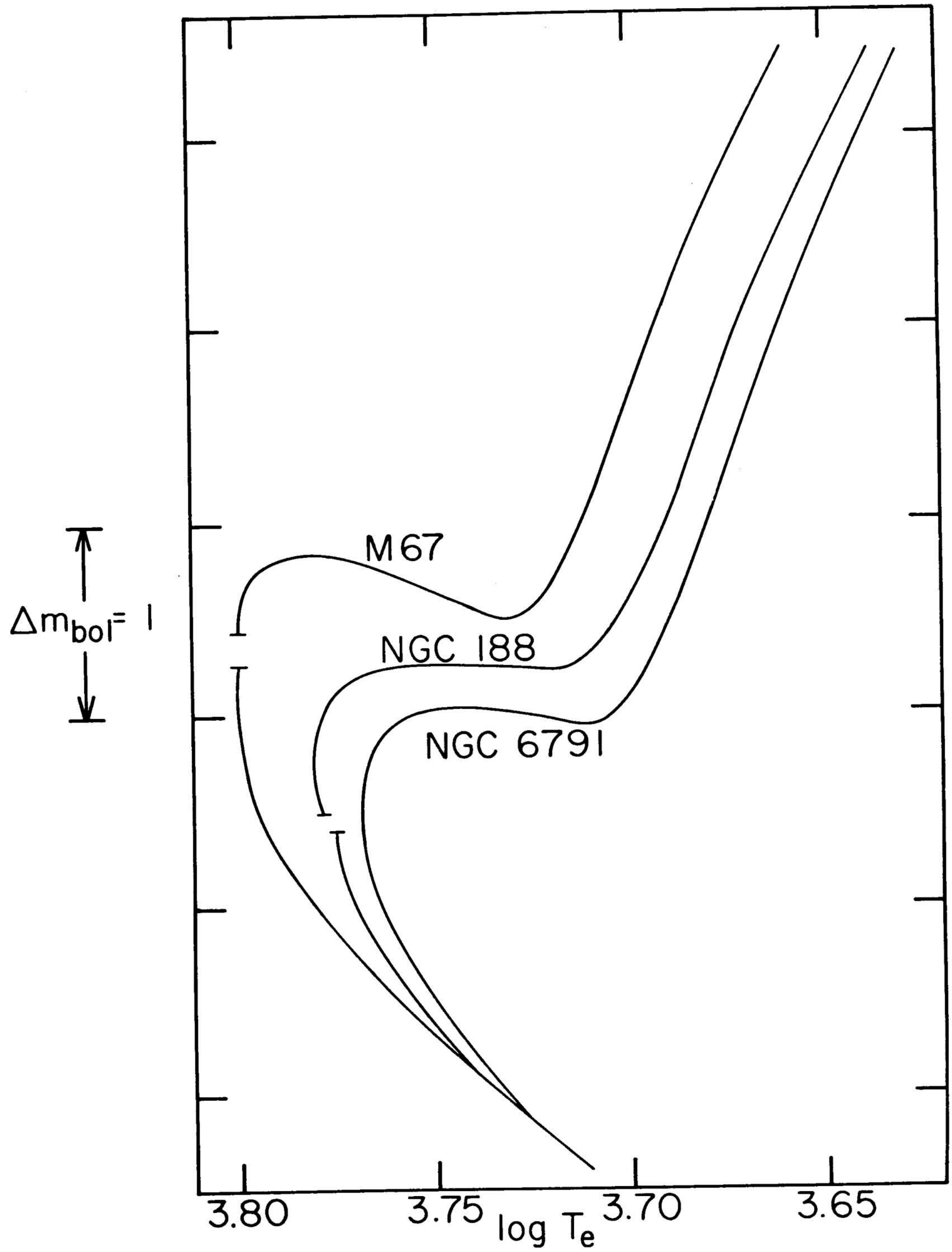


Fig. 3.—*Composite diagram with NGC 188, M67, and NGC 6791, assuming that the main sequence is in the same position for all three clusters. The reddening corrections applied are 0.15, 0.13, and 0.22 mag, respectively.*

The relative ages derived from this figure are uncertain since, as it will be explained in §V, the chemical composition of NGC 6791 is probably different from that of NGC 188 and M67. Based on the main sequence stars, M67 would be younger than NGC 188 if the chemical composition of both clusters were the same. A characteristic to be noted on this diagram is that the difference in magnitude from the upper boundary of the gap to the sub-giant branch is much smaller for M67 than for NGC 188.

There are two substantial differences present between the observed C-M diagram of M67 and *any* of the isochrones presented here: *a)* the observed sub-giant branch above the upper boundary of the gap is at least 0.3 mag fainter than predicted and *b)* the stars immediately above the gap are at the same effective temperature as those immediately below it, while the predictions are that the stars above the gap should be bluer than those below it; *c)* the observed difference in luminosity between the gap and the main sequence is larger than predicted. Table 2 contains the ages and distances derived from the comparison of the observations to the isochrones assuming that the observed effective temperature and magnitude of the gap correspond to the mean position of the predicted discontinuity.

TABLE 2

*Age and Distance of M67**

	$E(B-V) = 0.06$		$E(B-V) = 0.13$	
	Age (y)	$(m\text{-}M)_o$	*Age (y)*	$(m\text{-}M)_o$
$Z = 0.10$ $X = 0.60$	2.0×10^9	9.4	1.5×10^9	9.7
$Z = 0.05$ $X = 0.633$	3.0×10^9	9.0	2.5×10^9	9.2
$Z = 0.023$ $X = 0.651$	—	—	3.0×10^9	8.9

* According to the mean color of the gap.

In order to exhibit the disagreements between M67 and the predicted evolutionary sequences a composite diagram is shown in Figure 2*b*. On this diagram the fit has been made with the color and magnitude of the upper boundary of the gap. It should be noticed that the branch immediately above the gap is much flatter in M67 than in the predicted curves.

From the present study, it is not possible to assign any chemical composition, or range in chemical compositions, to M67, since *none of the isochrones* agree at all with the observations. Other studies have been made on the chemical composition of M67: Mannery, Wallerstein, and Welch (1968), from wide band photometry of KIII stars have determined the ratio of Fe/H to be 60% higher than that of the Hyades. Also, Sargent (1968), from a study of the cluster's horizontal branch stars F55 and F153, classified them as metallic line stars. And, from the line and molecular strength measurements of nine giant stars ($0.94 \leq B-V \leq 1.50$; $12.0 \geq V \geq 8.8$), Spinrad and Taylor (1969) determined that N, Na, Mg, Ca, and Fe are overabundant in these stars. In particular, from three objects of this cluster the average abundance ratios of Ca/H, Mg/H, and Na/H relative to solar are equal to 3.6, 2.9, and 2.2, respectively.

As has already been discussed, there is no possibility of assigning a likely chemical composition to this cluster from the comparison with the H—R diagram of M67 because the fits are unsatisfactory. If a composition of $Z = 0.06$, $X = 0.627$, and a reddening value of $E = 0.13$ mag are assumed, following the interpretation of supermetallicity by Spinrad and Taylor (1969) and Spinrad *et al.* (1970), then an age of *2.3* $\times$ *10⁹ years* and a distance of $(m\text{-}M)_0 = 9.3 \pm 0.3$ *mag* are the most likely values for this cluster. The cluster would be 720 pc away, and 380 pc above the galactic plane. The distance modulus determined by Sandage and Eggen (1969) is 9.38 mag.

We will mention explicitly the discrepancies between the theoretical tracks and the observations of M67. First, the isochrones predict that the brightest yellow subgiants should be at least 0.75 mag above the upper boundary of the gap, however M67 shows a maximum increase of 0.35 mag; essentially the same discrepancy is present from the models by Iben (1967) and Demarque and Heasley (1971). Second, the models predict that during contraction and shell development corresponding to a gap of $\Delta M_{bol} = 0.20$ mag there should be a change of $\Delta \log T_e = 0.01$; this expected increase in temperature is not observed since the upper and the lower boundaries of the gap of M67 have the same color.

These two discrepancies will not vanish by adopting different reddening and blanketing values since any change would be the same for stars above and below the gap. Furthermore, it is very unlikely that the shape of the C-M diagram would change significantly by the addition of new members since the regions of the diagram where the discrepancies are found correspond to stars brighter than $V = 13$ mag, where the surveys are relatively complete.

The possibility of stellar rotation to explain the peculiar shape of M67 was analyzed and rejected. According to Faulkner, Roxburgh and Strittmatter (1968) a rotating star seen equator-on appears brighter and redder. For a change in color of $\Delta(B-V) = 0.05$ mag, a 2 $M_{\odot}$ object would have $\Delta M_{bol} = 0.20$ mag and a surface equatorial velocity of 1500 km/sec, while a 1 $M_{\odot}$ would have $\Delta M_{bol} = 0.16$ mag and 1000 km/sec. It appears very unlikely to have rotation present only during the thick shell burning phase, especially for the extreme values required.

Up to now, the effect of mass loss has not been considered in the age determinations. If any mass loss has occurred the ages computed here would be upper limits to the real ages. This effect is indeed present to some degree. However it is difficult to improve the fit through the assumption of mass loss since a fit of the main sequence yields an age of about 2.0×10^9 years; alternatively, a fit of the post main sequence shape yields an age of 3.2×10^9 years. Another possibility would be a drastic decrease of the mass with a very short time scale only at the turnoff point, but not at other stages of evolution; this seems very unlikely too.

In view of all the difficulties already mentioned, the age determination of M67 is very uncertain; the presence of the gap sets an upper limit of 4×10^9 years although it is more plausible that it was formed earlier, probably about 2 or 3 billion years ago.

V. Comparison with NGC 6791

The photographic photometry by Kinman (1965) of this rich open cluster has been the basis of the comparison with the theoretical isochrones. A reddening value of $E(B-V) = 0.22$ mag was determined by Kinman for this cluster. The observed diagram of NGC 6791 contains a well defined main sequence, turnoff point, horizontal branch, and no apparent gap. Since NGC 6791 is a faint cluster, the observational errors are large and it is not possible to rule out the presence of a gap. Following the procedure by Aizenman *et al.* (1969) a plot of $N(V)$, the number of stars fainter than magnitude V, projected on a line of constant $B-V$ was constructed and is shown in Figure 4; a gap would appear on this diagram as a vertical discontinuity of the line connecting the points. The plot for NGC 6791 shows no evidence of a gap.

From the comparison of the shape of this cluster with the theoretical curves, values for the age and distance have been obtained for the three different chemical compositions available. The results are given in Table 3 that includes also the comparison with the cluster assuming a reddening correction of $E(B-V) = 0.11$ mag, in order to study the variations with reddening. The range in compositions that fit NGC 6791 is much wider than for NGC 188 since there are no restrictions due to the presence of a gap and its luminosity.

Spinrad and Taylor (1971) estimate that the metal abundance of NGC 6791 is from 1.5 to 2.0 times higher than the metal abundance of NGC 188, which is already about three times higher than the solar one.

A composition of $Z = 0.10$, $X = 0.60$ would agree with the "super-metallicity" of the cluster as well as with the shape of NGC 6791 as is shown in Figure 2*c*. The age that this fit yields is 4×10^9 *years* and the distance modulus $(m\text{-}M)_0$ *is* 13.30 ± 0.15 *mag*, which yields a distance to the cluster of 4600 pc, or 900 pc above the galactic plane. This fit predicts a gap of $\Delta V = 0.12$ mag at $M_{bol} = 4.5$ mag, that corresponds to $V_0 = 17.8$ mag or $V = 18.5$ mag. Since the observational mean errors are of 0.057 mag for the faint stars of the cluster, and the expected stellar density for narrow gaps is not reduced drastically (See Figure 1), it is plausible that a 0.12 mag gap could be present at that position, and yet be undetected.

VI. Conclusions

The comparison with theoretical models yields ages for the open clusters NGC 188, M67 and NGC 6791 in the range of 2.3 to 4×10^9 years. According to the models by Torres-Peimbert (1971) these ages cannot be increased substantially for NGC 188 or M67 since the C-M diagrams of these two clusters show a gap.

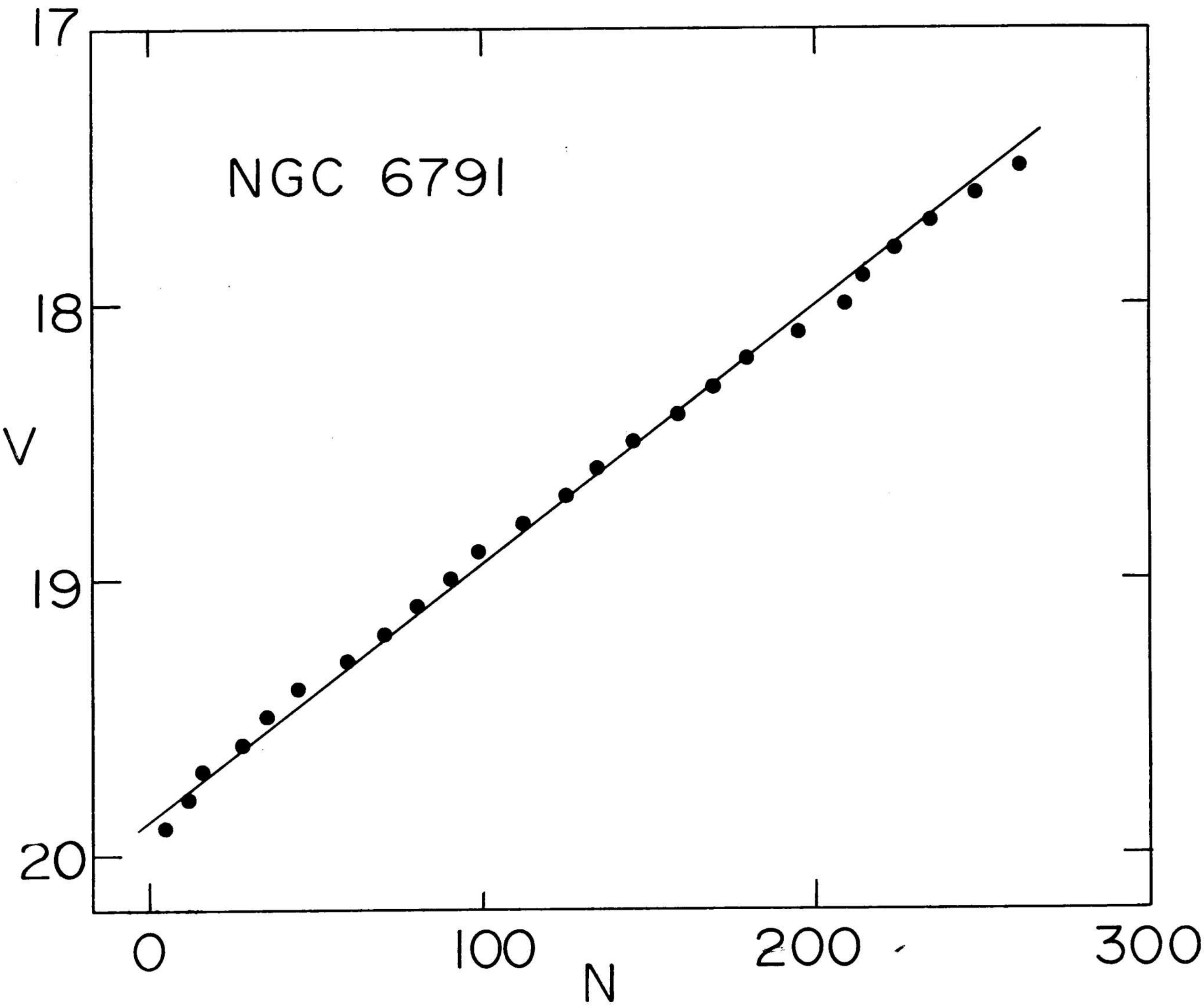


Fig. 4.—*Star counts for NGC 6791. The ordinate is the visual magnitude and the abscissa is the number of stars fainter than V and brighter than 20 mag. A vertical discontinuity would indicate the presence of a gap.*

TABLE 3

Age and Distance of NGC 6791

	$E(B-V) = 0.11$			$E(B-V) = 0.22$		
	Age (y)	$(m\text{-}M)_0$	*Comments*	*Age (y)*	$(m\text{-}M)_0$	*Comments*
$Z = 0.10$ $X = 0.60$	9×10^9	12.9	No gap predicted	4×10^9	13.3	Predicted gap. .12 mag wide
$Z = 0.05$ $X = 0.633$	$>9 \times 10^9$	—	No gap predicted	5.5×10^9	13.2	No gap predicted
$Z = 0.023$ $X = 0.651$	$>9 \times 10^9$	—	No gap predicted	6×10^9	13.1	No gap predicted

The theory of stellar structure agrees with a high metal abundance for NGC 188; this result confirms the spectral observations and excludes atmospheric effects to be responsible for the metal enrichment or the line anomalies.

The derived ages for the "old" galactic clusters NGC 188, M67, and NGC 6791 are considerably smaller than the value determined for the globular clusters of 8-9 $\times$ 10^9 years (Rood and Iben 1968); this difference might be due to the smallness of the sample of open clusters according to King (1968) and, therefore, the existence of galactic clusters as old as the galaxy should not be ruled out.

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