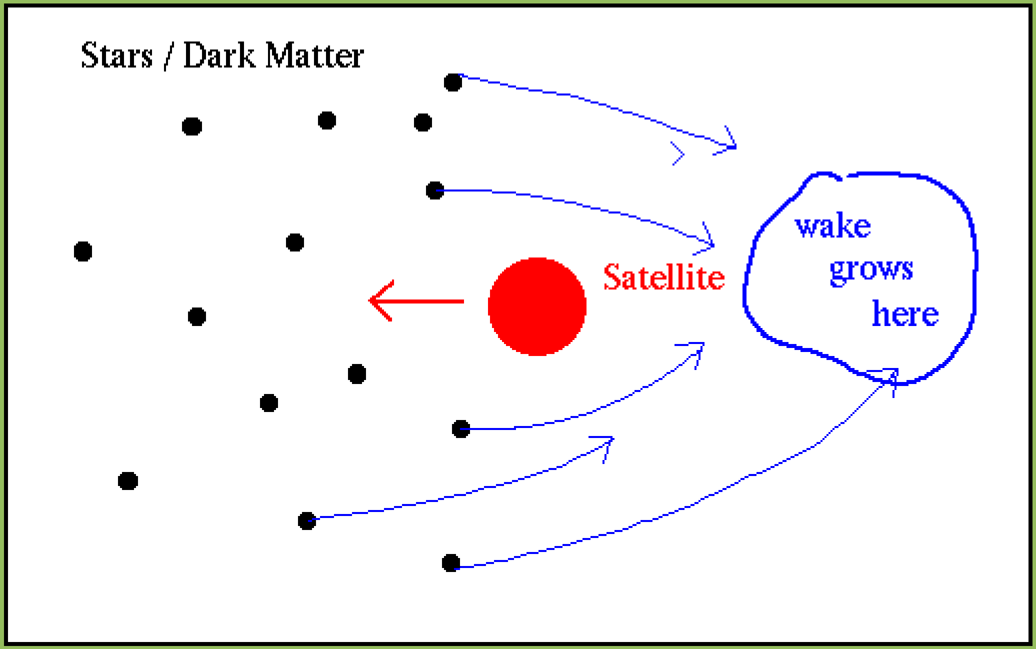
Dynamical Friction:

Anisotropic velocity distributions

Oscar Esquivel
Max-Planck Institute for Astronomy
Heidelberg, Germany

Introduction.

- Dynamical friction (DF) can be understood as the loss of momentum and kinetic energy of moving bodies through a gravitational interaction with the surrounding matter in space.



Context.

- Chandrasekhar (1943) envisaged the scenario of a sequence of consecutive two-body encounters of test and field stars in order to calculate the drag force

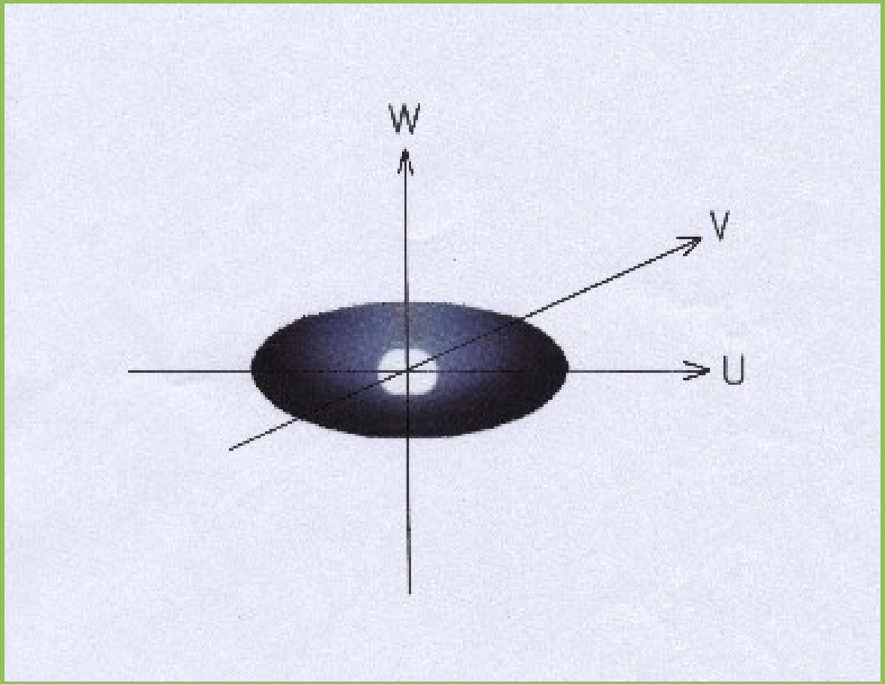
$$< \dot{\mathbf{v}} > \sim \frac{M^2 \ ln(\Lambda)}{v^2} \qquad (1)$$

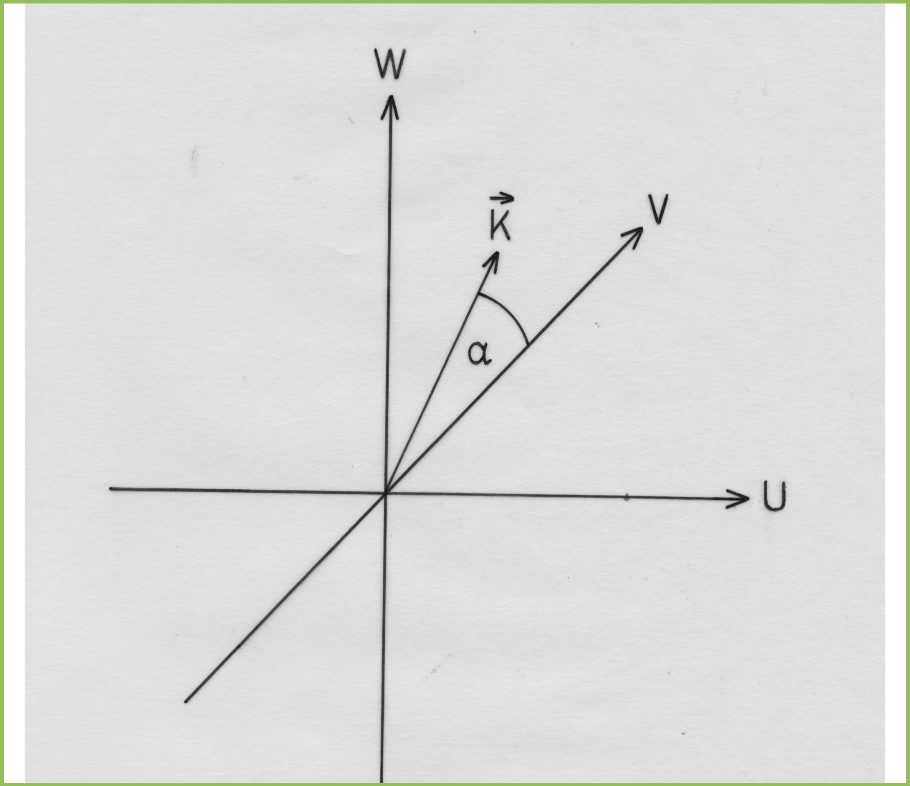
- In an alternative approach Marochnik ('68), Kalnajs ('72), and Esquivel ('07) determined the collective response as a polarization cloud decelerating the perturber, the last work considering various potentials in addition to that of a point mass.

- White ('76) obtained the DF by considering the impulse approximation where the principal result was a modification of the Coulomb logarithm so that it **does not diverge** anymore at small scales.

- Finally, Binney ('77) showed that the anisotropy of the distributions of the brighter galaxies in a cluster will be increased by the effect of DF by considering a 2D anisotropy in the velocity distributions of background particles.

The Treatment.





- The response of the system is determined by solving the linearized Boltzmann equation

$$\frac{\partial f_1}{\partial t} + \sum_{i=1}^{3} \mathrm{v_i} \frac{\partial f_1}{\partial \mathrm{x_i}} - \frac{\partial \Phi_1}{\partial \mathrm{x_i}} \frac{\partial f_0}{\partial \mathrm{v_i}} = 0\,, \qquad (2)$$

- Whose solution is greatly facilitated by Fourier-Transforming both f and Φ, thus finding

$$\omega f_{\omega,\mathbf{k}} + \upsilon k f_{\omega,\mathbf{k}} - k \Phi_{\omega,\mathbf{k}} \frac{\partial f_0}{\partial \upsilon} = 0 \qquad (3)$$

- For the field stars we consider a velocity distribution function that satisfies

$$\frac{\partial f_0}{\partial \upsilon} = -\frac{\upsilon}{\sigma_{eff}^2} \frac{n_\mathrm{b}}{\sqrt{2\pi}\sigma_{eff}} e^{-\frac{\upsilon^2}{2\sigma_{eff}^2}}\,, \qquad (4)$$

- where

$$\sigma_{eff} = \sqrt{\sigma_u^2 cos^2\alpha + \sigma_w^2\, sin^2\alpha} \tag{5}$$

- Integrating the next eq. over the *v*-velocity leads to the density distribution of the induced polarisation cloud

$$f_{\omega,\mathbf{k}} = -\frac{kv}{\omega + kv} \frac{n_\mathrm{b}}{\sqrt{2\pi}\sigma^3} e^{-\frac{v^2}{2\sigma^2}} \Phi_{\omega,\mathbf{k}} \,. \tag{6}$$

- As next we calculate the Fourier-Transform of the potential of a point mass which moves with a velocity v_0 along the y-axis

$$\Phi_\mathbf{k} = -\frac{Gm}{2\pi^2} \frac{1}{k^2} e^{-i\mathrm{k_y v_0}t} \,. \tag{7}$$

- Then from symmetry reasons the acceleration vector is expected to be oriented along the y-axis. Following Landau's rule we introduce $\omega = -K_y v_0 - i\lambda$ and we get

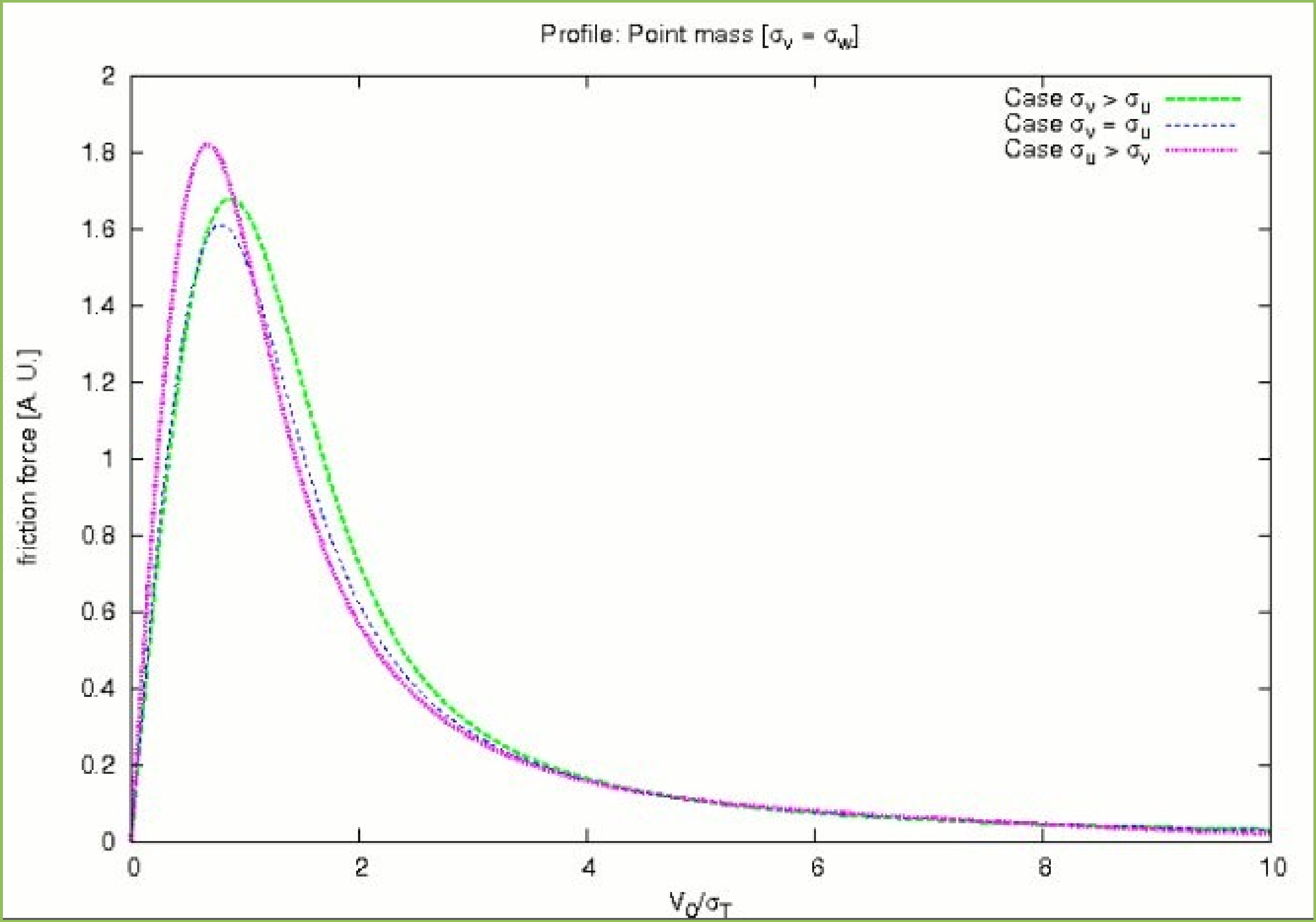
$$< \dot{\mathbf{v}} >= -\int d^3\mathrm{x} \int d^3\mathrm{v} f(\mathbf{x}, \mathbf{v}) \nabla\Phi \,, \tag{8}$$

- And we able to solve for the acceleration of the ensemble of stars caused by the perturber

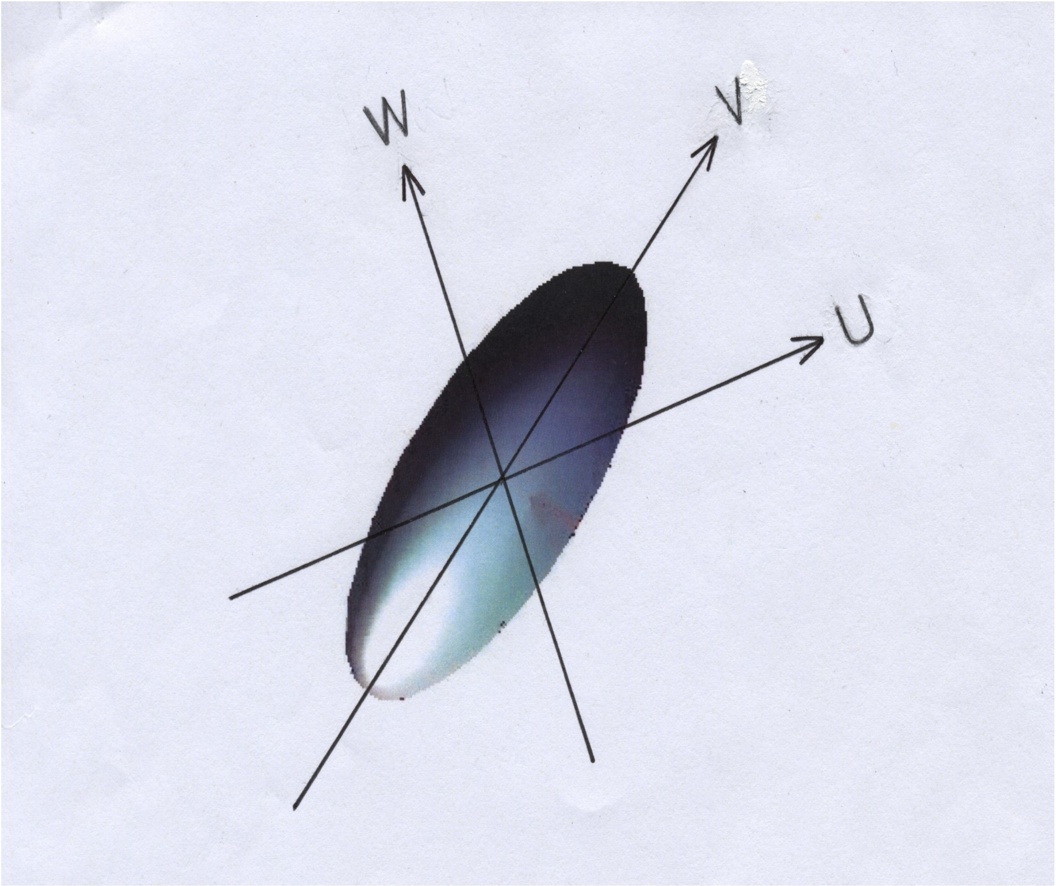
$$\begin{aligned} < \dot{\mathbf{v}} > &= -\frac{(2\pi)^{5/2} n_\mathrm{b}}{\sigma^3} \int d^3\mathrm{k} \int_{-\infty}^{\infty} dv \, \frac{ik\mathbf{k}v}{-\mathrm{k_y v_0} - i\lambda + kv} \\ &\qquad \times |\Phi_\mathbf{k}|^2 e^{2\lambda t} e^{-\frac{v^2}{2\sigma^2}} \\ &= \frac{(2\pi)^{5/2} n_\mathrm{b}}{\sigma^3} \int d^3\mathrm{k} \int_{-\infty}^{\infty} dv \, \frac{k\mathbf{k}v\lambda}{(kv - \mathrm{k_y v_0})^2 + \lambda^2} \\ &\qquad \times |\Phi_\mathbf{k}|^2 e^{2\lambda t} e^{-\frac{v^2}{2\sigma^2}} \,. \end{aligned} \tag{9}$$

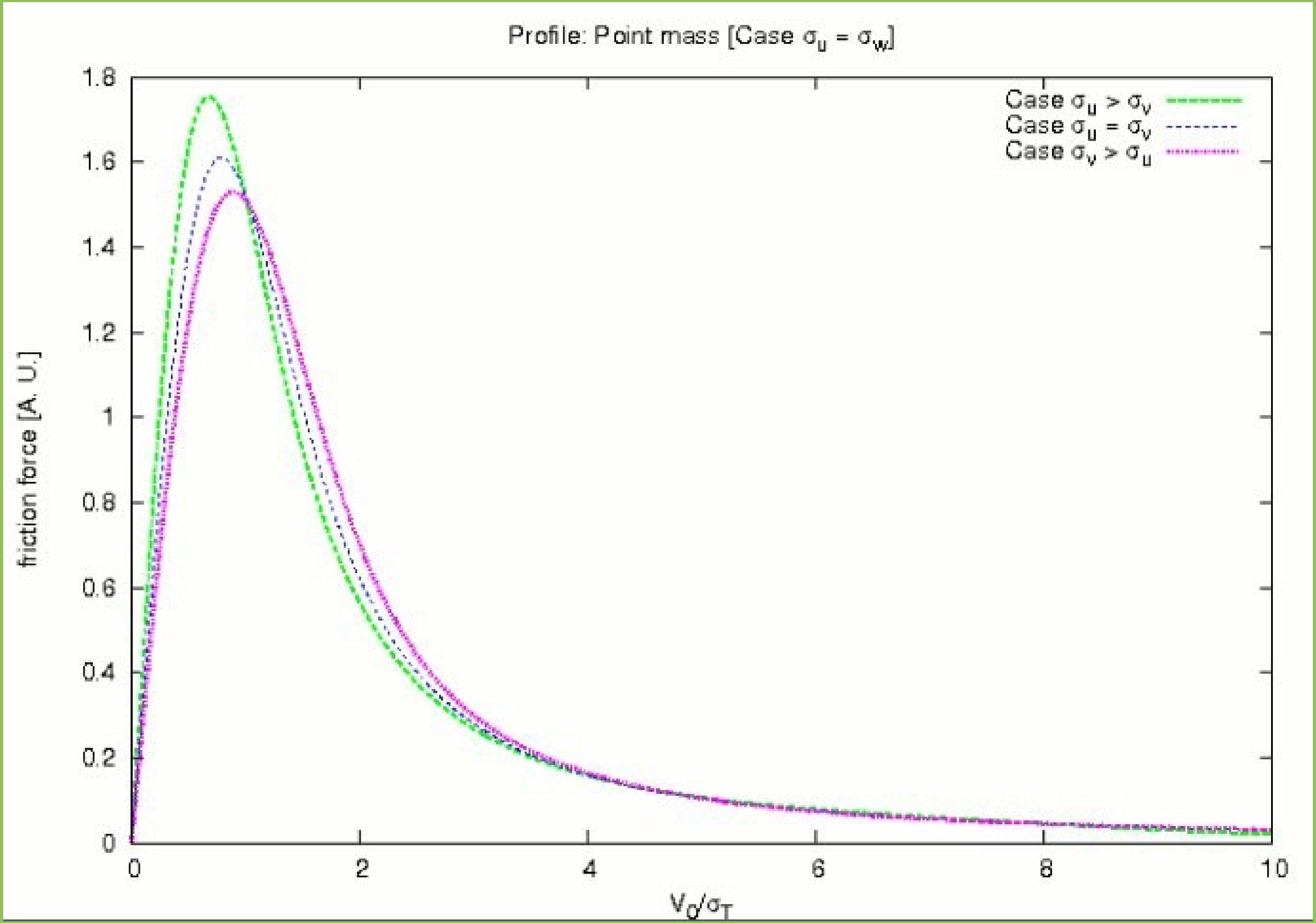
- Taking the limit as $\lambda \rightarrow 0$, and trying to solve as many integrals as possible we arrive at

$$< \dot{\mathbf{v}} >= -2\frac{\sqrt{2\pi}}{\sigma_T^2}\rho_b G^2 M^2 \frac{v_0}{\sigma_T}\frac{\sigma_T^3}{\sigma_w^3} ln(\Lambda) \times \int_0^1 dk_y^* \frac{\sqrt{k_y^*} e^{-\frac{k_y^* v_0^2 \sigma_T^2}{2\sigma_T^2 \sigma_w^2}}}{\sqrt{1+(\frac{\sigma_T^2}{\sigma_w^2}-3)k_y^*}} \tag{10}$$


Profile: Point mass [$\sigma_v = \sigma_w$]
Case $\sigma_v > \sigma_u$
Case $\sigma_v = \sigma_u$
Case $\sigma_u > \sigma_v$
friction force [A. U.]
V_0/σ_T
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
2
4
6
8
10

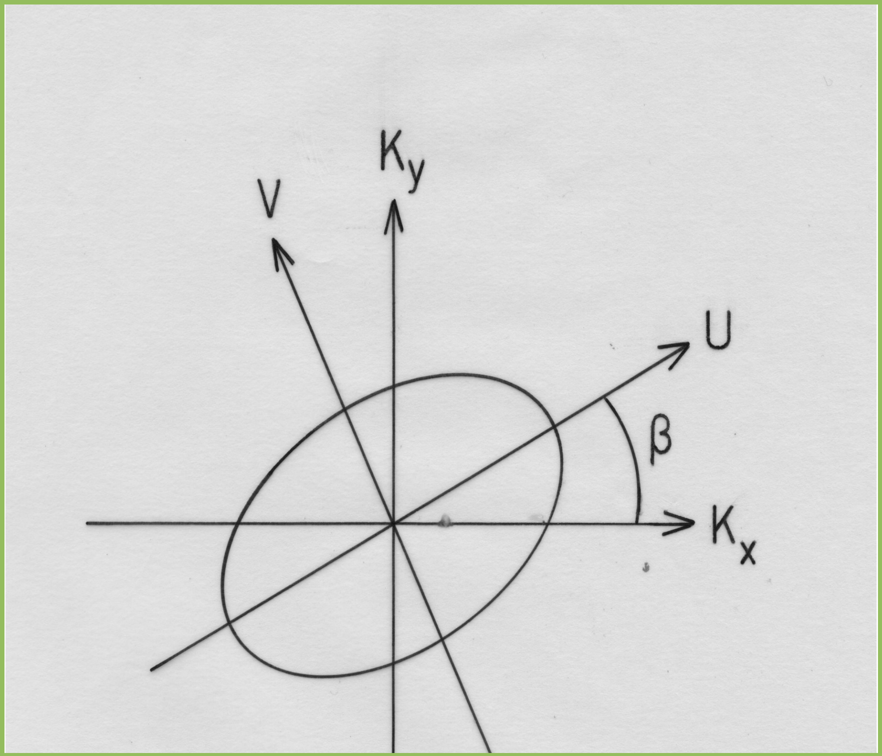
Next case

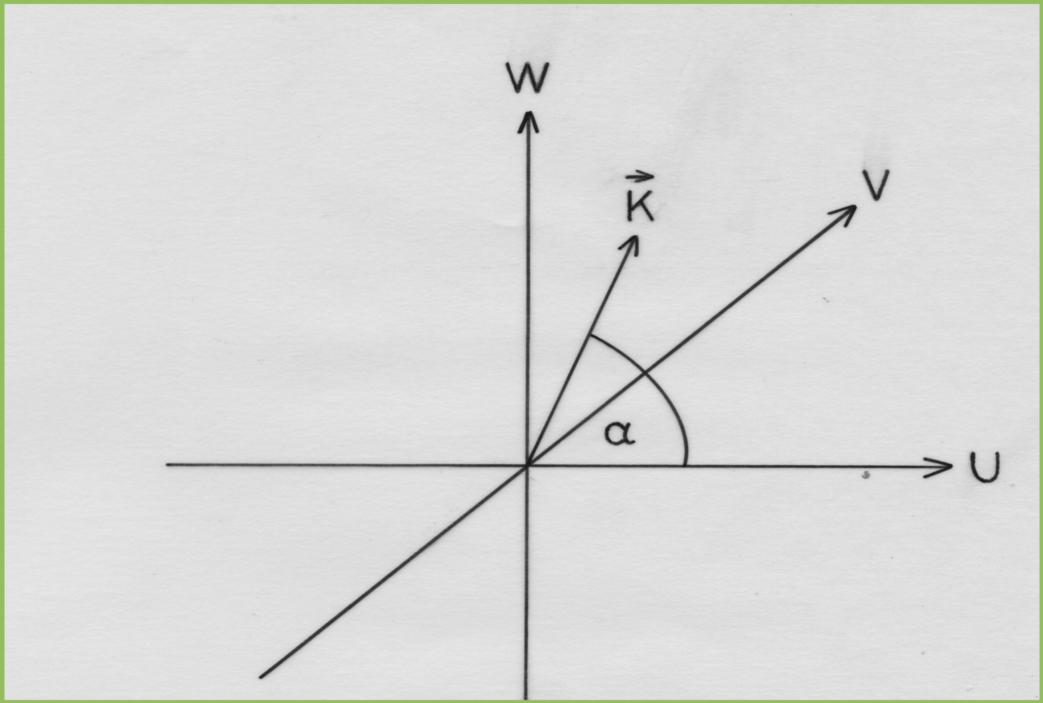




Profile: Point mass [Case σu = σw]
Case σu > σv
Case σu = σv
Case σv > σu
friction force [A. U.]
V0/σT
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
2
4
6
8
10

Future





$$\sigma_{eff}^2 = k_x^2(\sigma_u^2 cos^2\beta + \sigma_w^2 - \sigma_w^2 cos^2\beta)$$
$$+2k_xk_y(\sigma_u^2 sin\beta cos\beta - \sigma_w^2 sin\beta cos\beta)$$
$$+k_y^2(\sigma_u^2 sin^2\beta + \sigma_w^2 - \sigma_w^2 sin^2\beta) + \sigma_w^2 k_z^2 \qquad (11)$$

$$< \dot{\mathbf{v}} >= -\frac{4}{3\sigma_T^2}\sqrt{\frac{2}{pi}}\rho_b G^2 M^2 \frac{v_0}{\sigma_T}\frac{\sigma_w}{\sigma_T} ln(\Lambda)\int_{-\inf}^{\inf} dk^*$$
$$\times \frac{B(2, 0.5)}{\lambda^4}\Phi_1(2, 1, 2.5; -(kx^{*2} + 1 - \lambda^2)/\lambda^2; \frac{v_0^2}{2\sigma_w^2\lambda^2}) \qquad (12)$$