A simple accretion model of a rotating gas sphere onto a Schwarzschild black hole

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1 Summary

2 Spherical accretion

★ Model built “simply for mathematical curiosity” (Bondi 2005) by Bondi (1952).
★ It considers a central gravitational potential produced by a central mass $M$ embedded into an infinite gas cloud.
★ Cloud is at rest at $\infty$ with a pressure $p_\infty$ and a density $\rho_\infty$.
★ Radial symmetry and stationary situation.
★ Polytropic relation: $p \propto \rho^\kappa$.
★ Continuity equation applied to a streamline implies that:

$$\dot{m} = 4\pi r^2 \rho v = \text{const.} \quad (1)$$

★ Bernoulli’s equation can be integrated to obtain velocity field.
★ This model “provides a good estimate to the order of magnitude of the accretion rate in all cases of physical interest” (Bondi, 1952).
3 Spherical accretion with rotation (Ulrich 1976)

☆ Small perturbation to Bondi’s model: the gas cloud rotates as a rigid body in such a way that its specific angular momentum $h$ has a value $h_\infty$ far away from the rotation axis which satisfies $h_\infty \ll 1$.
☆ Since the accretion process is sufficiently slow ⇒ mass $M$ of the central object is constant.
☆ Self–gravity of the gas is negligible compared to the gravity of central object.
☆ Collisions between particles arriving to the equatorial plane ⇒ formation of an accretion disc of radius $r_d = h_\infty^2 / GM$.
☆ Assuming the mass of the disc $M_d \ll M$, and no viscosity ⇒ trajectory of a fluid element is described by a Newtonian central potential since its specific energy $E \approx 0$ along all the fluid’s particle trajectory.
Streamlines and density isocontours (Mendoza, Cantó & Raga 2005).
4 Spherical accretion on a Schwarzschild space–time (Michel 1972)

★ Gas cloud without rotation. Particles fall from $\infty$ to the black hole in a stationary way.
★ Accretion rate given by:

$$\dot{n} = 4\pi r^2 nu = \text{const.} \quad (2)$$

★ Relativistic Bernoulli’s equation can be integrated to find the solution (Huerta & Mendoza 2006).
★ Michel (1972) obtained his solutions using the time component of $\nabla_\mu T^{\mu\nu} = 0$ + continuity equation.
5  Relativistic accretion with rotation

- Generalisation of Ulrich’s accretion model in a Schwarzschild space–time.
- Convergence with Ulrich and Michel’s models.
- Problem: integrate equation of motion for material particles taking into account general relativity, without approximations, i.e. no perturbations and no pseudo–Newtonian potentials.
- Solution: exact (analytic) integral of motion in terms of Jacobi elliptic functions.
- Under these assumptions one can obtain analytical solutions for the streamlines, velocity field and accretion flow density (Huerta & Mendoza, 2007)
6 Results

The model describes “parabolic” orbits and so, it turns out that it depends on a parameter $\alpha$, s.t., when $\alpha \to 0$, then $c \to \infty$. In fact, $0 \leq \alpha \leq 1/8$ and when $\alpha \to 1/8$ the specific angular momentum $h = 2r_g$ reaches its minimum value.

Solutions converge to Ulrich’s model when $\alpha \to 0$, and to Michel’s one with null pressure in the absence of angular momentum, i.e. $h = 0$.

Why is it a coherent model?

- Resulting orbits satisfy: $E_{tot} = 0$.
- Orbits are the relativistic counterpart of the parabolic Newtonian model obtained when $\alpha = 0$.
- Exact analytic model that gives information about velocity field and density.
Unexpected results:

🌟 Softening of the density divergence at the border of the disc.
🌟 Streamlines are not packed at the border of the disc.
🌟 Accretion disc extends to all the equatorial plane since its radius $r_d \to \infty$ as $\alpha \to 1/8$. ¡This can be proved analytically!
Lee & Ramirez–Ruiz (2006) built a similar Pseudo-Newtonian accretion model with the same conditions as our model.

Notable differences at small and large scales in the streamlines. In the full relativistic model, particles approach faster the equatorial plane. The hole does not swallow directly all particles on its surroundings, they are previously injected to the disc.
7 Conclusions

☆ We had built a relativistic generalisation of Ulrich’s accretion model.
☆ Essential features are preserved.
☆ Accretion disc radius increases without limit when $h = 2r_g$.
☆ Divergence in the particle number density at the border of the disc occurs only for the Newtonian case.

Future: together with E. Tejeda y E. Ley Koo we are building an accretion model of a rotating gas cloud towards a Kerr black hole.