The evolutionary phase of B[e] stars is difficult to establish due to the uncertainties in their fundamental parameters. For instance, possible classifications for the Galactic B[e] star MWC 137 include pre-main-sequence and post-main-sequence phases, with a large range in luminosity. Our goal is to clarify the evolutionary stage of this peculiar object, and to study the CO molecular component of its circumstellar medium. To this purpose, we modeled the CO molecular bands using high-resolution K-band spectra. We find that MWC 137 is surrounded by a detached cool (T = 1900 +- 100 K) and dense (N = (3 +- 1) x 10^21 cm^-2) ring of CO gas orbiting the star with a rotational velocity, projected to the line of sight, of 84 +- 2 km/s. We also find that the molecular gas is enriched in the isotope 13C, excluding the classification of the star as a Herbig Be. The observed isotopic abundance ratio (12C / 13C = 25 +- 2) derived from our modeling is compatible with a proto-PN, main-sequence or supergiant evolutionary phase. However, based on some observable characteristics of MWC 137, we propose that the supergiant scenario seems to be the most plausible. Hence, we suggest that MWC 137 could be in an extremely short-lived phase, evolving from a B[e] supergiant to a blue supergiant with a bipolar ring nebula.