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THE KINEMATICS OF TRAPEZIUM SYSTEMS

CHRISTINE ALLEN AND ARCADIO POVEDA
Instituto de Astronomía
Universidad Nacional Autónoma de México

and

CHARLES E. WORLEY
U. S. Naval Observatory
Washington, D. C.

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RESUMEN

Se han analizado todas las observaciones de los trapecios de Ambartsumian, con el propósito de establecer las propiedades cinemáticas de este interesante grupo de estrellas múltiples. No se encontró evidencia alguna de que exista una expansión sistemática en ninguno de los 46 trapecios estudiados. En particular, el Trapecio de Orión, que es el caso mejor documentado, no está en expansión. Sin embargo, una fracción apreciable de los trapecios contiene estrellas con movimientos relativos. Las velocidades transversales de algunas de estas estrellas sobrepasan los 30 km seg^{-1}. Este hecho resulta importante para entender la dinámica de los trapecios.

ABSTRACT

An analysis has been made of all the observations available of Ambartsumian's trapezia, with the aim of establishing the kinematic properties of this interesting class of multiple stars. It was found that there is no evidence for a systematic expansion in any of the 46 trapezia that were studied. In particular, the Orion Trapezium, the best documented case, shows no expansion. However, a sizeable fraction of the trapezia show one or two stars with relative motions. A statistical test and a proper motion test have been applied to establish membership of these stars in their trapezia. Some of these stars have transverse velocities larger than 30 km sec^{-1}. This behaviour is important in understanding the dynamics of trapezia.

Key words: MULTIPLE STARS — TRAPEZIA — ORION TRAPEZIUM — RUNAWAY STARS — STELLAR KINEMATICS.

I. INTRODUCTION

In an extensive and fundamental paper on the Orion aggregate, Parenago (1953) concluded that the stars of the Trapezium were expanding and that the energy of this multiple system was greater than zero. Furthermore, by combining the present dimensions of the Trapezium with the velocity of expansion that he had obtained, Parenago found that the expansion age of the Orion Trapezium could not be larger than 10 000 years.

Parenago's conclusion on the Trapezium remained for many years an intriguing result, because of the very small age that it implied for the member stars. It was not until the study of the expansion age of the Orion Nebula —based on gas-dynamical considerations— indicated an age between 10 000 and 20 000 years (Kahn and Menon, 1961; Vandervoort, 1964) that the result of Parenago began to be taken more seriously in the West, particulary since independent —but unpublished— work by Franz (1965) and Strand (1967, 1973) seemed to confirm Parenago's result.

Before the gas-dynamical approach to the age of the Orion Nebula, Ambartsumian (1954), in a classic paper on trapezium systems, stressed the fact that multiple star systems of the trapezium type should be unstable, and that after a few crossing times the whole configuration should break apart. Using the results of Parengo for the Orion Trapezium, Ambartsumian concluded that most trapezium systems are multiple stars with positive energy and, therefore, in the process of expansion.

The previous considerations led several authors (*e. g.* Sharpless, 1965) to propose that the expanding motions of O-associations were a result of the expansion of the many trapezia born in the volume occupied by the stars of a given association.

From purely cosmogonical and dynamical considerations, Parenago's result is strange and unexpected, because the process of star formation implies a mass distribution in contraction, *i. e.,* with negative energy, from which the stars begin their dynamical history. Clearly, one would expect trapezia to be bound systems, which occasionally eject stars with positive energy as the result of dynamical interactions. The space motions of the trapezium stars, at least at the beginning of their dynamical history, should reflect the initial conditions of the cloud from which they were formed (*i.e.* contraction and negative energy). In an effort to reconcile Parenago's result with the negative energy of the cloud, various authors have devised ways to balance the alleged positive energy of expansion of trapezia with the negative binding energy of the close binaries existing in such systems (*e.g.* Sharpless, 1965; Poveda *et al.,* 1967).

Clearly, the kinematic behaviour of trapezium systems contains valuable information about the initial conditions in which stars are formed. Thus, at present, we have two physically distinct lines of dynamical evolution for trapezia: (a) Trapezia with positive total energy, which expand without significant interaction among their stars. (b) Trapezia with negative total energy, which, through energy exchanges of their stars, evolve by ejecting every once in a while a star with positive energy. As the result of these energy exchanges, the system evolves towards a more tightly bound and hierarchic configuration.

The purpose of the present paper is to ascertain which of the two modes of evolution mentioned above is followed by observed trapezium systems. In particular, we analyze all published observations of the Orion Trapezium to test its alleged expansion.

II. ANALYSIS OF THE OBSERVATIONAL MATERIAL

In order to proceed in a systematic way, we searched all the trapezia in Ambartsumian's (1954) list to detect which of them had more than four different "observations" listed in the various catalogues of double stars (BDS, ADS, IDS, and the U. S. Naval Observatory card catalogue). By an "observation" of a trapezium we understand a measurement, at a given epoch, of the position angles and separations of at least three stars in the system. By this procedure, we found 42 trapezia —out of 108 in Ambartsumian's list— for which more than four observation at different times existed. We shall designate the trapezia of this group as "well-observed trapezia". For each one of the 42 well-observed trapezia we plotted the measured separations s_{ij} between components i and j $(i, j = 1, 2, \ldots)$ versus the time of observation.

In these plots, observers were given different weights, which were estimated according to the internal consistency of their measurements, the equipment they used, etc. Maximum weight was given to the visual observations by W. Struve and Burnham, and to the photographic measurements made at the U. S. Naval Observatory after 1960. High weight was given to observations by Barnard, Aitken, van den Bos and Finsen.

In this way, we ended up with 128 graphs, which then contained the available information about the kinematics of well-observed trapezia. All the graphs were carefully examined for any possible changes in the separations s_{ij}. Much to our surprise, we found that the existence of an overall contraction or expansion could not be established —not even marginally— for any of the 42 trapezia examined. Only 16 trapezia showed some indication of relative motion of one or two member stars. In order that the reader may judge for himself, we present in Figures 1 through 25 plots of the separations of the stars versus time of observation, for a representative sample of trapezia. In many cases, these observations cover a time interval of over 100 years. Because it is very difficult to express mathematically the relative weights to be given to the various observations, a least squares solution would not be very meaningful; instead we fitted by eye a line through the observed points of highest weight, for those components showing some indication of relative motion.

We would like to draw particular attention to the Orion Trapezium (ADS 4186), which has been observed extensively in the last 150 years (Figures 1 through 7). In this particular case, we can see that only star E shows a clearly detectable motion, whereas stars B and C show only marginal displacements; from the available observations it is therefore impossible to conclude that there exists an overall expansion. In fact, we do not see how Parenago could have arrived at the result that the Orion Trapezium expands in the way shown in Figure 26, which is reproduced from Parenago's paper.

Another object of interest is ADS 6033, better known as VY Canis Majoris, the enigmatic infrared, OH and H_2O source. This system shows marginal displacements of some of its components; Figures 23 through 25 exhibit the available measurements. Modern observation of this object (Worley, 1972) have cast doubt on its conventional classification as a multiple star; however, the available data

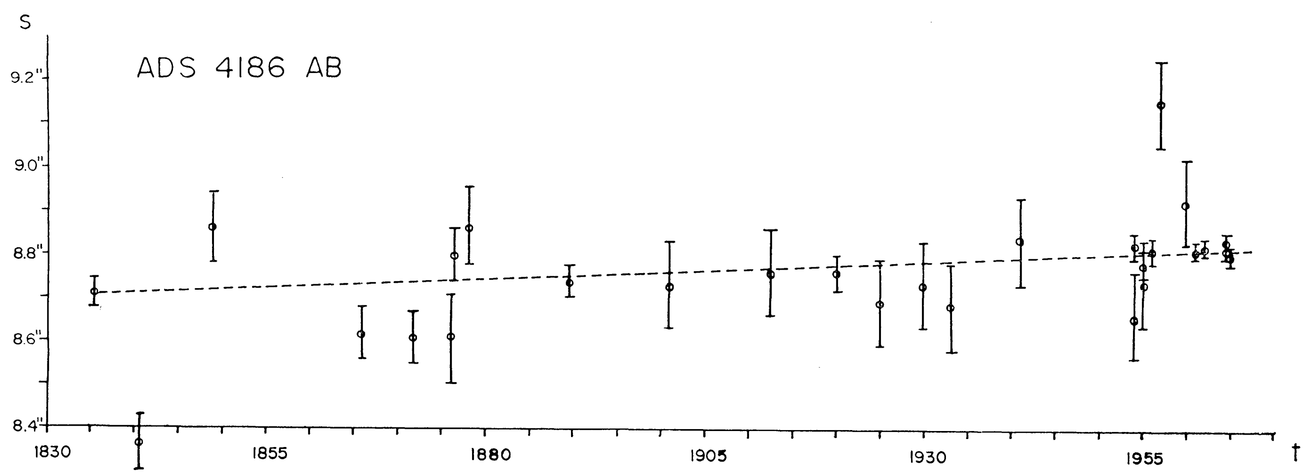

FIG. 1. Separation versus time for ADS 4186 AB (Orion Trapezium).

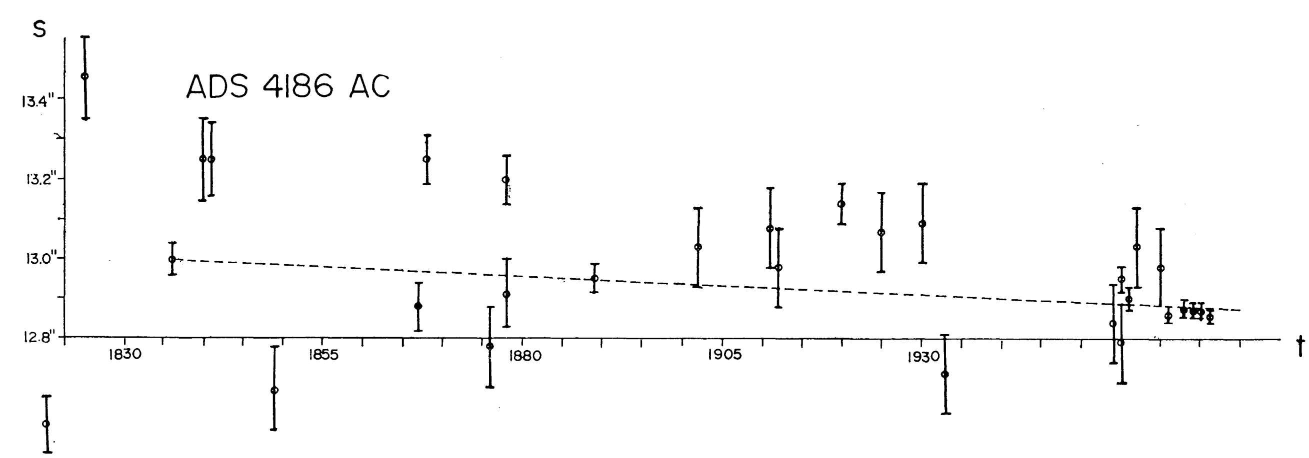

FIG. 2. Separation versus time for ADS 4186 AC (Orion Trapezium).

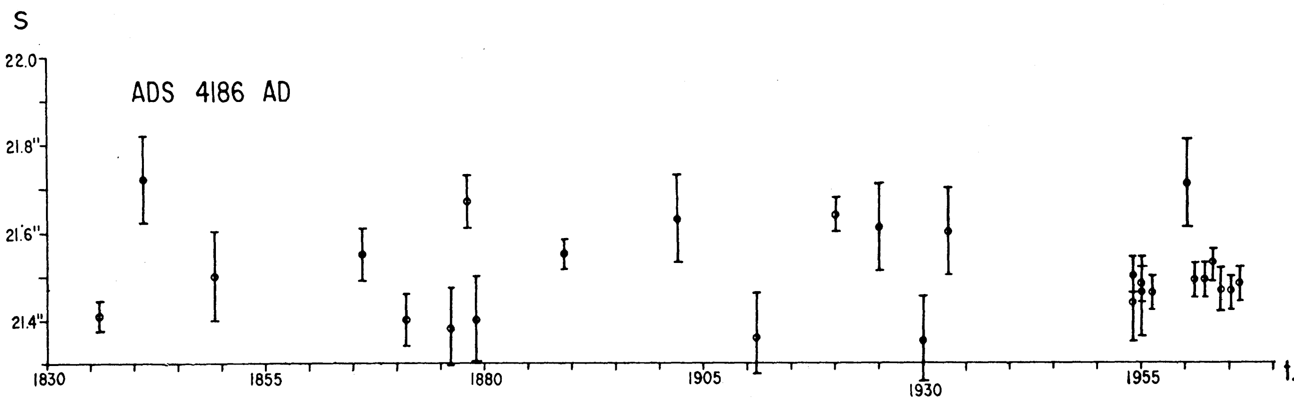

FIG. 3. Separation versus time for ADS 4186 AD (Orion Trapezium).

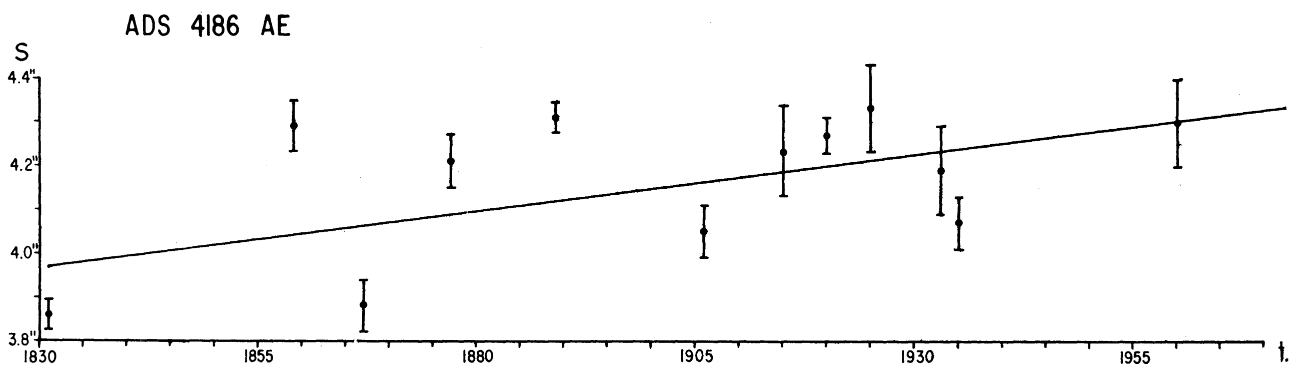

FIG. 4. Separation versus time for ADS 4186 AE (Orion Trapezium).

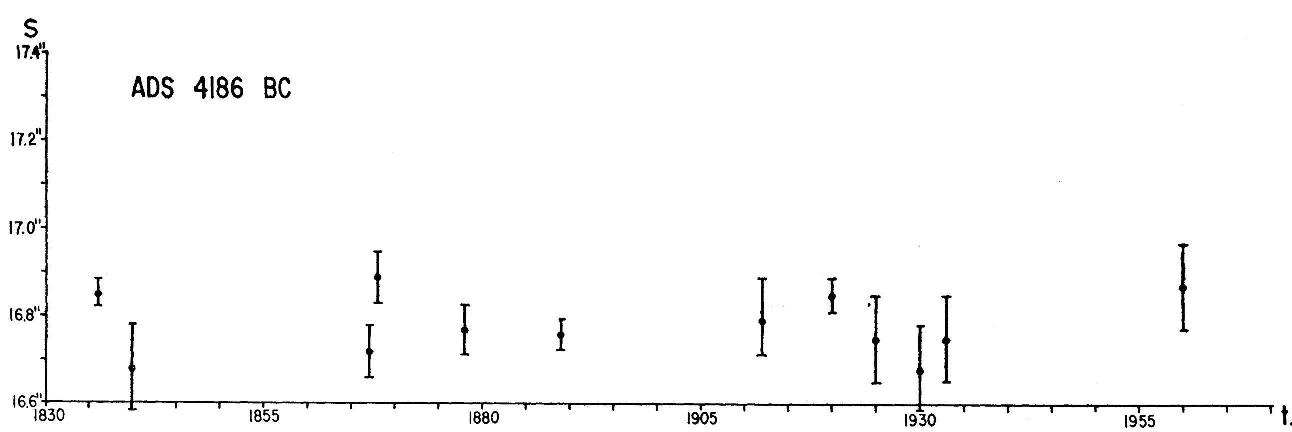

FIG. 5. Separation versus time for ADS 4186 BC (Orion Trapezium).

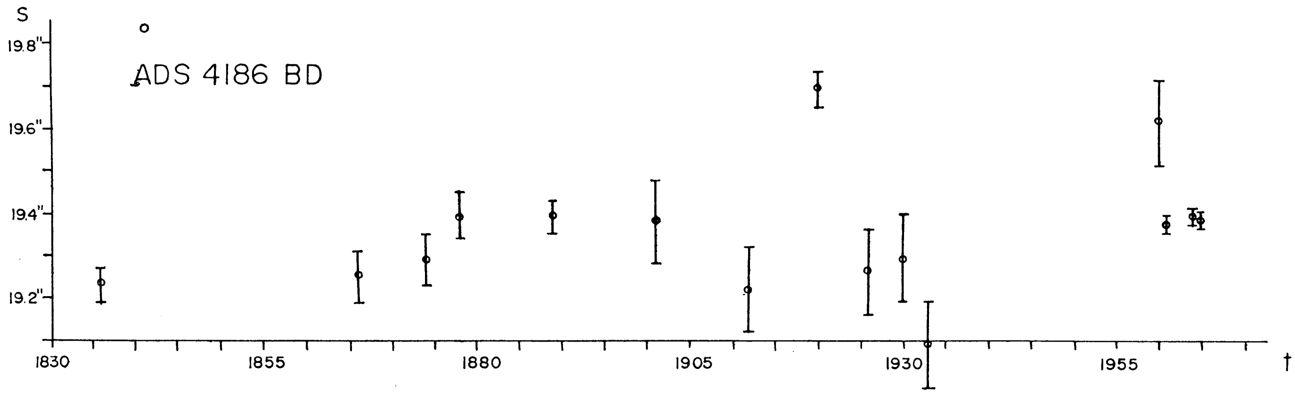

FIG. 6. Separation versus time for ADS 4186 BD (Orion Trapezium).

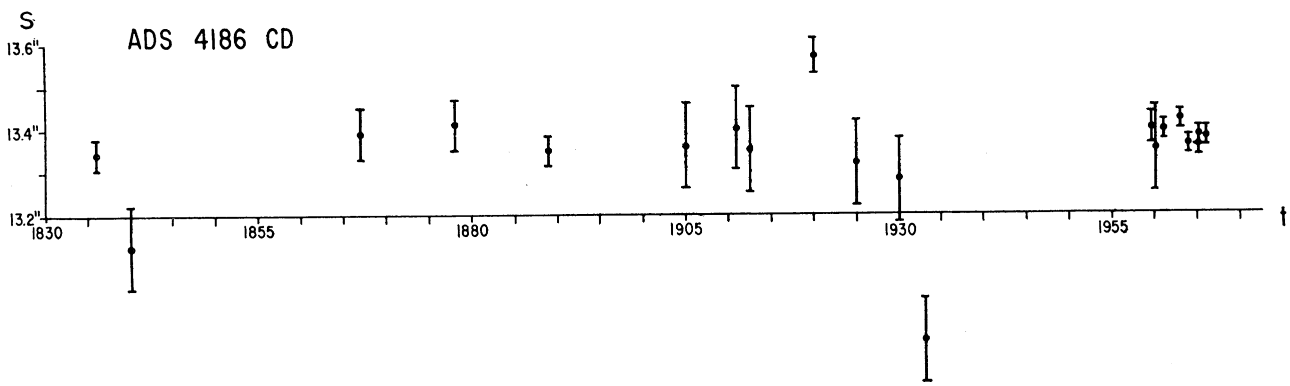

FIG. 7. Separation versus time for ADS 4186 CD (Orion Trapezium).

cannot yet rule out the possibility of VY Canis Majoris being a trapezium still embedded in its cocoon nebula.

Even though we did not find any indication of expansion, some trapezia exhibit one, or in exceptional cases, several, stars with a significant relative motion, as can be seen in Figures 4, 10 and 17. In total, from the 42 well-observed trapezia, 16 have at least one star showing a measurable relative motion. Table 1 contains a list of the group of well-observed trapezia and shows which of them contain relative-motion stars. In addition to the 42 well-observed trapezia we also list in Table 1 the systems ADS 6366 and ADS 15260, which, even though insufficiently observed, clearly have moving components.

The stars that show relative motion may do so for two reasons:

(a) They may be field stars, which generally would show a proper motion relative to the trapezium.

(b) They may be physical members of the trapezium, which have acquired a velocity large enough to be observable.

Clearly, in order to advance in our understanding of trapezia, we should find to which one of the cases (a) or (b) belong the relative-motion stars that have been found.

In order to clarify the nature of the relative-motion stars, we subjected these stars to a statistical test and a proper motion test. The statistical test proceeds as follows: let the separation of the moving star from the primary be s; if s is larger than $r(m)$ the moving star is considered to be an optical member; $r(m)$ is given by:

$$\pi N_{b,l}\,(m)\; r^2\,(m) \;=\; 10^{-2}, \qquad (1)$$

where $N_{b,l}(m)$ is the number of field stars per unit area brighter than magnitude m in the direction (b,l) of the trapezium in question, and m is the aparent magnitude of the moving star. Equation (1) establishes that the expected number of stars brighter than magnitude m inside a circle with radius $r(m)$ is only 10^{-2}. It follows from this that the expected number of *moving* field stars in the sample of 44 trapezia is much less than one.

To apply this test, we plotted r as a function of m (as defined in equation 1) for five suitably chosen intervals of l and b covering the areas of the sky that contain trapezia. Values for the stellar density $N_{b,l}$ were taken from the tables of Seares and Joyner (1928); for each area we used the largest value of $N_{b,l}$. With these graphs, it was an easy matter to verify if a moving component of magnitude m had a separation s smaller than $r(m)$ as required by equation (1). Moving components that did not pass this test do not appear in Table 2.

Because of random fluctuations in the distribution of field stars, it is possible that, among the 44 trapezia, one or two optical members may still survive the selection procedure described above. One would expect such optical survivors to have a proper motion different from that of the trapezium as a whole. Our second test consists, therefore, in a comparison of the vector $\bar{s}_{iA}(t)$ of the moving component i, relative to component A, against the displacement of component A due to its proper motion during the time the trapezium has been observed. (Usually the only proper motion available is that of component A). When the vector $\bar{s}_{iA}$ is approximately opposite to and of the same magnitude as the displacement vector of component A, we conclude that the moving component is a field star, because its aparent motion is only a reflection of the proper motion of component A. In Table 1 we indicate for which trapezia it was possible to apply such a test. Figures 8, 14 and 18 are examples of moving components that have passed the proper motion test. ADS 4186 E (Figure 8), ADS 13374 B, D (Fi-

TABLE 1

WELL-OBSERVED TRAPEZIA

Ambartsumian No.	ADS No.	Relative-Motion Stars	Proper Motion Test
3	364	—	
4	423	—	
5	719	—	
12	1877	—	
15	2159	AB, AC	yes, yes
20	2843	AB, AD, AE	yes, yes, yes
21	2984	—	
23	3579	—	
24	3684	—	
25	3940	AB	yes
26	3943	—	
27	4053	—	
29	4164	AC	no
30	4186	AE	yes
31	4241	—	
32	4728	—	
34	4962	—	
36	5322	—	
40	5977	—	
41	6033	AB	yes
44	6366	AB	yes
45	7372	AC	no
51	10991	—	
53	11168	—	
54	11169	—	
59	11344	—	
60	11421	AC	yes
66	13117	AB	yes
68	13312	—	
70	13374	AC	yes
72	13610	AE, BC	yes, no
73	13626	—	
77	14010	—	
82	14526	—	
85	14831	AC	yes
86	14885	—	
88	14969	—	
90	15184	—	
92	15260	AB	yes
100	15834	AB	no
101	15847	AC	yes
102	16095	—	
103	16474	AB	yes
104	16795	AE	yes

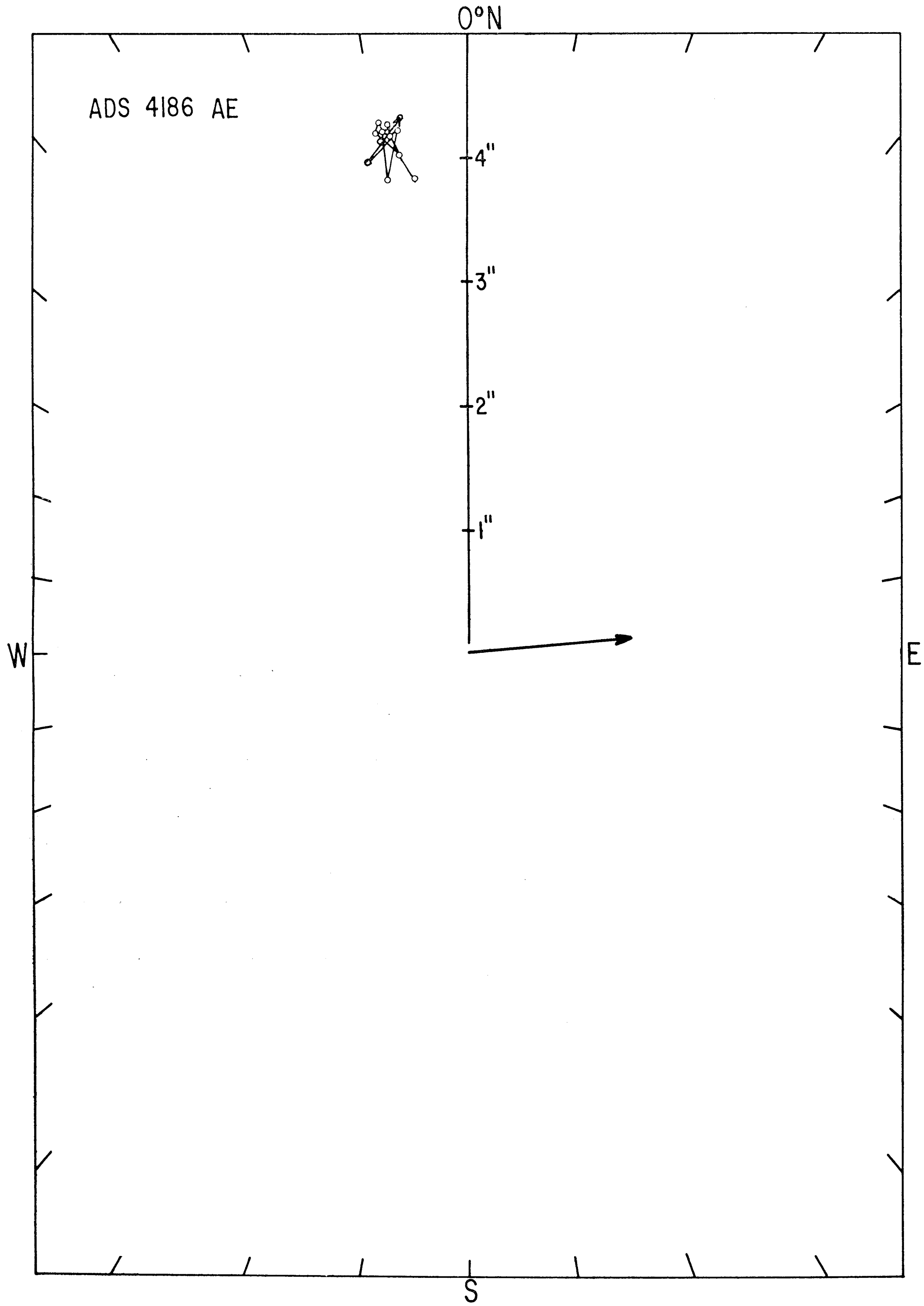

FIG. 8. Proper motion test for ADS 4186 AE (Orion Trapezium). The displacement vector of star A (due to its proper motion) during the time covered by the observations is plotted at the origin. The vector $\bar{s}_{EA}(t)$ is constructed by plotting (in polar coordinates) the position angles and separations of component E at different times of observation. The separations plotted correspond to those of Figure 4.

TABLE 2

TRAPEZIA WITH PHYSICAL MEMBERS SHOWING RELATIVE MOTION

ADS N°	Relative-Motion Stars	$\frac{ds}{dt}$ [" per 100 y]	Spectrum of Comp. A	Association	Distance [pc]	Transverse Velocity [km sec^{-1}]	Sources for Distance
2843	AB; AD	+1.8; +7.5	B1 Ib	ζ Persei	380	34.2; 142.5	Allen (1)
3940	AB	+0.2	B3	—	—	—	
4164*	AB; AC	+1.8; +2.9	—	—	—	—	
4186	AB; AE	+0.09; +0.25	O9.5 Vp	Orion	500	2.3; 6.4	
6033	AB	+0.7	M5 Ibp	NGC 2362	1500	52.5	Herbig (2)
6366	AB	−0.4	B9	—	≥330	≥6.6	
7372*	AC	+2.3	—	—	—	—	
11421	AC	+1.2	—	—	—	—	
13117	AB	−0.3	B5 V	—	295	4.4	
13374	AC	+3.1	WN5 + O9.5 III	NGC 6871	1600	248.0	Rubin *et al.* (3)
14831	AC	+0.4	B2 Ve	66υ Cygni	345	6.9	Rubin *et al.* (3)
15260	AB	+0.5	G0	—	≥87	≥1.7	
15834*	AB	+0.4	—	Cep OB1	3600	72.0	Morgan *et al.* (4)
15847	AC	+7.6	B5III	—	325	123.5	

REFERENCES TO TABLE 2

(1) Allen, C. W. 1963, *Astrophysical Quantities* (London: The Athlone Press).
(2) Herbig, G. H. 1970, *Ap. J.*, **162**, 570.
(3) Rubin, V. C., Burley, J., Kiasatpoor, A., Klock, B., Pease, G., Rutscheidt, E., and Smith, C. 1962, *A. J.*, **67**, 491.
(4) Morgan, W. W., Whitford, A. E., and Code, A. D., 1953 *Ap. J.*, **118**, 318

gure 14), and ADS 2843 D (Figure 18) passed the test because their vectors $\bar{s}_{iA}$, are not anti-parallel and not of the same magnitude as the displacement vector of their primaries; ADS 13374 C (Figure 14) passed because the vector $\bar{s}_{CA}$, although anti-parallel, is significantly smaller than the displacement vector of component A; therefore, the relative motion of component C with respect to A is not merely a reflection of the proper motion of A.

The stars that survive these two tests have a very high probability of being members of their trapezia, some of them with an unusually large motion. The trapezia with physical members showing relative motion are listed in Table 2. The entries in the Table are self-explanatory. In a few of the trapezia with moving stars it was not possible to apply the proper motion test, and therefore these stars have a smaller probability of being physical members. These components are marked with an a asterisk in Table 2.

The proper motion test should, however, be viewed with reservation. In fact, if as a result of dynamical interactions a physical member of a trapezium happens to be ejected with high velocity,

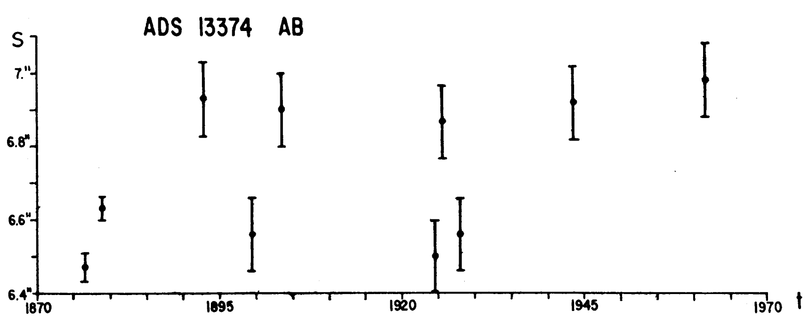

FIG. 9. Separation versus time for ADS 13374 AB.

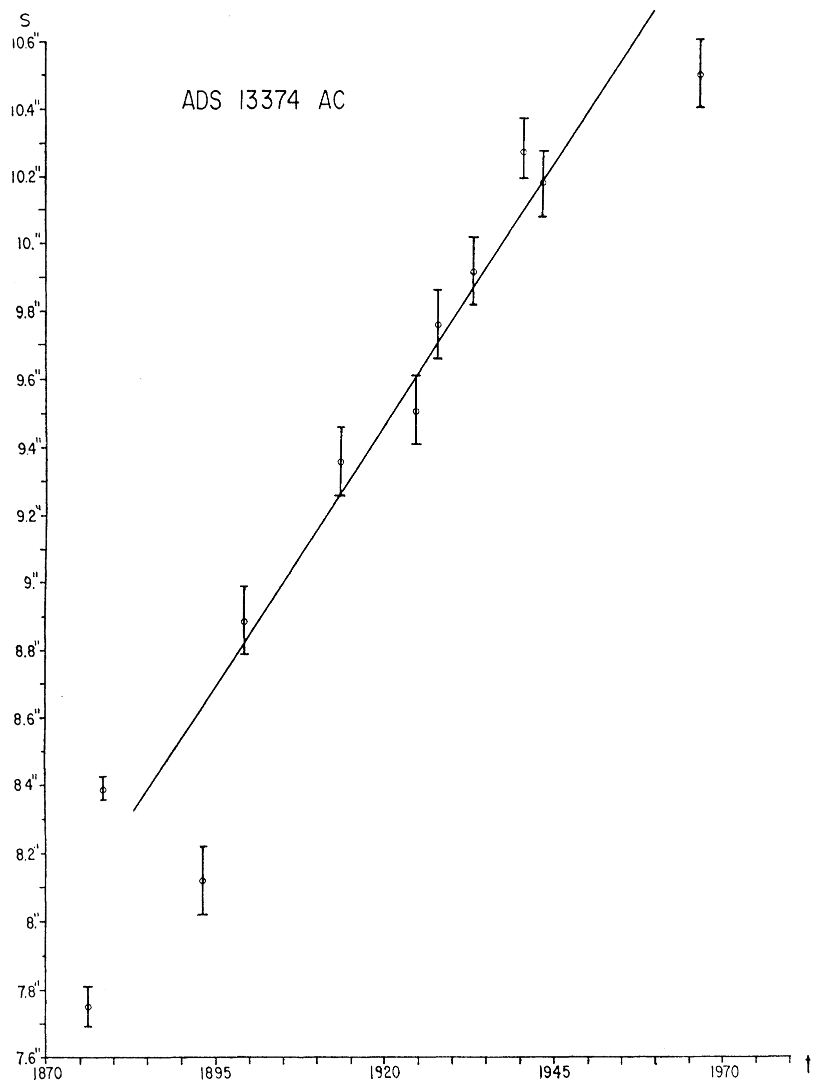

FIG. 10. Separation versus time for ADS 13374 AC.

then the whole trapezium (or some of its members) will recoil, by conservation of momentum. In such a case the proper motion test, within the errors of observation, may reject a physical member.

From the above treatment we can see that the relative-motion stars that have passed both tests are expected to be physical members of their trapezia. In this way, we are left with 14 trapezia, each one of them containing at least one relative-motion star which is very likely a physical member.

An inspection of the distribution of signs of ds/dt listed in Table 2 shows a remarkable preponderance of positive values. This is a significant piece of information, because it gives a further proof that the great majority of the relative-motion stars found cannot be field stars; if they were, we would expect equal numbers of positive and negative velocities.

An important matter follows, namely to calculate the space velocity of the relative-motion stars with respect to the trapezium. Unfortunately, there are no radial velocities available for these stars, so that the best we can do is to estimate the transverse velocity, which, of course, is subject to the uncertain-

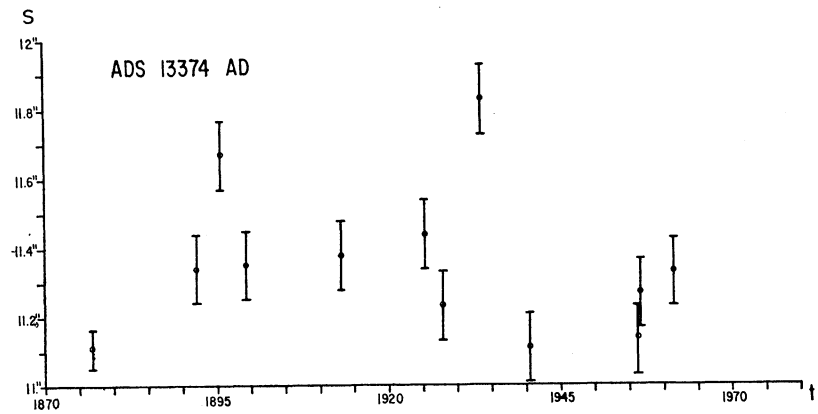

FIG. 11. Separation versus time for ADS 13374 AD.

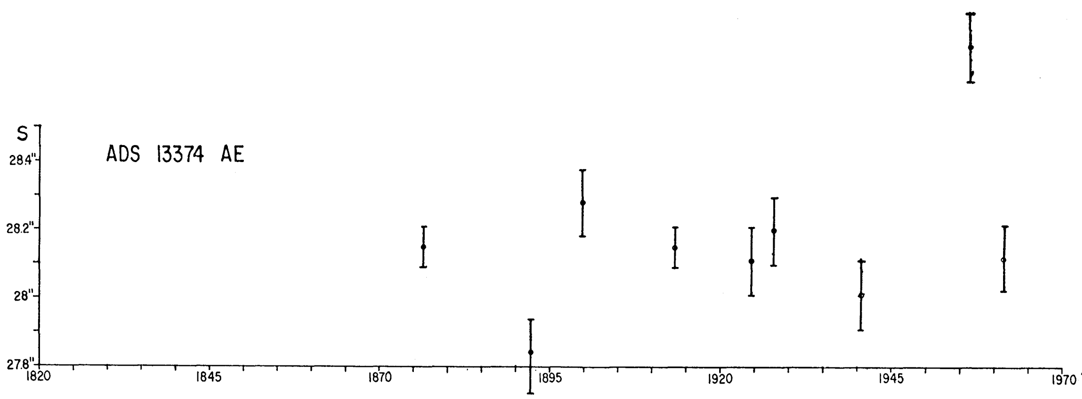

FIG. 12. Separation versus time for ADS 13374 AE.

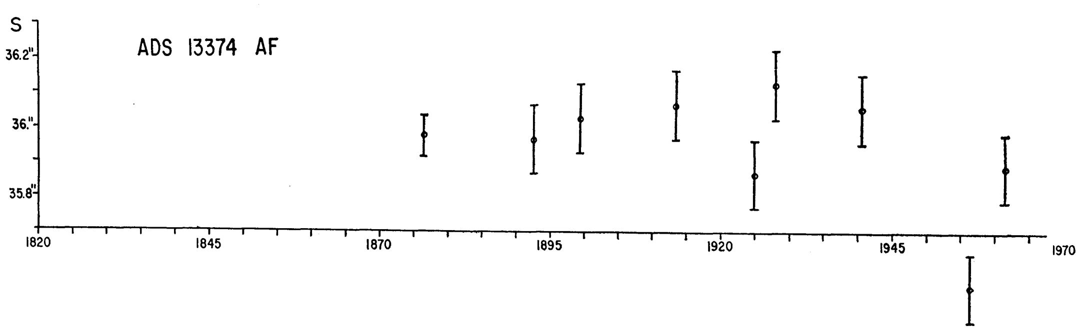

FIG. 13. Separation versus time for ADS 13374 AF.

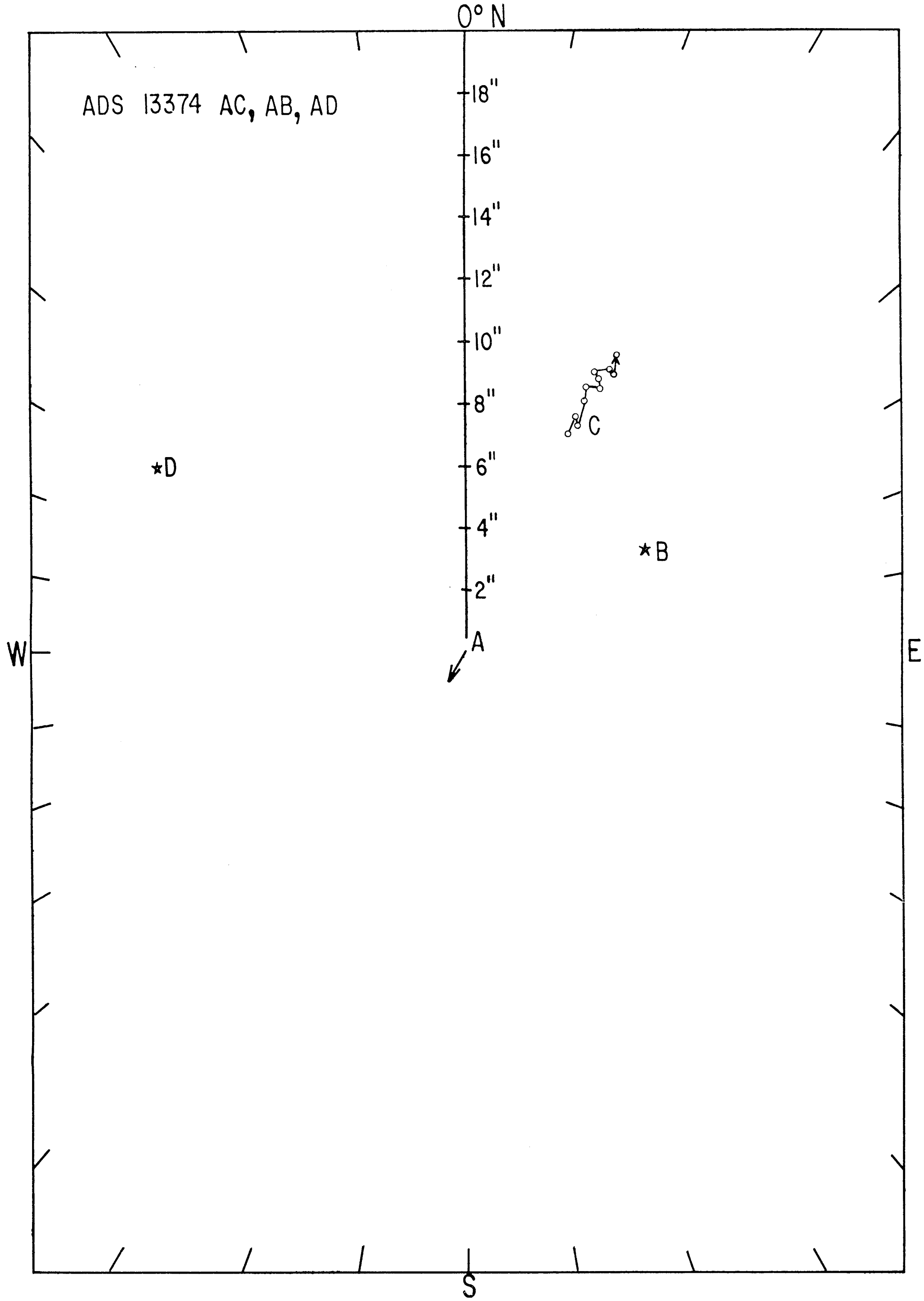

FIG. 14. Proper motion test for ADS 13374 AB, AC, AD. For explanation see caption, Figure 8. The separations plotted correspond to those of Figures 9, 10, and 11.

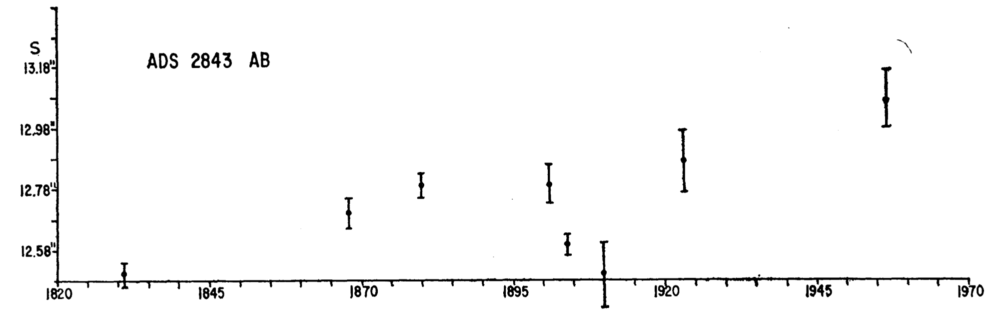

FIG. 15. Separation versus time for ADS 2843 AB.

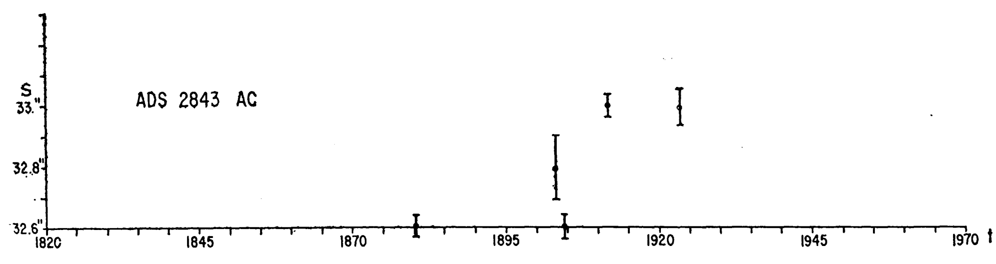

FIG. 16. Separation versus time for ADS 2843 AC.

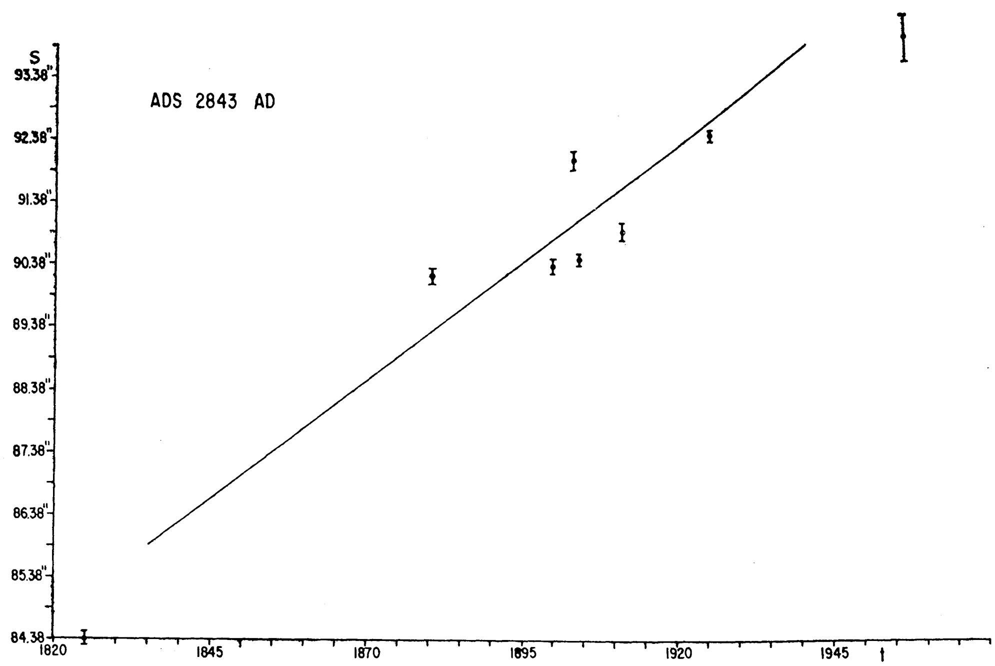

FIG. 17. Separation versus time for ADS 2843 AD.

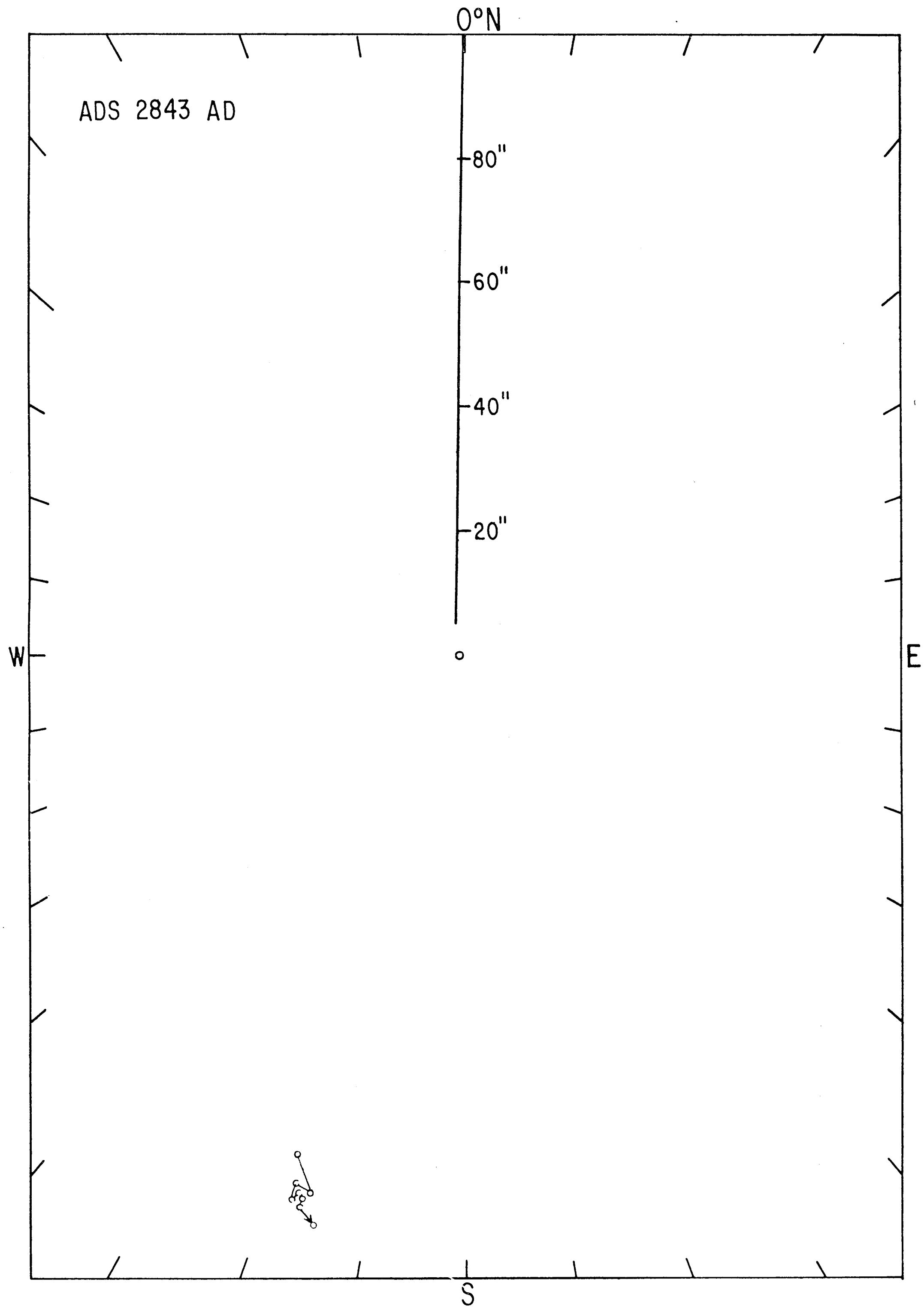

FIG. 18. Proper motion test for ADS 2843 AD. For explanation, see caption, Figure 8. The separations plotted correspond to those of Figure 17.

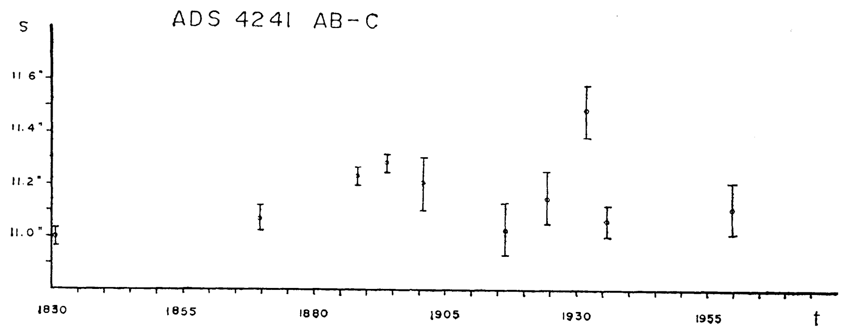

FIG. 19. Separation versus time for ADS 4241 AB-C.

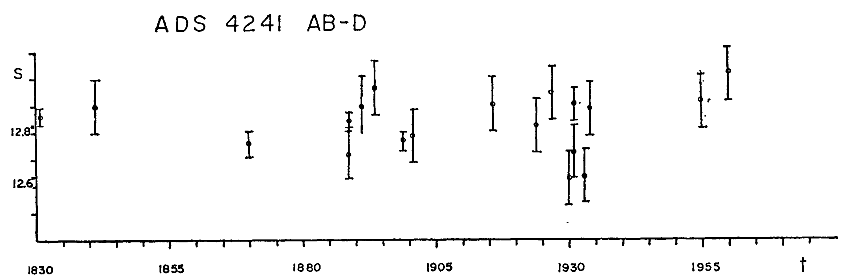

FIG. 20. Separation versus time for ADS 4241 AB-D.

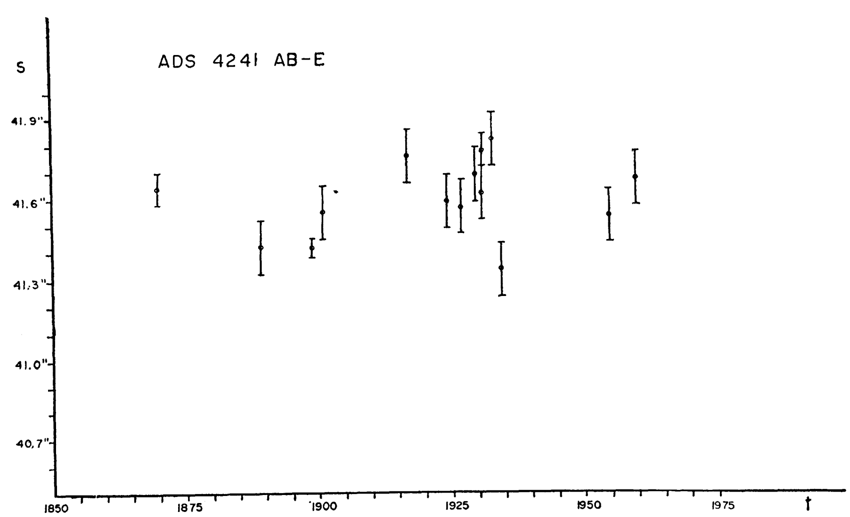

FIG. 21. Separation versus time for ADS 4241 AB-E.

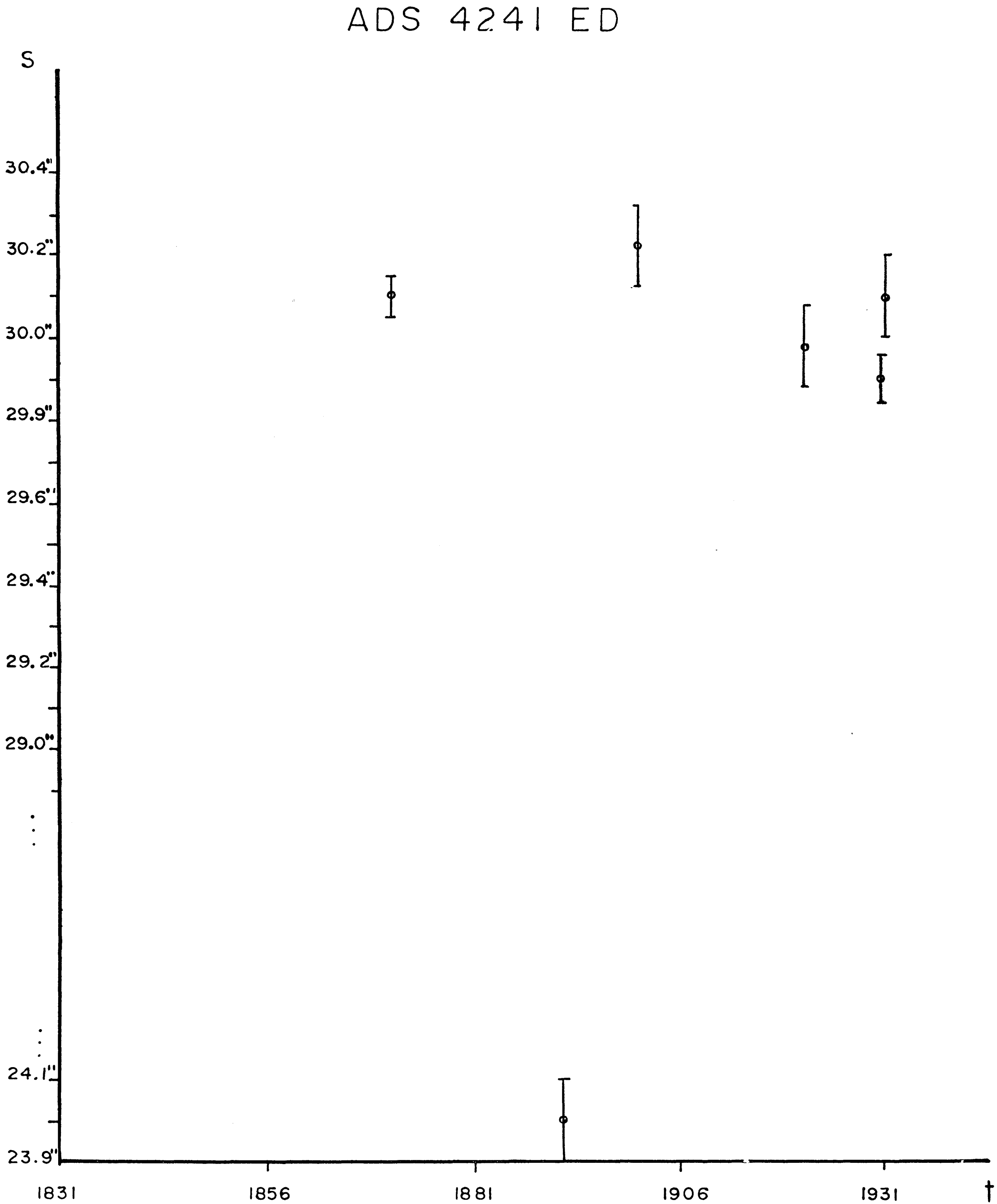

FIG. 22. Separation versus time for ADS 4241 ED.

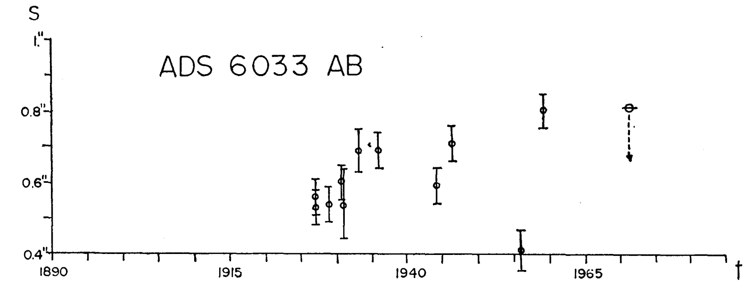

FIG. 23. Separation versus time for ADS 6033 AB (VY Canis Majoris).

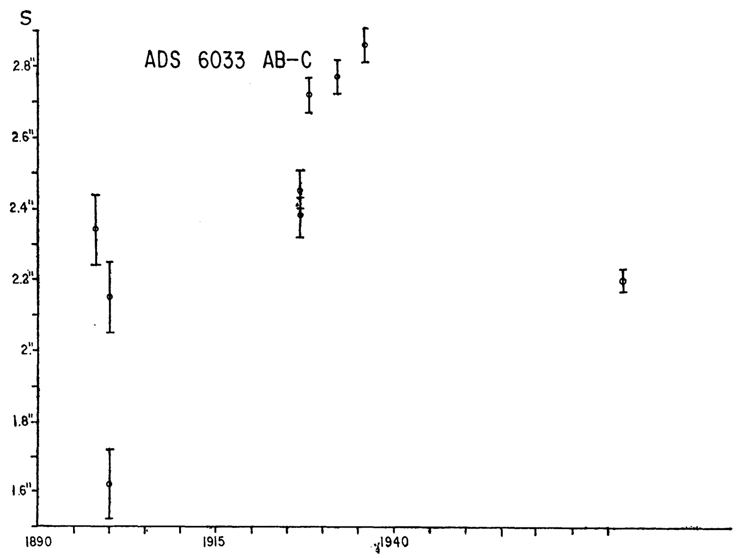

FIG. 24. Separation versus time for ADS 6033 AB-C (VY Canis Majoris).

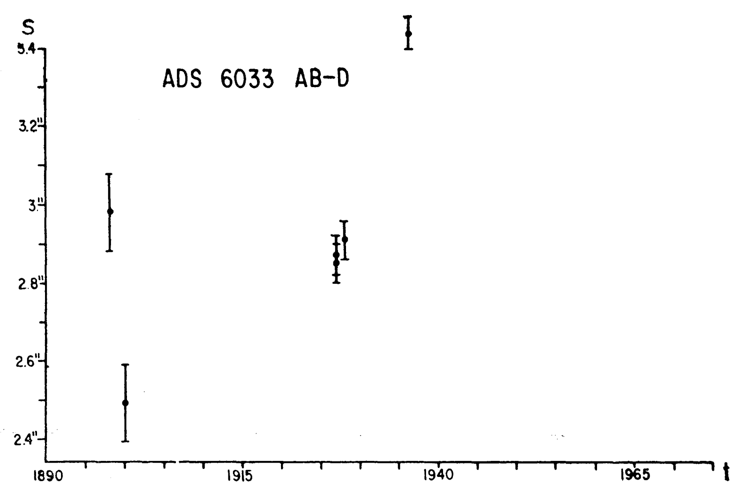

FIG. 25. Separation versus time for ADS 6033 AB-D (VY Canis Majoris).

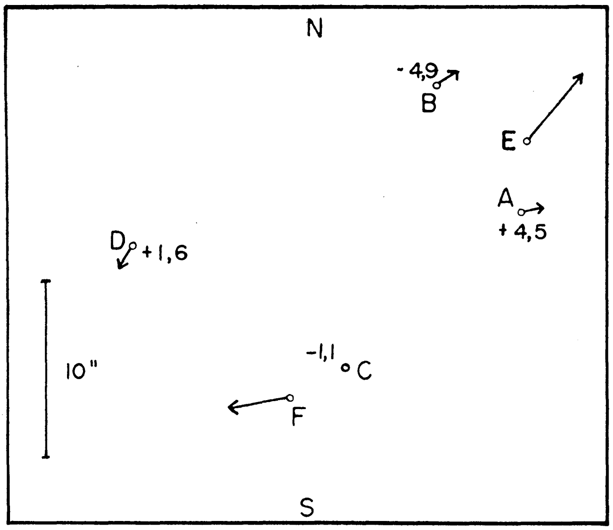

FIG. 26. Expansion of the Orion Trapezium (ADS 4186) relative to component C, according to Parenago (1953). The numbers given are radial velocties in km sec^{-1}.

ties of the distance determination. The distances of the systems ADS 6366 and ADS 15260 are inadequately known, and so in Table 2 we can only list a lower limit for the transverse velocity of their relative-motion stars.

III. DISCUSSION AND CONCLUSIONS

After a careful inspection of all the available regressions for the 44 trapezia listed in Table 1, we find no way to justify the claim that some trapezia exhibit expanding motions. The case of the Orion Trapezium (ADS 4186) is the best documented example of our negative conclusion. In fact, according to Parenago, the expansion rate of ADS 4186 is such that if in 10 000 years it has expanded to the present average radius of ten arc seconds, then in 130 years the expansion relative to the center of mass should have been 0″13, projected on the plane of the sky. Thus, on the average, opposite stars should be receding from each other at the rate of 0″26 per 130 years. The reader may convince himself by looking at Figures 1 through 7 that only one of the regressions for the system ADS 4186, namely that of components AE, indicates a rate of expansion that resembles the value given by Parenago. The remaining stars show no motions comparable to this one.

Furthermore, we note that for a trapezium like Orion to be in expansion, the various stars must have velocities larger than the escape velocity. From a semi-empirical mass-luminosity relation, with adequate corrections for absorption, the total mass of the stars of the Orion Trapezium can be shown to be at least 100 $M_{\odot}$. Taking a radius of 5 000 AU we find a velocity of escape $v_e \geqslant 6$ km sec^{-1}, which on projection, converts into 0″31/130 years, relative to the center of mass. Again, with the exception of star E, none of the remaining stars in Orion shows any indication of a motion approaching the velocity of escape from the system. In fact, the marginal displacements of stars B and C are compatible with the transverse velocities expected for this system from the Virial Theorem. We are forced to conclude, therefore, that there is *no evidence* for the expansion of the Orion Trapezium.

We have found, however, that a sizeable fraction of the well-observed trapezia have at least one star with significant motion. Star E in Orion, for instance, shows a transverse velocity of 6.4 km sec^{-1}; it would be particularly valuable to obtain new high weight observations for this star. Moreover, the distribution of signs of ds/dt in Table 2 shows that the observed motions do not in general correspond to the internal motions of the systems, because these motions should produce roughly equal numbers of positive and negative values of ds/dt. The internal motions would seem to be below the present level of detection. It can be concluded, therefore, that most of the relative-motion stars of Table 2 are member stars in the process of being ejected by the systems.

Some of the relative-motion stars show large transverse velocities, as can be seen in Colum 7 of Table 2. A few of these stars, such as ADS 2843 D and ADS 13374 C, should have space velocities larger than 30 km sec^{-1}. This behaviour is of particular interest, because it suggests that dynamical interactions among the members of a trapezium may ocassionally result in the ejection of high velocity stars; clearly, this fact may be relevant to our understanding of the origin of runaway stars.

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REFERENCES

Ambartsumian, V. A. 1954, *Bull. Obs. Byurokan,* **15,** 3.
Franz, O. G. 1965, cited by Sharpless, 1965.
Kahn, F. D., and Menon, T. K. 1961, *Proc. Nat. Acad. Sci.,* **47,** 1712.
Parenago, P. P. 1953, *A. J. U.R.S.S.,* **30,** 249.
Poveda, A., Ruiz, J. and Allen, C. 1967, *Bol. Obs. Tonantzintla y Tacubaya,* **4,** 86.
Seares, F. H., and Joyner, M. C. 1928, *Contr. Mt. Wilson Obs.,* No. 346.
Sharpless, S. 1965, *Vistas in Astronomy,* ed. A. Beer (London: Pergamon Press) Vol. 8, p. 127.
Strand, K. Aa. 1967, private communication to A. Poveda.
Strand, K. Aa. 1973, cited by Heintz, W. D. 1973. *J.R.A.S.C.,* **67,** 49.
Vandervoort, P. O. 1964, *Ap. J.,* **139,** 869.
Worley, C. E. 1972, *Ap. J.,* **175,** L93.