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ON THE STRUCTURE AND EVOLUTION OF A SHOCK WAVE MODEL OF QSO'S

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RESUMEN

Se proponen aproximaciones analíticas a los principales parámetros que describen el comportamiento del modelo de onda de choque de un QSO propuesto por Daltabuit y Cox (1972) para concentraciones entre 10^7 y 10^8 partículas por cm^3 y velocidades entre 700 km sec^{-1} y 10 000 km sec^{-1}.

Se estudia, además, el efecto de la presión de radiación sobre la evolución de dicho modelo.

ABSTRACT

Some rough analytical approximations are proposed to describe the behavior of the shock wave model of QSO'S proposed by Daltabuit and Cox (1972) for 10^7 $cm^{-3} \leq n \leq 10^8$ cm^{-3} and 700 km $sec^{-1} \leq v \leq 1\,000$ km sec^{-1}. The possible effect of radiation pressure on the evolution of this model is studied.

Key words: HYDRODYNAMICS — QUASI-STELLAR SOURCES — SHOCK WAVES.

I. INTRODUCTION

A detailed description of the structure of the shock wave model of QSO's proposed by Daltabuit and Cox (1972) requires the computation of the physical parameters of the cooling region that exists behind the shock, of the spectrum of the radiation emitted in this cooling region, of the physical parameters of the H II region that exists ahead of the shock and of the spectrum of the radiation emitted in this H II region. This computation has been carried out by Daltabuit, MacAlpine and Cox (1974) for a set basic parameters, $v = 1\,000$ km/s and $n = 10^7$ cm^{-3}, where v is the velocity of the gas that enters the shock and n is its density. However the machine time required by this calculation is fairly long, and the description of the behavior of the model when these parameters change would require a prohibitive amount of machine time. It is therefore of interest to find analytical approximations that contain the main characteristics of this model. In this paper we propose some rough analytical approximations and describe the behavior of the model as the basic parameters change. We also study the possible effect of a physical process that might determine the evolution of this model, that is, the effect of radiation pressure.

II. THE BASIC EQUATIONS

In the model we are studying (see Figure 1) the kinetic energy of two gas clouds that collide is converted first to thermal energy at the shock fronts that form during the collision, and second to hydrogen-ionizing electromagnetic energy within the cooling regions behind the shock fronts. This radiation is then converted before it escapes to non-ionizing radiation in the H II regions that form ahead of the shocks.

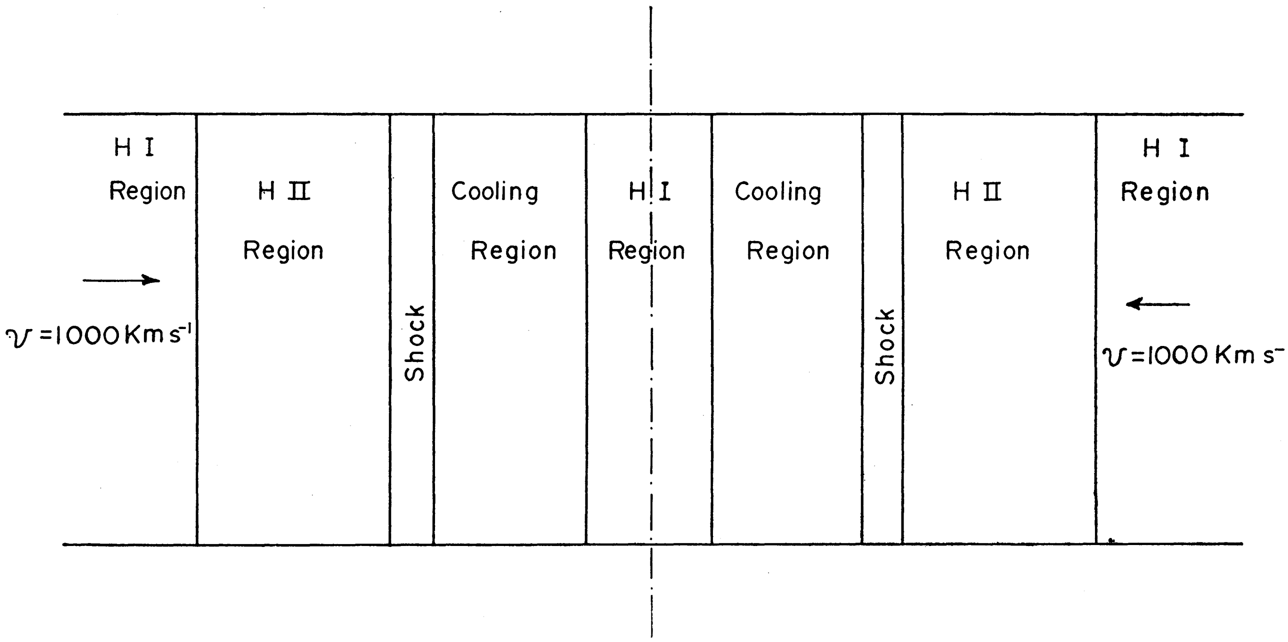

FIG. 1. Schematic diagram of the structure of the shock wave model of QSO's. Scales greatly distorted.

To describe the flow of the gas and its discontinuities we use the three hydrodynamic conservation equations, neglecting the effects of viscosity and thermal conductivity (Daltabuit 1972). We take the radiative emissivity to be a function of the temperature, density and chemical composition of the gas. We consider the hydrodynamic effects as being dominated by hydrogen and the heavy elements in a gas with cosmic abundances only as contributing to the emission of radiation by the gas.

To describe the physical parameters of the gas immediately behind the shock we assume that the gas is completely pre-ionized and does not have time to radiate a significant amount of energy while crossing the discontinuity.

It is well known that these equations admit two solutions; the continuous solution and

$$v_i = v_0/4$$
$$n_i = 4\, n_0$$
$$T_i = \frac{3}{16}\frac{m_H v_0^2}{2k},$$

where we have assumed $kT_0 \ll m_H v_0^2$ and where n_0, v_0 and T_0 are the parameters before the gas enters the shock and n_i, v_i and T_i are the parameters of the gas immediately after it has crossed the shock.

After the gas has reached this point it continues to flow, radiating energy until it cools and stops. The emissivity $L(T)$ in ergs cm^3 s^{-1} for a gas with cosmic abundances of heavy elements and in collisional equilibrium given by Cox and Daltabuit (1971), can be used to describe the cooling as a function of temperature. At high temperatures (greater than 10^6 °K) there is no density dependence and in this case we can fit a power law to the emissivity to obtain an analytical approximation. The result of this procedure is

$$L(T) = 1.6 \times 10^{-23} \text{ ergs cm}^3 \text{ s}^{-1}, \quad 6 \times 10^6\,°K \leq T \leq 5 \times 10^7\,°K \qquad (1.a)$$

$$L(T) = 2.29 \times 10^{-27} [T(°K)]^{1/2} \text{ ergs cm}^3 \text{ s}^{-1} \quad T > 5 \times 10^7\,°K \qquad (1.b)$$

The cooling regime described by Eq. (1.a) corresponds to emission by free-free and free-bound transition for heavy elements in high stages of ionization. The cooling regime described by Eq. (1.b) corresponds to free-free transition for completely ionized heavy elements. Kafatos (1973) has computed the emissivity for a gas not in equilibrium and for $T > 10^6$ °K; the difference between the equilibrium and non equilibrium cases is negligible for the purposes of our work.

The energy conservation equation can be written, if the gas is pre-ionized, as

$$n_0 v_0 \frac{\partial}{\partial x}\left(5kT + \frac{1}{2} m_H v^2\right) = -n^2 L(T).$$

Since the flow is subsonic after the gas has passed through the shock, we can neglect the term $\frac{1}{2} m_H v^2$ to simplify the integration of this equation. The result is

$$\frac{T}{T_i} = \left(1 - \frac{x}{x_c}\right)^{1/(3-\beta)} \qquad (2)$$

where x_c is the distance that the gas travels before it cools and is given by

$$x_c = \left(\frac{3}{16}\right)^3 \left(\frac{5}{3-\beta}\right) \frac{\frac{1}{2} m_H v_0^2}{n_0 L(T_i)} v_0 \qquad (3)$$

and β is the exponent of T in Eq. (1).

Let $P_\nu(T)$ be the spectral emissivity of a gas at a temperature T in ergs cm^3 s^{-1} Hz^{-1}. The spectrum of the cooling region in ergs cm^{-2} s^{-1} Hz^{-1} is then

$$F_\nu = \int_0^{x_c} P_\nu n^2 dx. \qquad (4)$$

In the regime described by Eq. (1.a) the exact spectral emissivity is hard to compute. We take

$$P_\nu = \frac{1.6 \times 10^{-23}}{kT_i} \exp\left(-10 \frac{h\nu}{kT_i}\right)$$

since this allows us to obtain the appropriate emissivity and fits approximately the spectrum given by Daltabuit (1972) for v = 1000 km/s and $n = 10^7$ cm^{-3}. For the regime described by Eq. (1.b) it is well known (see for instance Cox 1970) that

$$P_\nu = 1.09 \times 10^{-37} [T(°K)]^{-1/2} \exp(-h\nu/kT).$$

If we define E_H as the energy emitted by the cooling region per unit area per unit time in hydrogen ionizing photons, and N_f the number of such photons emitted per unit area, we have

$$E_H = \int_{\nu_H}^{\infty} F_\nu d\nu \qquad (5)$$

$$N_f = \int_{\nu_H}^{\infty} \frac{F_\nu}{h\nu} d\nu \qquad (6)$$

where $h\nu_H = 13.60$ eV, and the average energy of the ionizing photons is

$$\overline{h\nu} = \frac{E_H}{N_f}. \qquad (7)$$

The average energy of the photons emitted by the cooling region should be approximately equal to $kT_i = 978.5\ v_{08}^2$ (eV), where $v_0 = v_{08} \times 10^8$ cm/s. For velocities of the order of 1000 km/s the cooling region will emit hydrogen ionizing photons, and hence the gas that enters the shock is already ionized, according to our previous assumption.

If $\alpha \sim 2 \times 10^{-13} T_4^{-1/2}$ cm^3 s^{-1}, ($T = T_4 \times 10^4$ °K), is the recombination coefficient (Hummer and Seaton 1963), and we fit a power law to the emissivity given by Cox and Daltabuit (1971) for temperatures near 10^4 °K, obtaining $L \sim 1.2 \times 10^{-23} T_4^{1.92}$ ergs cm^3 s^{-1}, and remembering that this emissivity does not include the contribution of hydrogen, although this contribution is included in the form of one Lyman α photon per recombination, we get the approximate average temperature of the H II region

$$T_{4_{H\,II}} = 0.22\,[\overline{h\nu}\,(eV) - 10.96]^{0.41} \qquad (8)$$

where the Lyman α photon energy is 10.96 eV and its approximate size is given by

$$l_{HII} = N_f/(\alpha(T_4) n_0^2) \qquad (9)$$

III. THE DEPENDENCE OF THE MODEL ON THE BASIC PARAMETERS

The amount of energy emitted by a given volume of gas in the cooling region from the time it crosses

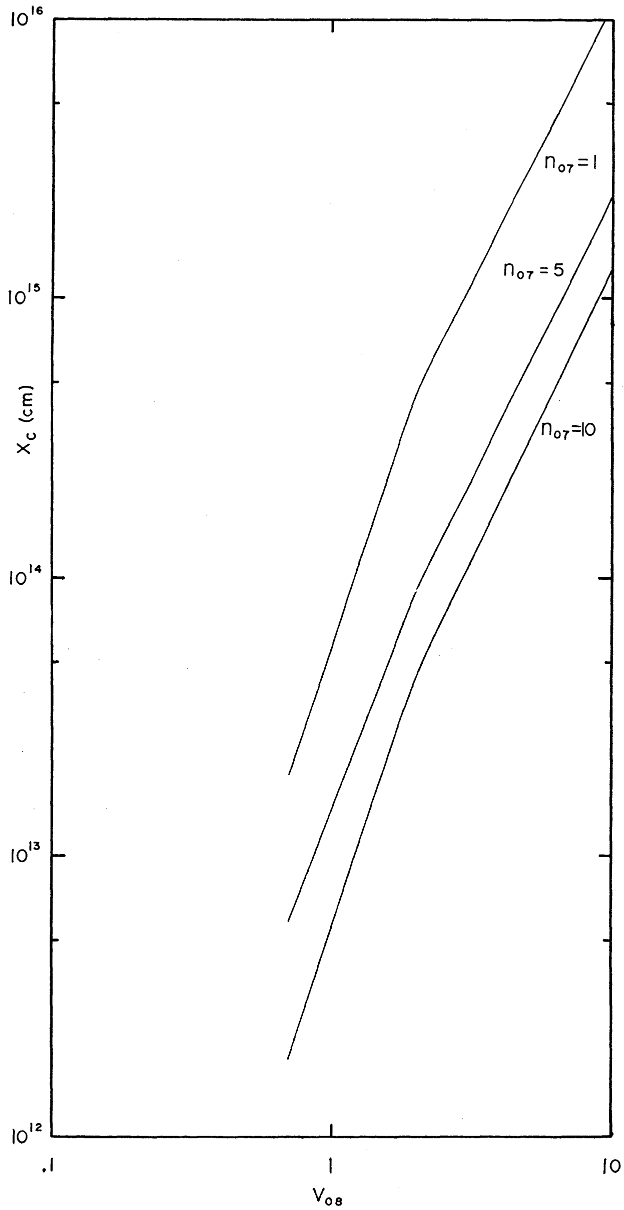

FIG. 2. Size of the cooling regions as a function of density and collision speed.

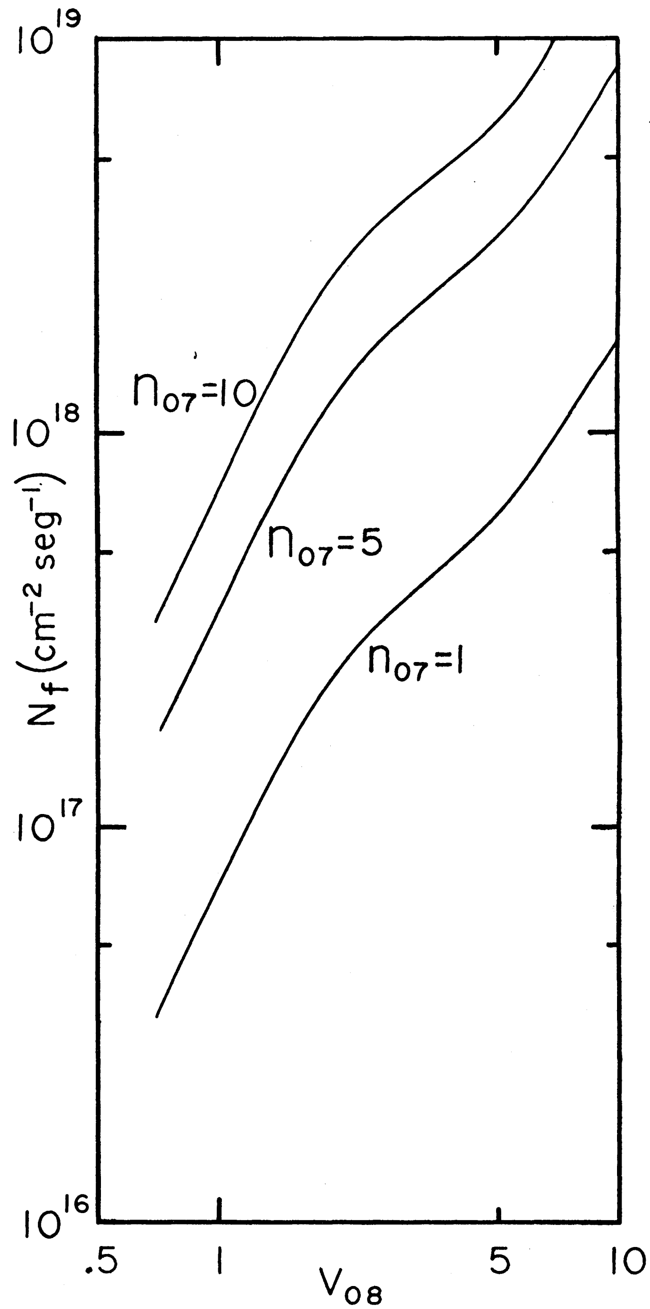

FIG. 3. Ionizing photon fux emitted by the cooling regions as a function of density and collision speed.

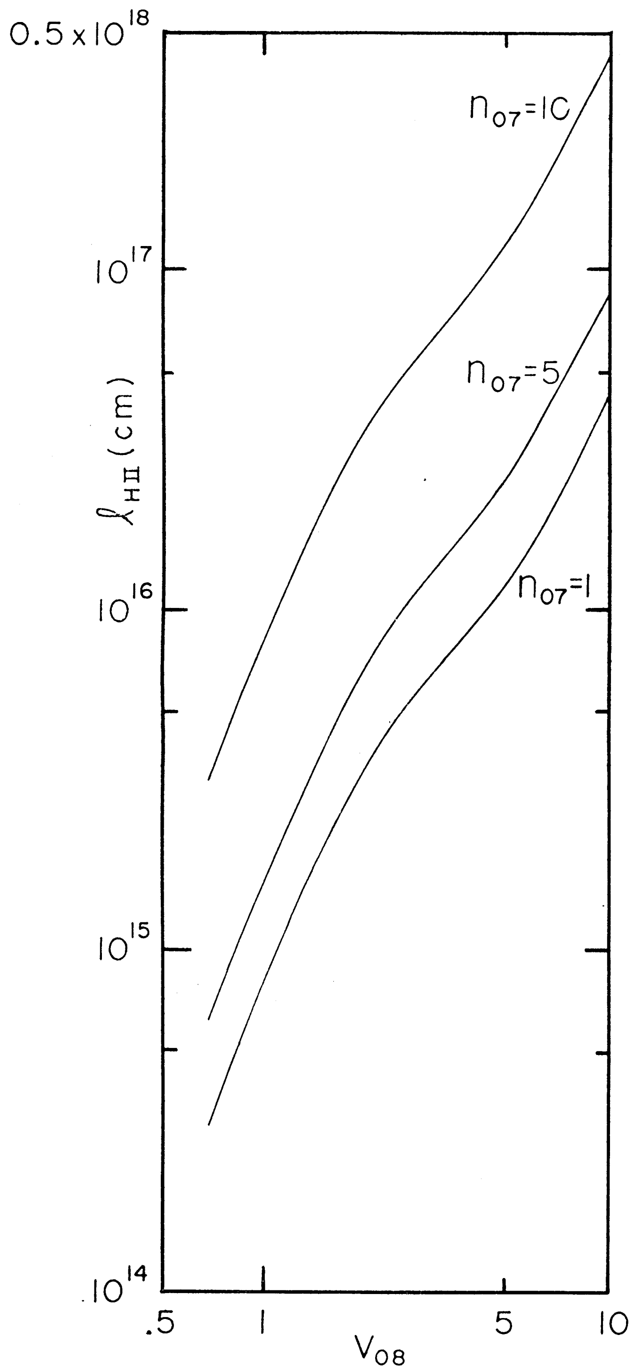

FIG. 4. Size of the H II regions as a function of density and collision speed.

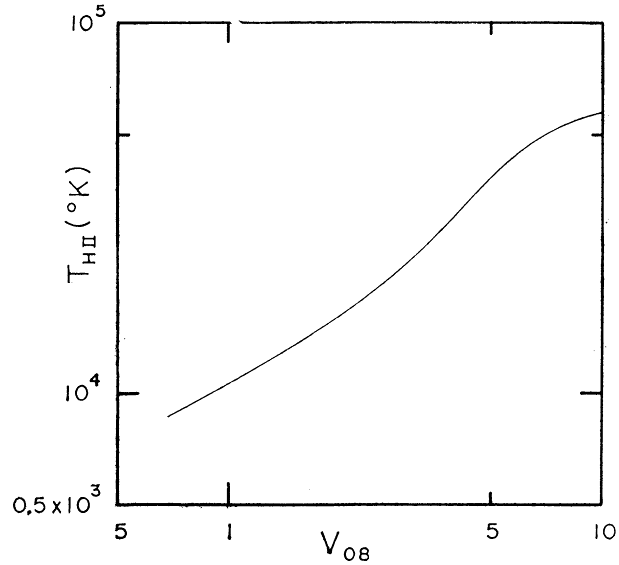

FIG. 5. Average temperature of the H II regions as a function of collision speed. This temperature is density independent.

the shock to any time smaller than the cooling time $t_c = \frac{4x_c}{v_0}$, depends only on the temperatures the gas has at the extremes of the time interval considered. The fraction of its available energy that has been emitted when the gas has reached a temperature T is proportional to $(1 - T/T_i)$. For instance, when the gas has cooled to, say, a fourth of its initial post-shock temperature, it has radiated about ¾ of its available energy. This means that if its initial post-shock temperature is above 8×10^6 °K, but below 5×10^7 °K, it will cool in a regime described by Eq. (1.a) almost entirely, while if $T_i > 8 \times 10^7$ °K, Eq. (1.b) can be used to describe the cooling. For temperatures between 5×10^7 °K and 8×10^7 °K, an intermediate emissivity should be used. We will thus study two cooling regimes and obtain by interpolation results corresponding to the intermediate case.

We will consider that initial post-shock temperatures within the regime described by the emissivity given by Eq. (1.a) are reached for $.7 \leq v_{08} \leq 2$. If $v_{08} > 2$ the initial post-shock temperature is such that we can use the emissivity given by Eq. (1.b) to compute the cooling of the gas. For $v_{08} > 10$ our computation is not valid since additional cooling mechanisms appear and relativistic effects might become important.

In Figures 2 to 5 we present curves drawn smoothly throught the points computed using Eqs. (3), (6), (8) and (9) for the corresponding parameters as functions of v_{08} and n_{07} where $n = n_{07} \times 10^7$ cm^{-3}.

The temperature computed for the H II region using Eq. (8), that appears in Figure 5, is of course an average temperature, since the temperature of each volume element is determined by the average energy of the photons that arrive at that volume element and we have not included the effects of radiative transfer. This average temperature is not very sensitive to variations in the flow velocity or density, as can be seen in Figure 5.

Notice also that the size of the cooling region is approximately two orders of magnitude smaller than the size of the H II region. There exists a trailing H II region (Daltabuit and Cox 1972) in the central part of the collision. The dimensions of this trailing H II region are much smaller than those of the cooling region, and since the corresponding contribution to the volume of ionized gas is not important, we have neglected it here.

The aproximate computation of the structure presented in this section agrees with the exact calculations of Daltabuit, MacAlpine and Cox (1974) within a factor of 2. It seems then that the analytical approximation are useful.

IV. THE ESCAPE OF LYMAN α PHOTONS

Each recombination in the H II region yields approximately one Lyman α photon. To escape, these photons have to travel through the neutral hydrogen clouds that are colliding whose size L is of the order of 1 pc. (Daltabuit and Cox 1972). There are two ways in which a Lyman α photon can escape from the ionized gas volume: it can either diffuse across the cloud or it can be converted to the two photon hydrogen continuum (see for instance Osterbrock 1962).

For temperatures of about 100 °K for the H II region the average number of collisions required for escape through diffusion, taking into account the Doppler shift that occurs in each collisions, is approximately (Osterbrock 1962)

$$Q \simeq 1.57 \times 10^{21} n_{07}{}^2 L_{pc}{}^2$$

where $L = L_{pc} \times 3.2 \times 10^{18}$. The escape time Qt_f (t_f is the mean free time) is much longer than the collision time

$$t_c = \frac{L}{v_0} = 3 \times 10^{10} \frac{L_{pc}}{v_{08}} \sec$$

so diffusion is not important.

The Lyman α photons can be converted to the two photon continuum when they collide against the neutral clouds. Let λ be the probability of a Lyman α photon being destroyed when it collides with the boundary of the H I region. This probability is quite different from the probability of destruction during a collision with a neutral hydrogen atom. This is due to the fact that when a photon encounters the H I region boundary, it will diffuse into it for a time before returning to the H II region. During this diffusion it will encounter many neutral atoms, thus increasing its probability of destruction.

To compute approximately the probability of destruction, we use the one dimensional radiative transfer equation, in the Schuster-Schwarschild approximation (see for instance Chandrasekhar 1949) with an albedo for single scattering, $p = 1.7 \times 10^{-13}$ Ne (this value is computed for an H I region temperature of 120 °K, Osterbrock, 1962, Ne is the electron density in the H I region. Thus we obtain for $\tau_m \sqrt{p} \gg 1$ (τ_m being the total optical depth of the cloud)

$$\lambda \simeq 2\sqrt{p} = 9.24 \times 10^{-7} \, Ne^{1/2}$$

We can then compute the density of Lyman α photons n_α, in the H II region, by solving

$$\frac{dn_\alpha}{dt} = R_\alpha - \frac{n_\alpha}{t'}\lambda \tag{10}$$

where R_α is the Lyman α production rate in cm^{-3} s^{-1} and $t' = l_{H\,II}/c$ is the time a photon takes to cross the H II region, n_α/t' being the collision rate of the Lyman α photons with the H I region boundary. Considering $n_\alpha(0) = 0$ we integrate (10) and obtain a radiation pressure P_r that is exerted only at the boundary of the H I region.

$$P_r = P_{rm}\,(1 - e^{-\lambda t/t'}) = \frac{1}{3} h\nu_\alpha n_\alpha \qquad (11)$$

where

$$P_{rm} = \frac{1}{3} h\nu_\alpha R_\alpha \frac{t'}{\lambda} = \frac{1}{3} h\nu_\alpha \frac{N_f}{c\lambda}$$

V. THE HYDRODYNAMIC EFFECTS OF THE RADIATION PRESSURE

To compute the effect of a pressure discontinuity at the H I region boundary, produced by the trapped Lyman α photons, we take the hydrodynamic equat ions, adding a pressure P_r that acts only at the boundary, that is

$$nv = n_0 v_0$$

$$nv^2 + \frac{P_g}{m_H} + \frac{P_r}{m_H} = n_0 v_0^2 + \frac{P_{g0}}{m_H}$$

$$\frac{1}{2} m_H v^2 + \frac{5}{2}\frac{P_g}{n} + \frac{P_r}{n} = \frac{1}{2} m_H v_0^2 + \frac{5}{2}\frac{P_{g0}}{n_0}.$$

Here P_g is the gas pressure. If we measure the velocity in units of v_0, the pressures in units of $m_H n_0 v_0^2$, and the density in units of n_0, and we solve for v, we obtain

$$v = \frac{1}{8}(5 - 3P_r)\left[1 \pm \left(1 - \left(\frac{4}{5 - 3P_r}\right)^2\right)^{1/2}\right]. \qquad (12)$$

For values of P_r between 0 and ⅓ solutions exist, as can be seen in Figure 6.

If P_r is a slowly growing function of time, and we can assume that steady flow conditions are maintained as P_r grows, and if we further specify that $P_r(0) = 0$, we can then ask which of the two solutions will be valid at the discontinuity, at any given time. It seems natural to assume that the first solution (+ sign) (quasicontinuous) will be valid as long as P_r is not too large. Otherwise the second solution (discontinuous) will hold.

Let P_{rc} be the value of P_r coinciding with the value of the viscous pressure P_v that is a result of the velocity discontinuity that appears in the second

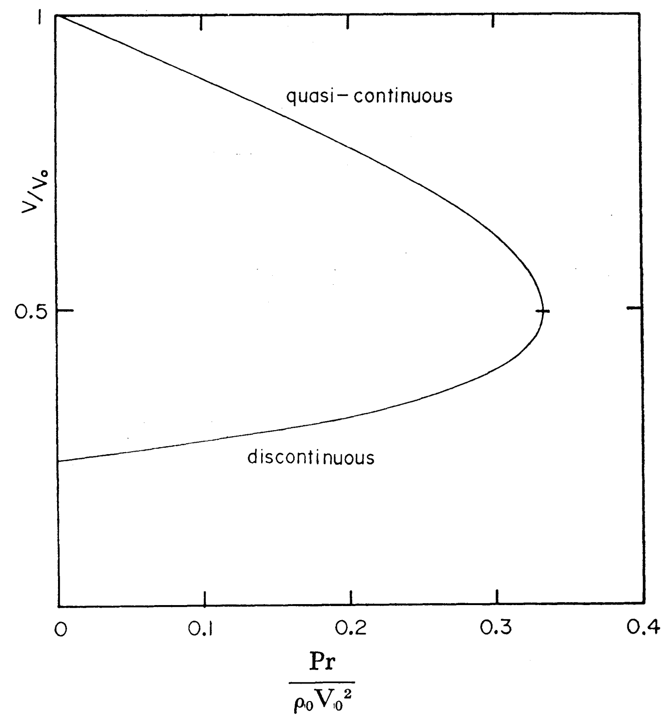

FIG. 6. Velocity of the gas immediately after the ionization front as a function of radiation pressure.

(discontinuous) damped shock solution. We have approximately

$$P_v \sim \frac{1}{6}\frac{v_T - v_{0T}}{v_0}$$

where v_T is the thermal velocity of the gas particles. We assume that if $P_r < P_{rc}$ the discontinuity is described by the first solution, while if $P_r \geq P_{rc}$ the second solution holds.

If we make $P_r = P_v = P_{rc}$ in Eq. (12), we can obtain the value of the post-shock velocity for which the conditions are satisfied. This allow us to compute the critical pressure

$$P_{rc} = 0.0092\; n_{07} v_{08}^2 \text{ ergs cm}^{-3}$$

If the radiation pressure established by the trapped Lyman α photons is to reach this critical value we must have $P_{rm} \geq P_{rc}$. If we take $n_{07} = v_{08} = l_{H\,II_{16}} = T_4 = 1$ $(l_{H\,II} = l_{H\,II_{16}} \times 10^{16})$ this conditions means

$$\lambda \leq 3.8 \times 10^{-3} = \lambda_0.$$

If in Eq. (11) we make $P_r = P_{rc}$ and let $\lambda = x\lambda_0$ we can solve for t/t' to obtain

$$\frac{t^*}{t'} = \frac{1}{\lambda_0 x} \ln\left(\frac{1}{1-x}\right)$$

where $0 \leq x < 1$. This gives us the critical time t^* for which the radiation pressure reaches the critical value. In Figure 7 we plot t^*/t' as a function of x. This figure shows that for values of x slightly

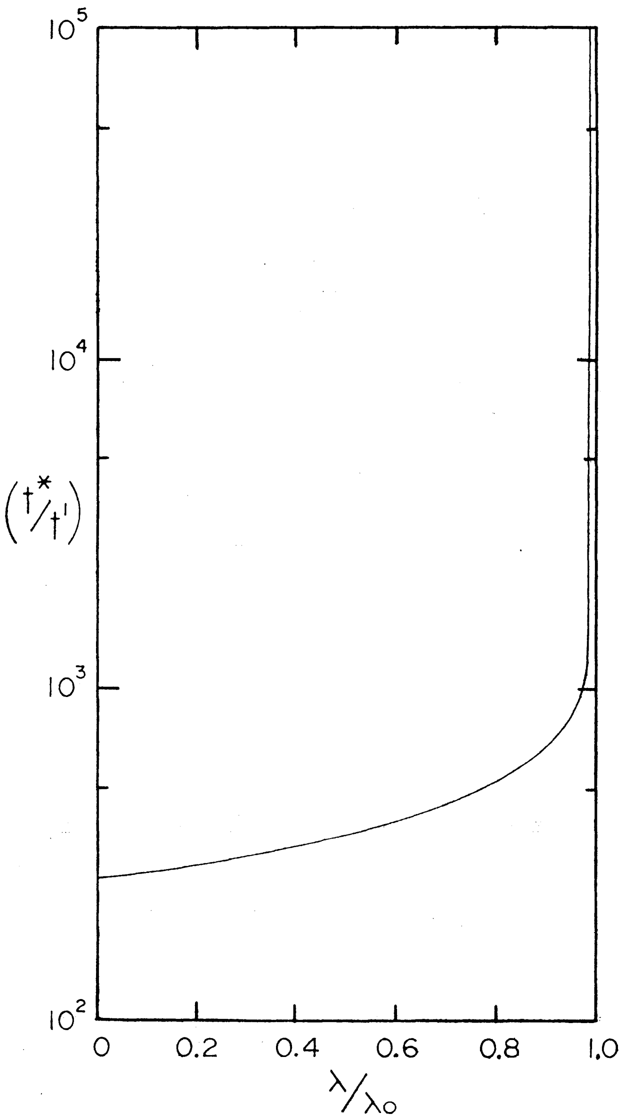

FIG. 7. Critical time as a function of the destruction probability.

smaller than 1, the critical pressure is reached for times smaller than 10 years since $t' \sim 10^{-2}$ years.

For the model we are considering, the transition between the H II and the H I regions can be gradual, in which case Ne would take a value between 10^7 cm^{-3} and 10^4 cm^{-3}. This last value corresponds to an H I region where the hydrogen is completely neutral, but where the heavy elements are ionized by the high energy photons, emitted by the cooling region, that are not absorbed in the H II region. For the purposes of this work it is sufficient to take Ne = 10^4 cm^{-3}. Then

$$\lambda = 8.24 \times 10^{-5} < \lambda_0$$

and the radiation pressure reaches the critical value in 2.57 years. For Ne = 10^6 cm^{-3} this happens in approximately 3 years.

VI. CONCLUSIONS

We have presented analytical approximations that allow us to compute the structure of the shock wave model of QSO's proposed by Daltabuit and Cox (1972), for collision velocities ranging from 700 km/sec to 10,000 km/s and densities ranging from 10^7 cm^{-3} to 10^8 cm^{-3}. Using these results we have shown that the Lyman α radiation produced is trapped, and exerts a considerable pressure at the ionization front. It is plausible to assume that this pressure reaches within a few years, values high enough to produce a shock at the ionization front. This of course doubles the luminosity of this model over a period of time.

The further evolution of this model has to be computed, taking into account the hydrodynamical effects of the trapped radiation. We have not performed this computation but we suspect that the flow will be unstable, shocks perhaps appearing in a certain time sequence. This would lead to a possible irregular variability of the luminosity of this model with amplitudes of at least one magnitude and characteristic times of less than 5 years.

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