THE VELOCITY DEPENDENCE OF THE ACCELERATION SPECTRUM OF SOLAR PROTONS

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RESUMEN

En un análisis preliminar del espectro en la fuente de los protones solares, se investiga el caso en que las pérdidas de energía de las partículas en la región de aceleración son insignificantes durante el proceso de aceleración. Se utiliza un espectro en forma de ley de una potencia de la energía, bajo la suposición de que la tasa de aceleración es proporcional a la energía misma y *dependiente de la velocidad* de las partículas. Los espectros calculados para varios conjuntos de parámetros de aceleración son comparados con el espectro de energía experimental del evento de protones solares del 28 de enero de 1967. Los resultados obtenidos indican que el espectro de aceleración más aproximado a la curva experimental, puede corresponder a un conjunto de parámetros en la fuente tales que la eficiencia de aceleración sea $\alpha \simeq 0.4$ seg^{-1} y el tiempo medio de confinamiento $\tau \simeq 0.4$ seg. Esto implica la existencia de un campo magnético de $\simeq 500$ gauss y una concentración del medio del orden de $10^{11} - 10^{13}$ cm^{-3} dentro de la región de aceleración. El flujo inicial de protones que participan en el proceso de aceleración que se prevee por este método representa solamente una fracción muy pequeña $(10^{-12} - 10^{-14})$ de las partículas locales.

ABSTRACT

In a preliminary analysis of the source spectrum of solar protons, we investigate the case in which energy losses of protons in the source are negligible while they are undergoing Fermi-acceleration. A power law in energy is adopted, under the assumption of an acceleration rate proportional to the energy and *dependent on the velocity* of the protons. The calculated spectra for several sets of accelerating parameters are compared with the experimental spectrum of the solar proton event of January 28, 1967. The results obtained indicate that the theoretical Fermi-curve which approaches best to the experimental curve corresponds to a parameter set at the source with $\alpha \simeq 0.4$ s^{-1} and $\tau \simeq 0.4$ s (acceleration efficiency and mean confinement time respectively). These values imply a source region where the strength of the magnetic field is of the order of $\simeq 500$ gauss and the concentration of plasma particles $\simeq 10^{11} - 10^{13}$ cm^{-3}. The initial flux of protons participating into the acceleration stage is predicted as being only a very small fraction $(10^{-12} - 10^{-14})$ of the local plasma particles.

Key words: ENERGY SPECTRUM—FERMI-ACCELERATION—SOLAR FLARES.

I. INTRODUCTION

The determination of the energy spectrum of solar particles is one of the most difficult topics in the study of solar cosmic rays. Among other difficulties there is the fact that the experimental data do not furnish a direct measure of the whole energy spec-trum. Thus it is necessary to construct the spectrum from a set of measurements derived from a large number of different detectors, for the entire energy interval. It follows that any description of the integral spectrum at a given moment during the event will result in some spread of spectral shape representations, according to the different investigators. The most

plausible spectral shapes are described either by power laws in kinetic energy, momentum or magnetic rigidity or an exponential law in magnetic rigidity (see for example Krimigis 1965). In the particular case of the solar proton event of January 28, 1967, for which experimental measurements (derived from satellite, balloon and neutron monitors) at a given time and through a wide range of energy $(E > 10$ MeV), are available, four different integral energy spectra have been proposed. Lockwood (1968) analyzes an exponential law in rigidity, $\{\sim \exp\,(-R/0.6$ GV$)\}$ as well as an inverse power law in rigidity $(\sim R^{-5})$. Barcus (1969) proposes a law of the form $(\sim R^{-4})$ and Heristchi and Trottet (1971) propose an inverse power law in energy $(\sim E^{-2})$ and an inverse power law in rigidity $(\sim R^{-3.1})$, both with an upper cutoff $(E_m = 4.3$ GeV and $R_m = 5.3$ GV$)$. It is readily shown by the latter authors, that in this solar proton event at least, the best fit with the experimental points throughout the energy domain explored, is represented by the inverse power law in kinetic energy with an upper cutoff E_m.

We shall proceed here to compare the experimental spectrum with the acceleration spectrum, in order to make inferences about the generation process of solar particles at the source. For simplicity we shall assume that modulating effects in interplanetary space do not introduce an important modification in the particle spectrum through their propagation to the earth.

Since the source of particle acceleration is most probably found in the fluctuating fields of the turbulent motion of the medium, which entails a process of statistical character, we consider a Fermi-type mechanism of acceleration, which is apparently a very widespread phenomenon in turbulent plasma.

Concerning the Fermi-mechanism, it has been shown by Wentzel (1965), that several spectral shapes may be obtained from this kind of process, depending upon the different assumptions regarding the mean rate of increase in total energy (dW/dt), momentum (dP/dt) or rigidity (dR/dt), and upon the characteristic confinement time τ, in the acceleration region, which may be a function of velocity, momentum, energy or magnetic rigidity. The case to be discussed here is for a mean confinement time, τ, and for a rate of energy gain which is proportional to the energy and dependent on the velocity of the particles.

$$\frac{dW}{dt} = \alpha\beta W = \alpha(W^2 - (M_p c^2)^2)^{\frac{1}{2}} \qquad (1)$$

where α is the parameter characterizing the efficiency of acceleration (which is taken here as constant), $\beta = v/c$ the velocity of protons in units of the velocity of light, $M_p c^2$ the rest energy of the protons and W the total energy of particles. We have chosen the Fermi-statistical acceleration rate (1) because it furnishes a source spectrum after acceleration in the form of an inverse power law in energy, such as that suggested by Heristchi and Trottet (1971) in the case of solar protons. The consideration of a rate of energy gain with dependence on the velocity follows from the experimental results obtained by Bryant et al. (1965) from studies of solar proton events, observed by Explorers XII and XIV. It is shown by these authors that in interplanetary space: 1) the rate of propagation of solar protons is linearly dependent on particle velocity, 2) the observed differential intensity is a function of the product of the particle velocity by the time elapsed since flare maximum, and that at the source level: 1) the source spectrum shows a velocity dependence, 2) in each event studied, the spectrum is a power law in energy.

II. THE ACCELERATION SPECTRUM

To calculate the energy spectrum of solar particles which are accelerated by a Fermi-process, we made the following hypotheses: we assumed that N_0 particles, with similar energy W_i are present in the region where the Fermi-mechanism is operating, and are then accelerated at the same rate (1) but escape with mean probability τ^{-1} per second from the accelerating region (i.e. $\tau \simeq$ constant). Furthermore, energy loss processes are negligible in the region of acceleration, to the extent that the rate of energy change in the source is only determined by (1).

Following Fermi (1949) we assume that the number of particles $N(W)\,dW$ with energy between W and $W + dW$ is similar to the number of particles of age between t and $t + dt$, which by analogy with radioactive decay is assumed as an exponential distribution in age

$$N(W)\,dW = N(t)\,dt = \frac{N_0}{\tau}\exp\,(-t/\tau)\,dt \qquad (2)$$

where N_0 is the initial number of particles available for acceleration.

Using explicitly the energy dependence of β, and transforming the time dependence into an energy dependence, we immediately obtain, from equations (1) and (2), the following differential energy spectrum

$$N(W) = \frac{N_0}{\alpha\tau} (M_pc^2)^{1/\alpha\tau} \frac{(1+\beta)^{-1/\alpha\tau}}{\beta} W^{-(1+1/\alpha\tau)} \tag{3}$$

where we have assumed for the initial energy of protons the extreme lower limit, $W_i = M_pc^2$, neglecting thus the energy losses. Expressing β in terms of total energy, equation (3) may be rewritten as

$$N(W) = \frac{N_0}{\alpha\tau} (M_pc^2)^{1/\alpha\tau} \frac{\{W + [W^2 - (M_pc^2)^2]^{\frac{1}{2}}\}^{-1/\alpha\tau}}{[W^2 - (M_pc^2)^2]^{\frac{1}{2}}} \tag{4}$$

expressing now equation (4) in terms of kinetic energy and integrating up to a maximum value of the energy of the accelerated protons, E_m, we obtain immediately the integral energy spectrum.

$$J(> E) = N_0 (M_pc^2)^{1/\alpha\tau}$$
$$\{[E + M_pc^2 + (E^2 + 2M_pc^2E)^{\frac{1}{2}}]^{-1/\alpha\tau} -$$
$$- [E_m + M_pc^2 + (E_m^2 + 2M_pc^2 E_m)^{\frac{1}{2}}]^{-1/\alpha\tau}\} \tag{5}$$

Equations (3), (4) and (5) represent the acceleration spectrum of particles when the velocity dependence is explicitly taken into account in the time-energy transformation. When the parameter β is assumed constant in the integrations, a somewhat different expression for the differential spectrum is obtained (e.g. Ramudarai and Biswas 1971),

$$N(W) = \frac{N_0}{\alpha\beta\tau} (M_pc^2)^{1/\alpha\beta\tau} W^{-(1+1/\alpha\beta\tau)} \tag{6}$$

Integrating as in the latter case, the following integral energy spectrum is obtained:

$$J(> E) = N_0(M_pc^2)^{1/\alpha\beta\tau} [(E + M_pc^2)^{-1/\alpha\beta\tau} - (E_m + M_pc^2)^{-1/\alpha\beta\tau}] \tag{7}$$

For the case of relativistic particles, $\beta \simeq 1$, equations (3) to (7) reduce to the original spectrum derived for relativistic cosmic rays (Fermi 1949). Although these expressions have been deduced without considering their relevance to any source region, we shall proceed here to apply them to the generation of energetic protons in solar flares. With that aim we have performed our calculations with four different combinations of the product $\alpha\tau$ (Table 1) appearing in equations (5) and (7). These values may correspond to the different sets of acceleration parameters (listed in the third and fourth columns in Table 1) which are dependent on the source media ($n = 10^9$ cm^{-3} — 10^{13} cm^{-3}). We have chosen these source media because they are the two extreme values for concentration of particles, usually associated with solar flare regions. We have assumed that the source medium is composed only of hydrogen nuclei, and that these are completely stripped of their surrounding electrons. Set I and II may correspond to a region of magnetic field strength $\simeq 500$ gauss, and a mean distance between magnetic inhomogeneities, l, of the order of $10^5 - 10^6$ cm (Pérez-Peraza 1975); set III may correspond to the possible assumption of a random acceleration step $l \simeq 10^7$ cm in the same

TABLE 1

THE SOURCE PARAMETERS FOR STATISTCAL ACCELERATION IN SOLAR FLARES

Set	$\alpha\tau$	α(s^{-1})	τ(s)	n(part/cm^3)
I	0.17	0.4	0.42	$10^{11} - 10^{13}$
II	0.19	0.4	0.47	$10^{11} - 10^{13}$
III	0.80	0.4	2.0	$10^{11} - 10^{13}$
IV*	0.96	3.2×10^{-2}	30.0	10^9

* see e.g. Hayakawa et al. 1964.

magnetic field. Set IV corresponds to a region of $H \simeq 30$ gauss and $l \simeq 10^9$ cm (Hayakawa *et al.* 1964) which can be considered as a typical set of parameters in the corona, where the acceleration region is often assumed to be located (e.g. Melrose 1974). In all four cases it is assumed that magnetic scatter centers travel with the characteristic hydromagnetic velocity of the medium.

For the purpose of intercomparing the integral theoretical spectra obtained from equations (5) and (7) we have used the experimental spectrum of the January 28, 1967 event at 1600 UT, represented as an inverse power law in kinetic energy ($\sim E^{-2}$) with a high energy cutoff at 4.3 GeV, (Heristchi and Trottet 1971). In both cases we have performed our calculations under two different assumptions: first that the source spectra follows a perfect power law ($E_m = \infty$) as is usually adopted in cosmic ray physics and second, that a high energy cutoff ($E_m = 4.3$ GeV) exists in the acceleration process, according to the measured upper cutoff in the spectrum of solar protons, (Heristchi and Trottet 1971). This comparison is illustrated in Figure 1 with the integral spectrum (5) calculated under the first assumption for some values of the product $\alpha\tau$. In Figure 2 we plot expression (5) integrated up to a maximum energy E_m corresponding to the upper cutoff observed. Figures 3 and 4 show the integral spectrum (7) plotted for the same values of $\alpha\tau$; we have taken β as constant in each of the different energy bands and equal to its mean value in each interval.

In order to illustrate the theoretical spectra in the same scale as that of the experimental spectrum, we have normalized at 10 MeV, in such a way as to obtain at this energy the maximum flux of particles in each case. Although this energy limit of 10 MeV is related to the lower threshold of satellite detection, still it is a value which is not so far from the critical kinetic energy for Fermi-acceleration, $E_i \simeq 2.5$ MeV, when ionization losses are taken into account in a medium of $n \simeq 10^{13}$ cm^{-3} (Pérez-Peraza 1975). The normalization performed follows from the fact that our equations (5) and (7) do not furnish directly the source integral spectrum but $J(> E)/N_0$. We have therefore deduced in this manner, a factor K_0 which is proportional to the number N_0 in our expressions. In Table 2 we give the values obtained for the number $K_0 = pN_0$ (where

p is the normalization factor) according to the different source spectra calculated in this work.

III. RESULTS

The results obtained from this preliminary work are the following:

1) The best approach to the experimental curve is obtained with the particle spectrum (5) where the velocity of protons is explicitly included (Figures 1 and 2), while the theoretical spectra deduced from the assumption of $\beta \simeq$ constant, equation (7), is considerably deflected from the experimental spectrum (Figures 3 and 4).

2) The curves obtained with the sets of parameters I and II give the best systematic fit with the experimental curve, that is, $\alpha\tau = 0.17$ and $\alpha\tau = 0.19$ furnish the optimal theoretical spectra corresponding to $E_m = \infty$ and $E_m \simeq 4.3$ GeV respectively. [These *optimum* values of $\alpha\tau$ were obtained from the coupling of the conditions $J(> E)/K_0 \leq 1$ at 10 MeV and $J(> E) = 0$ at the upper cutoff, and as discussed in Section II, they may correspond to the parameter sets tabulated in Table 1].

3) Both the velocity-dependence spectrum, equation (5) and the acceleration spectrum with $\beta \simeq$ constant, equation (7), show that the parameter set IV is less appropriate for describing the acceleration parameters at the source, than the other sets (Figures 1 to 4).

4) The initial number of protons $N_0 (\propto K_0)$ susceptible to be accelerated is the same either by including or by neglecting a maximum energy in the acceleration spectrum, when the appropriate parameter sets are considered (second or third column in Table 2). This result must be expected because N_0 is independent of the acceleration process, however, the method used to calculate $J(> E)/N_0$ in equations (5) and (7) prevents us from separating the dependence on $\alpha\tau$. Nevertheless it can be appreciated in the second and third column in Table 2, that K_0 is substantially the same ($K_0 = 8.6 - 9.4 \times 10^3$ protons cm^{-2} s^{-1} steradian^{-1}). Thus we shall assume $K_0 = 9.0 \pm 0.4 \times 10^8$ protons cm^{-2} s^{-1} steradian^{-1}.

5) The acceleration region of energetic protons during the solar flare considered here, is found in a medium where $n \sim 10^{11}$ cm$^{-3} - 10^{13}$ cm^{-3}.
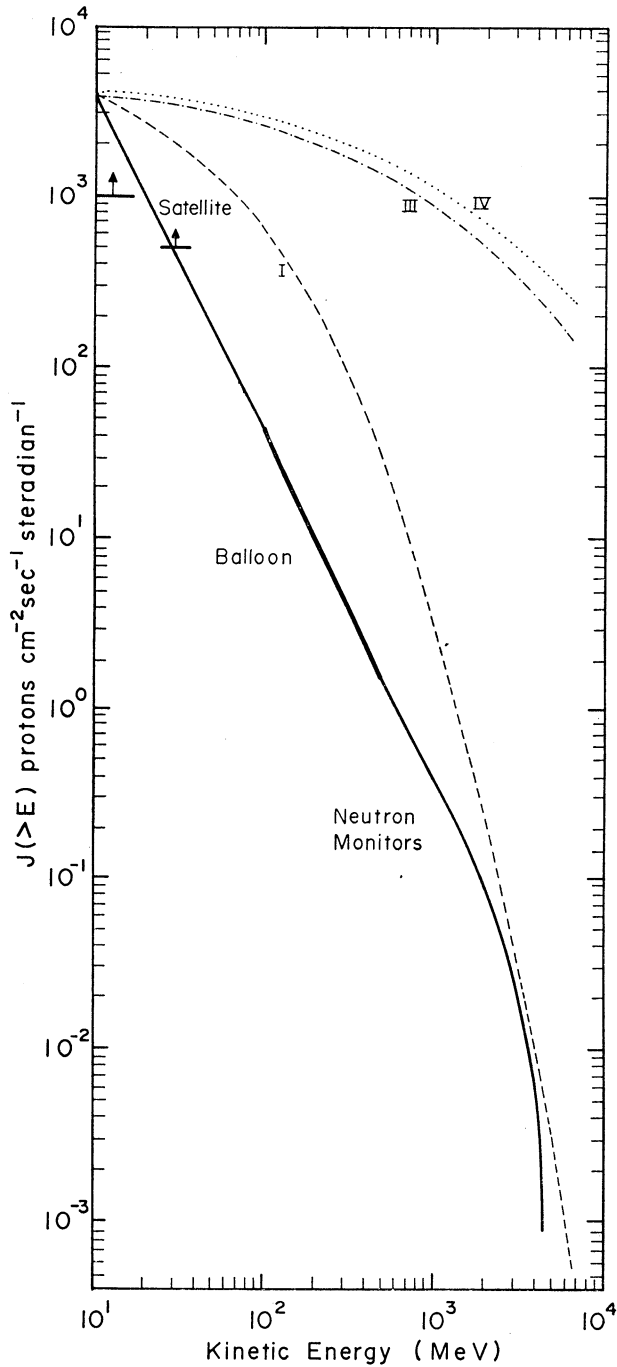
FIG. 1. Comparison of the experimental energy spectrum of the solar proton event of January 28 1967, with the predicted theoretical spectrum by Fermi-acceleration (Eq. (5) with $E_m \simeq \infty$).
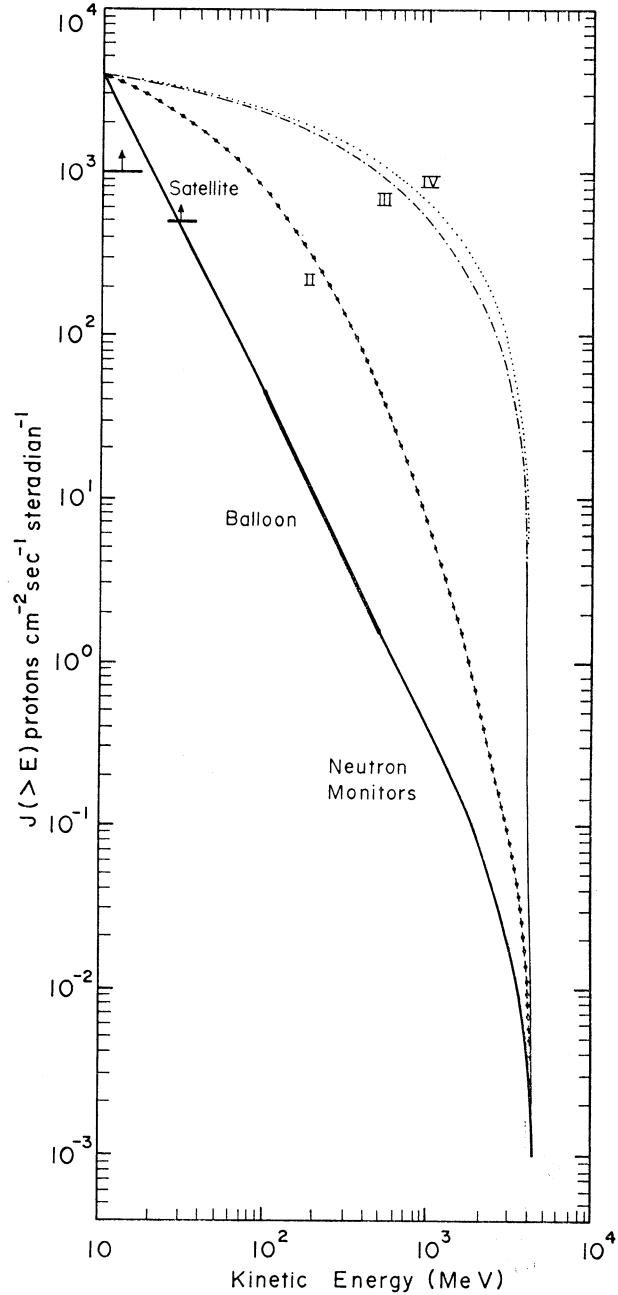
FIG. 2. Comparison of the experimental energy spectrum of the solar proton event of January 28 1967, with the predicted theoretical spectrum by Fermi-acceleration (Eq. (5) with $E_m = 4.3\,\mathrm{GeV}$).
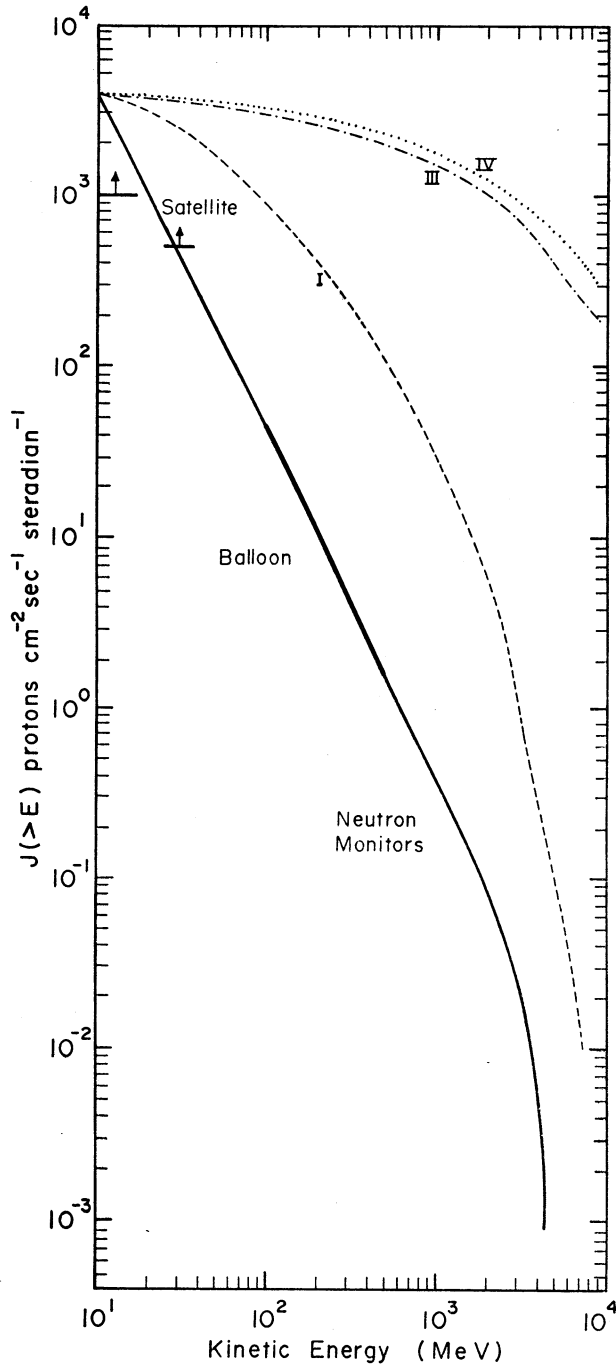
FIG. 3. Comparison of the experimental energy spectrum of the solar proton event of January 28 1967, with the predicted theoretical spectrum by Fermi-acceleration (Eq. (7) with $E_m \simeq \infty$).
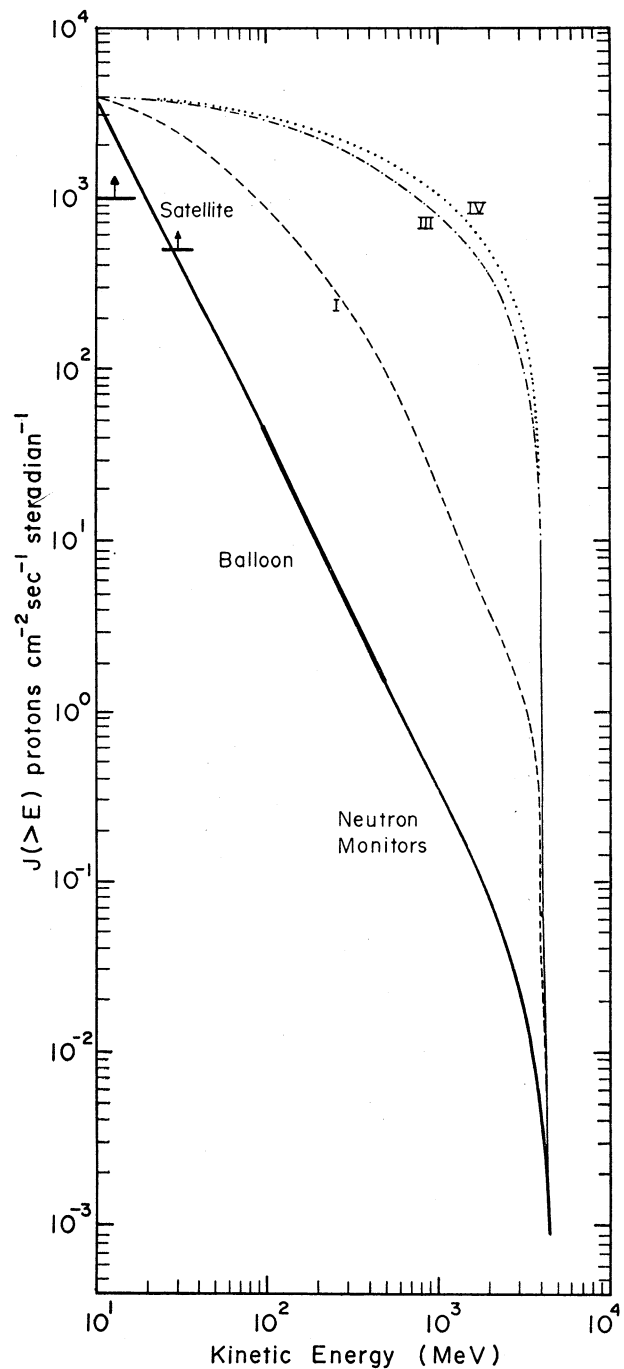
FIG. 4. Comparison of the experimental energy spectrum of the solar proton event of January 28 1967, with the predicted theoretical spectrum by Fermi-acceleration (Eq. (7) with $E_m = 4.3\,GeV$).

TABLE 2

THE NORMALIZED FLUX K_0 $(\propto N_0)$ FOR FERMI-ACCELERATION IN
SOLAR FLARES (protons cm^{-2} s^{-1} steradian^{-1})

Set	I	II	III	IV
Velocity-dependence acceleration (Eq. (5)); $E_m = \infty$	9.43×10^3	8.61×10^3	4.80×10^3	4.65×10^3
Velocity-dependence acceleration (Eq. (5)); $E_m = 4.3$ GeV	9.43×10^3	8.62×10^3	5.20×10^3	5.18×10^3
Acceleration with $\beta \simeq$ constant (Eq. (7)); $E_m = \infty$	4.99×10^3	.....	4.19×10^3	4.16×10^3
Acceleration with $\beta \simeq$ constant (Eq. (7)); $E_m = 4.3$ GeV	4.99×10^3	.....	4.19×10^3	4.16×10^3

IV. DISCUSSION

We have considered that the experimental spectrum measured for a particular event (January 28, 1967) and a given time (1600 UT) is typical for solar multi-GeV proton events and that $J(>E)_{\text{source}} \simeq J(>E)_{\text{earth orbit}}$. From these assumptions the comparison of the theoretical spectrum with the experimental curve points to the following features:

1) As it was expected, the consideration of an upper cutoff at 4.3 GeV in equations (5) and (7), such as in the experimental spectrum, gives the best fit with this curve. However, it must be pointed out that although the high energy cutoff in the acceleration process is somewhat higher than the experimental value measured at ground level (Pérez-Peraza 1975), we have not included this assumption in our calculations because the source spectra (5) or (7) only relate to the acceleration stage, disregarding energy losses during acceleration or a plausible energy degradation step after acceleration. Nevertheless it can be appreciated that the acceleration parameters α and τ are little affected by the assumptions made about the upper limit in the source energy spectrum as can be seen from the fact that $\alpha\tau = 0.17 - 0.19$. Therefore the parameter E_m in the acceleration spectrum takes greater importance, for instance, in the estimation of the time-scale for acceleration.

2) We believe that the normalization performed with the theoretical and experimental fluxes at 10 MeV in order to infer N_0, as explained in Section II, is a correct procedure. This can be seen from

the fact that our calculation with equations (5) and (7) which evaluate directly $J(>E)/N_0$ furnish systematically $J(>E)/N_0 < 1$ at all energy, for all the parameter sets. This result was expected because the number of Fermi-accelerated particles cannot be greater than the number of particles available for acceleration.

3) The flux obtained $K_0 = pN_0 \simeq 9 \pm 0.4 \times 10^3$ protons cm^{-2} s^{-1} steradian^{-1} indicate that only a very small fraction of the local plasma particles ($p \simeq 10^{-12} - 10^{-14}$) need to be picked up by the Fermi-statistical process in order to explain the observed spectrum, if we suppose that the initial number spectrum N_0 is determined in a region of n = 10^{11} cm^{-3} $- 10^{13}$ cm^{-3} and T $\simeq 10^4$ °K $- 10^5$ °K, by a thermal distribution, as $N_0 = [9/(2\pi)^{3/2}] (k/M_p)^{1/2}$ e$^{-3/2}$ nT$^{1/2}$ = 1.15843×10^3 nT$^{1/2}$ protons cm^{-2} s^{-2} steradian^{-1}.

4) Although the parameters α and τ are not subject to direct observational determination, we believe that the values deduced from the appropriate combinations of the factor $\alpha\tau$ in equations (5) and (7), represent an accurate description of these acceleration parameters in solar flare conditions:

Concerning the acceleration efficiency α, it is widely known at present that hydromagnetic velocities of the order of $10^7 - 10^8$ cm s^{-1} have been found in flare regions. Moreover the mean distance between magnetic mirrors cannot be larger than the linear dimension of the flare ($\leq 10^9$ cm); hence our parameter $\alpha \simeq 0.41$ s^{-1} cannot be very far from the appropriate value.

With respect to the mean life of particles in the acceleration regions, τ, the actual observational data do not permit, as yet, to infer about the nature of the particle loss process from the acceleration volume, losses which very likely occur by means of leakage through thin or thick scattering or by curvature drifts, or by drifts provoked by spatial and temporal variations of the magnetic field, or even by a disruption of a certain closed magnetic structure containing the accelerated particles when the kinetic energy density exceeds the magnetic energy density. Besides this, parameter τ may be slightly influenced by nuclear transformations from more than seven different interactions of protons with the hydrogen and helium of the source and, at high energies $(E > 285 \text{ MeV})$, by the nuclear reaction $p + H^1 \rightarrow H^2 + \pi^+$, which entails a loss of some protons from the acceleration process. Nevertheless, several indirect procedures permit to obtain an approximate evaluation of the orden of this parameter (Pérez-Peraza 1975); as for example, from the comparison of the theoretical spectrum slope with the observational slope. These estimations permit to verify the accuracy of our parameter $(\tau \sim 0.4 \text{ s} - 0.5 \text{ s})$ estimated here from the magnitude of the factor $\alpha\tau$ and scattering considerations into the frame of a thick geometry.

5) As can be seen in Figures (1) and (2) the calculated spectra do not give the desired agreement with the observed spectrum; we believe that the discrepancy between the theoretical spectra predicted by the Fermi mechanism and the experimental curve follow from three main reasons:

a) The existence of two or more sources in the active centers of the solar atmosphere might also be supposed to have a somewhat different physical structure and hence with different accelerating efficiencies in which case our theoretical spectrum should be the superposition of two or more curves. This would be the case if one supposes that low-energy protons are accelerated at a certain level of the atmosphere, and subsequently escape into interplanetary space, while a small fraction of them remains in the field of the active centers, to be re-accelerated at another level, up to relativistic energies. We have not explored here this possibility.

b) Neglect of the modulation effects in interplanetary space (convection, adiabatic deceleration and diffusion of protons) which introduces, as is well known, an exponential depression in the proton fluxes with dependence upon the rigidity and velocity of particles. In fact the influence of these effects is certainly of great importance during periods of maximum solar activity and is expected to affect mainly the low-energy portion of the source spectrum. Moreover it is not excluded that along with interplanetary modulation, the source spectrum could be modified owing to the production of H^2, H^3 and H^3_e by inelastic collisions of the energetic protons with interplanetary hydrogen and heilum.

c) Acceleration of solar particles is not performed in vacuum but in a highly ionized plasma of the flare region. Therefore, along with the Fermi-acceleration, the moving protons will experience braking because of the ionization losses, proton-proton collisions and eventually adiabatic deceleration if the source is in expansion. We think this feature accounts for the strongest discrepancy between the theoretical and the experimental energy spectra; the problem of energy losses during acceleration, in the high density medium of the source, is being investigated and results will be reported elsewhere.

From that analysis we hope to find a better adjustment of the Fermi-spectrum (5) with the experimental curve, for the following reasons:

(i) the influence of ionization losses is stronger on non-relativistic protons (which constitute the main portion of the energy spectrum). This leads to a decrease of the proton flux at low energies because of the thermalization of an important fraction of them. Therefore, the theoretical spectra will be depressed to bring it close to the experimental curve in the non-relativistic region.

(ii) In the relativistic portion of the energy spectrum the discrepancy is not very considerable (Figures 1 and 2); however, it is also expected that the energy degradation process from nuclear proton-proton interactions will serve to push the theoretical spectrum towards the experimental curve, as well as an eventual contribution of adiabatic deceleration.

Finally we may add that no definitive conclusion can so far be obtained regarding the generation of solar protons in the light of which this preliminary approach to the theoretical work may be altered as previously discussed. Moreover, an extension to other source spectral shapes (e.g. in momentum or mag-

netic rigidity) must be investigated before any definitive conclusión concerning this phenomenon can be drawn. Nevertheless in spite of the preliminary character of our results the velocity dependence of the acceleration process in solar flares seems to be established and on the other hand, we believe that the set of parameters I and II entails an approximate description of the solar proton source, and little fluctuations over them can be expected from the extension of the analysis to other multi-GeV proton events. In particular, it is very likely that the local plasma concentration at the source varies from one event to another but generally in such a way that $n \geq 10^{11}$ cm^{-3} where energy losses must be considered in connection with the Fermi-acceleration process.

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