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COMMENTS ON THE OPTICAL APPEARANCE OF WHITE HOLES

DEBORAH DULTZIN-HACYAN AND SHAHEN HACYAN

Instituto de Astronomía
Universidad Nacional Autónoma de México
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RESUMEN

La apariencia óptica de un gas dentro de la esfera de Swarzschild, y saliendo de ella, se ha estudiado. Se toman en cuenta fotones que se mueven con momento angular no trivial. Un análisis semicualitativo muestra que líneas espectrales bien definidas no pueden ser detectadas por un observador lejano si el gas emite de una región cercana al radio de Schwarzschild. Basados en nuestro análisis, sugerimos que los quásares y los objetos BL Lacertae son diferentes manifestaciones de un mismo proceso físico: la expansión de un hoyo blanco.

ABSTRACT

The optical appearance of gas inside and emerging beyond the Schwarzschild sphere is studied. We emphasize the importance of taking into account photons moving with non-trivial angular momentum. A semiqualitative analysis shows that no spectral lines could be seen by a distant observer if gas emits from a region near the Schwarzschild sphere. On the basis of our analysis we suggest that quasars and BL Lac objects are somewhat different manifestations of the same physical process, namely, a white hole expansion.

Key words: BL LACERTAE OBJECTS — GENERAL RELATIVITY — QUASARS — WHITE HOLES.

I. INTRODUCTION

The hypothesis that quasars can be interpreted as the result of the expansion of a gravitating sphere beyond its Schwarzschild radius was proposed by Novikov (1964) and, independently, by Ne'eman (1965); this expanding solution of Einstein's equations is called a "white hole". Novikov (1964) worked out a cosmological model in which these objects, called delayed cores, are retarded in their expansion for an external observer, and they emerge from beyond their Schwarzschild radius at different times for different cores. Subsequently, Ne'eman (1965) suggested that the energy of the system originates from the expansion of a central delayed core (white hole).

The optical appearance of an expanding central core, as the one described above, was studied by Faulkner, Hoyle and Narlikar (1964), and in more detail, by Narlikar and Apparao (1975). These authors considered radially moving photons only. However, as we will see below, photons with non-trivial angular momentum produce an important distortion of any spectral feature. This fact is already evident from the work of Ames and Thorne (1968), who studied the optical appearance of the collapsing surface of a star, and found that spectral lines would appear smeared out over a large range of frequencies, with the intensity diminishing exponentially with time.

As it is well known, the most accepted explanation for the very large redshifts of quasars spectral lines is that they are at comological distances. There are some problems, however, related to the large energies associated with these objects, the periodic variations of optical and radio flux, the association of quasars with nearby clusters, the anomaly in the redshift-apparent magnitude relation, and the recently discovered "superrelativistic expansion" (Shapiro 1976). These problems have given rise to the idea that quasars might have some intrinsic redshift and different models have been proposed (see e.g., Hoyle and Fowler 1967, Durgapal 1975). In this work we consider, among other things, the possibility of a "local white hole" hypothesis for quasars. For this reason, and in order to explain spectral lines, we shall not be interested in the emission of the core itself as it expands (this emission would contribute only to the continuum), but instead we shall consider the emission of a low density ionized gas emitting from a region near the Schwarzschild sphere. The results obtained show that no lines could be observed at all. Since this is in obvious contradiction with the observed quasars spectra, we conclude that it is not possible to formulate a "local white hole" hypothesis for quasars.

In the discussion it is pointed out that the absence of observable spectral lines predicted by the case studied here could explain, in principle, the spectra of BL Lac objects. Some general ideas are put forward that relate quasars and BL Lac objects to different types of white holes models. Detailed mathematical calculations are carried out in appendixes A and B, and we shall refer to them in what follows.

II. THEORETICAL CONSIDERATIONS

A major problem concerns the physics of the central core: for how long can matter remain inside the Schwarzschild radius? Indeed, no matter can stay at rest inside a white hole without violating causality. Consider, for simplicity, a sphere of dust (pressure $p = 0$) expanding symmetrically inside the Schwarzschild radius. The particles in the boundary of the sphere will move as free particles in an empty Schwarzschild geometry; therefore, as it is shown in Appendix A, the sphere of dust will reach the Schwarzschild radius in a time

$$U(2M) - U(O) = \int_0^{2M} dr\,[E - (E^2 - 1 + 2M/r)^{1/2}]/(1 - 2M/r) \qquad (1)$$

as seen by a distant observer (we set $G = 1 = c$; M = total mass of the white hole). E in equation (1) is the energy per unit mass of the particles ($E = 1$ corresponds to parabolic motion). The integrand of equation (1) is well behaved, even at $r = 2M$, and thus $U(2M) - U(O) \sim M/E \sim 10^{-5} (M/M_\odot) E^{-1}$ secs: this would be a few years for the mass of a galaxy if $E \sim 1$. It is only for $E = 0$ that the particles will remain eternally inside the Schwarzschild radius, as seen by a distant observer. On the other hand, if some matter is ejected at large distances from the white hole, as it is expected in the light of the model discussed below, then most likely $E \sim 1$ for this case, and the particles reach the Schwarzschild radius in a very short time.

Zel'dovich, Novikov and Starobinskii (1974), (ZNS), have pointed out that, due to quantum effects, particles must be created near the singularity, and this should affect the subsequent evolution of the white hole. Furthermore, these authors claim that a distant observer will see the matter inside a white hole either approaching asymptoticaly the Schwarzschild radius or blowing up in a time $\sim 10^{-5} (M/M_\odot)$ secs. This mathematical result is based on the simplifying assumption that the metric in the reference frame of the expanding particles admits a spatially-flat hypersurface. This, however, is not true in general, but only in a first approximation near the singularity, where the Schwarzschild metric in synchronous coordinates looks like a Kasner metric. In the idealized case of zero pressure, the two possible types of expansion of matter studied by ZNS, asymptotic expansion and fast explosion, correspond formally to the cases $E = 0$ and $E > 1$, respectively, in our eq. (1); clearly intermediate choices are possible, at least in principle. Unfortunately, it is still not possible to formulate a more detailed, and well justified theory of white holes before elucidating such problems as the behaviour of matter created inside the white hole at densities as high as 10^{84} gr/cm^3 (according to ZNS), the birth of white holes in the early universe, and, perhaps most important, the effect of rotation, since the topology near the singularity is drastically changed when rotation is present, as an analysis of the Kerr metric shows (Boyer and Lindquist 1967). So far there exists only guesses about these problems.

III. MODEL DESCRIPTION

From the previous considerations, the following model can be proposed: consider a dense central core whose expansion is slowed down by some still unknown mechanism; from the surface of this central core an "atmosphere" of particles evaporates. If this model corresponds to a quasar, it can be expected that the central core will liberate a great amount of energy in the form of radiation, and the spectral lines must be produced by the gas of evaporating particles. This gas would consist of ionized atoms, and its density must be very low in order to account for the existence of forbidden lines in the quasars' spectra (Schmidt 1971); therefore the gas must be transparent. Obviously, the atoms of this low density gas will move as free particles if radiation pressure and atomic collisions are neglected [in fact, the radiation pressure may be important (Burbidge and Perry 1976) but its contribution to the following semi-qualitative discussion is irrelevant]. Now, still within this model, two possibilities arise: i) the spectral lines are formed inside (or just outside) the Schwarzschild radius, and ii) they are formed in a distant region where the gravitational field is weak. The second possibility, or some variants of it, has already been considered (Ne'eman 1965); clearly, in this case, the redshift cannot be explained as a gravitational effect. We will show in what follows that the first possibility is incompatible with observed quasars' spectra, independently of the physical conditions of the gas, such as temperature, opacity, etc, which we shall not consider.

As it is well known, a Schwarzschild black (white) hole has a photon orbit of radius 3M. The quantity characterizing photon orbits is the *impact parameter at infinity, l.* Photons emitted from radii $r < 3M$ will escape to infinity only if their impact parameter is smaller than $l_{crit} = 3\sqrt{3}\,M$. If $l \ll 1$, the photon will move almost radially, and if $l \sim< l_{crit}$, the photon will spend a long time orbiting the hole, at 3M, before escaping (see e.g., Ames and Thorne 1968). On the other hand, photons emitted from $r > 3M$ can escape if their $l > [r^3/(r-2M)]^{1/2}$. It must be noted that the geometry outside the Schwarzschild radius is the same for either a black or a white hole.

In order to understand the importance of non-radial photons, consider a spherical layer of gas expanding from the central core. Radially moving photons emitted from this layer at different times cannot arrive simultaneously to a distant observer, and therefore they produce well defined spectral lines which vary with time: the most strongly blue shifted photons are emitted (and arrive) earlier. When non-radial photons are taken into account, the picture changes radically; a distant observer will receive simultaneously photons emitted at different stages of the layer expansion, and these photons will arrive within a very large range of frequency shift.

The frequency shift of a photon emitted at r by a radially moving out-going particle depends of the place of emission and the impact parameter of the photon (see Appendix A):

$$1 + z = [E - \sqrt{(E^2 - 1 + 2M/r)} \times \sqrt{(1-(1-2M/r)l^2/r^2)}]/(1-2M/r). \qquad (2)$$

(Notice that at $r = 2M$, the redshift has the finite value $1 + z\,|_{r=2M} = (1 + E^2l^2/r^2)/2E)$. A simple examination of this formula shows that, for $r \sim 3M$, the frequency shift varies over an extremely wide range, the maximum observed frequency, $\nu_{0_{max}}$ (blue-shift), corresponding to photons with $l = 0$, and the minimum observed frequency, $\nu_{0_{mim}}$ (red-shift), to photons with $l \simeq l_{crit}$ (typical values are $\nu_{0_{max}} = 2\nu_e$ and $\nu_{0_{mim}} = 0.26\,\nu_e$, where ν_e is the emitted frequency, as calculated for $r = 2M$ and $E = 1$). Obviously, as the distance of emission $\cdot$ r increases, the difference between the upper and the lower frequency shift becomes smaller. As $r \to \infty$, equation (2) reduces to the formula for a pure Doppler shift, the velocity of the particle at infinity beeing given by E (clearly, in this case: $z = 0$ for $E = 1$).

In order to determine the intensity per unit frequency of the observed spectral line produced within the Schwarzschild radius, we consider a fixed spherical shell of thickness dr and radius r enclosing the dense core of the white hole. Through this shell there is a continuous flux of atoms which are emitting photons with different l, at some frequency ν_e. Photons emitted with this frequency will be observed by a distant observer in a very wide spectral range between frequencies $\nu_{0_{mim}}$ (r) and $\nu_{0_{max}}$ (r). In this interval, the number of photons received per unit frequency interval ν_0, J_{ν_0}, is independent of ν_0, as it was shown in Appendix B. Thus, the net contribution of all possible shells around the compact core will correspond to the contribution of all the atoms flowing outside the central core and located inside (or just outside) the Schwarzschild radius: the spectral feature observed at large distances will be a superposition of flat spectral "lines" completely smeared out over a very large interval of frequencies. Furthermore, if the flow is stationary, the observed "line" will not vary in time. Due to the smearing out effect explained above, it is clear that a spectral line produced within (or just outside) the Schwarzschild radius *cannot be observed.*

The following question arises naturally: Can photons emitted at $r > 3M$ be observed as well defined spectral lines, yet gravitationally redshifted? As we have seen, such photons will have an impact parameter between $l = [r^3/(r - 2M)]^{1/2}$ and $l = 0$; therefore, according to eq. (2), they can be observed with wavelengths varying between

$$\frac{\lambda_0}{\lambda_e} = E/(1 - 2M/r) \tag{3a}$$

and

$$\frac{\lambda_0}{\lambda_e} = [E - (E^2 - 1 + 2M/r)^{1/2}]/(1 - 2M/r) \tag{3b}$$

For photons emitted by gas at a distance $r > r_{min}$ (where $r_{min} > 3M$, but otherwise arbitrary), the width of the observed line would be approximately: $1 + Z \sim E^{-1} > 1$. This means that a well defined gravitationally redshifted line could be produced only by gravitationally bounded atoms emitting at (or very near) their maximum r, all the atoms having the same energy E. This very peculiar situation is highly unrealistic. In conclusion we can say that, within the scheme of the model described above, a well defined, observable line cannot be intrinsically redshifted; on the other hand gravitationally redshifted photons cannot produce observable spectral lines.

IV. CONCLUSIONS AND DISCUSSION

a) If quasars are not at cosmological distances, (see e.g., Field, Arp and Bahcall 1973) the observed redshift might be explained as gravitational redshift. For a white hole model of quasars this implies that the spectral lines must be produced by atoms flowing out, but still near enough to the Schwarzschild sphere to be gravitationally redshifted. In this case, the study of line profiles and intensity distribution fails to explain the observed line spectra of quasars.

The results obtained here are in accordance with the effect of "limb reddening" foud by Novikov and Ozernoy (1963) using numerical techniques.

b) If quasars are at cosmological distances, the observed redshift might be explained as a cosmological effect. In this case a white hole model such as that proposed by Ne'eman (1965) could, in principle, explain the observed line spectra.

The BL Lac objects are very similar to quasars except for the fact that they do not exhibit spectral lines (see e.g., Oke, Neugebauer and Becklin 1969; Du Puy *et al.* 1969). The spectrum of these objects consists of a non-thermal continuum. A study of spectral flux distribution by O'Dell *et al.* (1977) shows that spectral shape does not, by itself, explain the absence of emission lines in BL Lac objects. All possible explanations have failed to account for this type of spectrum.

One can often find in the literature the idea that BL Lac objects merely represent a somewhat different manifestation of the same physical process that must give rise to quasars —and perhaps other compact nonthermal sources— (see e.g., Racine 1970; Strittmatter *et al.* 1972; Penston and Penston 1973; Stein, O'Dell and Strittmatter 1976). With all reserve, we would like to advance the following general idea: that the physical processes responsible for the quasar and BL Lac type phenomena are basically the same: expansion of superdense matter beyond its Schwarzschild radius. However these two types of objects correspond to different spectral production zones. Quasars would correspond to the case in which the spectrum is produced by matter far away from the Schwarzschild sphere, and so their redshifts must be cosmological. On the other hand, BL Lac objects could correspond to the case in which the spectrum is produced by matter very near the Schwarzschild sphere and, due to gravitational redshifting, no lines could be seen in this case. Moreover, from the final consideration in Section III it follows that no intermediate cases can be observed (the redshift of an observed spectral line cannot be due to a combined gravitational-cosmological effect). The very rapid variability of BL Lac objects gives an even higher restriction on the size of these objects than in the case of quasars; this is not in contradiction with the above mentioned ideas.

Ambartzumian (1965) has long ago proposed the existence of significant masses of matter that can remain in a state of superdense configuration. These masses are called D-bodies, and, according to Ambartzumian, explosions of such bodies are responsible for the existence of stellar associations, and for many of the violent phenomena that occur in nuclei of galaxies. From this point of view one could possibly regard the BL Lac objects as the result of the expansion of these hypothetical D-bodies in the nuclei of galaxies. There is strong evidence that, in at least a few cases, BL Lac objects exist in, or as, the nuclei of elliptic galaxies (see e.g., Ulrich *et al.* 1975; Bolton, Carke and Ekers 1965; Westerlund and Wall 1969; Disney *et al.* 1974; Stein, O'Dell, Strittmatter 1976 and Miller and Hawley 1977). The possibility that they may be ejected from the nuclei of galaxies (Craine, Tapia and Tarenghi 1975) must be further explored, but it receives support at least from such a case as OX 029 (Craine and Warner 1976).

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APPENDIX A

It will be most convenient to use the Schwarzschild metric in the outgoing Eddington-Finkelstein coordinates (Misner, Thorne and Wheeler 1973):

$$ds^2 = (1 - 2M/r)dU^2 + 2dUdr - r^2\, d\Omega^2 \qquad \text{(A1)}$$

(signature: -2), where

$$dU = dt - dr/(1 - 2M/r). \qquad \text{(A2)}$$

A photon moving radially has a world line given by U = const. Therefore if a radially moving particle emits two radially outgoing photons at events with coordinates U_1 and U_2, respectively, these photons will be reecived by a distant observer with a (proper) time difference $U_2 - U_1$, independently of the motion of the emitting particle. In this sense, U may be interpreted as a "retarded-time" coordinate.

From standard methods (see e.g., Landau and Lifschitz 1962), is can be seen that the equation for a (radial) time-like geodesic is given by

$$U(r) = U(a) + \int_a^r dr\,[E - (E^2 - 1 + 2M/r)^{1/2}]/(1 - 2M/r) \qquad \text{(A3)}$$

and the unit vector tangent to the geodesic is, therefore,

$$u^\mu = \{[E - (E^2 - 1 + 2M/r)^{1/2}]/(1 - 2M/r),\ (E^2 - 1 + 2M/r)^{1/2}, 0\}\,. \qquad \text{(A4)}$$

(Coordinates are set in the order $\{U, r, \phi\}$; the irrevelant coordinate θ will be set $\theta = \pi/2$).

E is the energy per unit mass of the geodesically moving particle. The proper time of this particle is

$$d\tau = dr/(E^2 - 1 + 2M/r) \qquad \text{(A5)}$$

As for a photon, the tangent vector to its trajectory is

$$k_\mu = \nu_e\{1,[1 - (1 - (1 - 2M/r)l^2/r^2)^{1/2}]/(1 - 2M/r), l\} \qquad \text{(A6)}$$

where l is the impact parameter and ν_e the frequency as seen by the photon emitter.

The redshift is given by

$$1 + z \equiv \frac{\nu_e}{\nu_0} = \frac{(k_\mu u^\mu)_e}{(k_\mu u^\mu)_0}, \qquad \text{(A7)}$$

where the subscripts e and o refer to the photon emitter and observer, respectively. Equation (2) in the text follows from (A4), (A6) and (A7).

APPENDIX B

We choose an orthonormal triad such that the time-like vector is the unit vector tangent to the world-line of a freely moving particle:

$$\partial_{(t)} = u^{\mu}\partial_{\mu}$$
$$\partial_{(r)} = -[E - (E^2 - 1 + 2M/r)^{1/2}](1 - 2M/r)^{-1}\partial_U + E\,\partial_r$$
$$\partial_{(\phi)} = \frac{1}{r}\partial_\phi . \quad \text{(B1)}$$

Then, from (A6), the trajectory of the photon in the particle rest frame is

$$k_{(t)} = \nu_e\{E - (E^2 - 1 + 2M/r)^{1/2}[1 - (1 - 2M/r) l^2/r^2]^{1/2}\}/(1 - 2M/r)$$
$$k_{(r)} = \nu_e\{(E^2 - 1 + 2M/r)^{1/2} - E[1 - (1 - 2M/r) l^2/r^2]^{1/2}\}/(1 - 2M/r) \quad \text{(B2)}$$
$$k_{(\phi)} = \nu_e\, l/r .$$

Thus, in the emitting particle rest frame, a photon with impact parameter l will be seen emitted at an angle α given by:

$$\tan\alpha = k_{(\phi)}/k_{(r)} . \quad \text{(B3)}$$

Using equations (B2) it follows that

$$\cos\alpha = (E - \nu_0/\nu_e)/(E^2 - 1 + 2M/r)^{1/2} . \quad \text{(B4)}$$

The number of photons emitted between α and $\alpha + d\alpha$ is proportional to senα dα; therefore if an atom emits photons isotropically and at a constant rate as seen in its own reference frame, then the number of photons emitted by this atom as it moves from r to $r + dr$, and received with frequency $\nu_0, \nu_0 + d\nu_0$ by distant observers is $J(\nu_0, r)\,d\nu_0 dr$, where

$$J(\nu_0, r) = (\text{const.}) \times (E^2 - 1 + 2M/r)^{-1}/\nu_e . \quad \text{(B5)}$$

Notice that $J(\nu_0, r)$ is independent of ν_0.

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