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# SPECTROSCOPIC OBSERVATIONS OF HD 93521: AN O-TYPE STAR OF POPULATION II

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## RESUMEN

Se asigna un nuevo tipo espectral (O9.5 Ib-II) a HD 93521. Su carácter de supergigante acentúa la discrepancia entre la distancia de la estrella al plano galáctico y su aparente pertenencia al grupo de Población I. Del análisis comparativo de anchos equivalentes para líneas espectrales de He I en la estrella y en 62 estrellas tempranas, se encuentra que HD 93521 es ligeramente sobreabundante en He. Si la estrella fuera de Población I, el tiempo de vida nuclear resultaría incompatible con su posición y cinemática actuales. En consecuencia, se propone que HD 93521 se encuentra en la misma fase evolutiva de las estrellas ultravioletas brillantes de Población II.

## ABSTRACT

The espectral type of HD 93521 is revised and is found to be O9.5 Ib-II. Its bright luminosity class stresses the contradiction between the star's distance to the galactic plane and its Population I apparent membership. A comparative analysis of the equivalent widths of He I lines in the star and 62 early-type stars yielded a detectable He overabundance in HD 93521. If the star is assumed to belong to the Population I group, its maximum nuclear lifetime is incompatible with its present position and kinematics. Therefore, HD 93521 is proposed to be in the evolutionary phase in which the Population II UV-bright stars are.

*Key words:* **STELLAR SPECTROSCOPY — BLUE HALO STARS — UV-BRIGHT STARS.**

## I. INTRODUCTION

HD 93521, an $m_v = 7.04$ star (Guetter 1974), was classified in the MK system as O9 Vp by Morgan, Code and Whitford (1955); no explanation regarding the peculiarity is given by the authors. This star is conspicuous because, being otherwise a normal O-type star, its galactic latitude ($b^{II} = +62^\circ\!.1$) and distance modulus place it at an abnormally large distance to the galactic plane. From its appearance this star could be included in the sample of the apparently normal blue halo stars studied by Greenstein and Sargent (1974). From the comparative analysis of the Ca II K interstellar components in HD 93521 and in $\rho$ Leo, Münch (1952) determined a lower limit of 750 pc for the distance of the former to the galactic plane, suggesting that the star is possibly a Population II object. This suggestion was first made by Ahmad (1952), who found relatively large equivalent widths in HD 93521 for the He I lines $\lambda$4471 and $\lambda$4144, though that of $\lambda$4387 was found to be normal. His analysis was done by comparing He I lines measured by Williams (1936), using a sample of thirty five O7 to B9 stars.

In order to analyze the problem with more detail, we have obtained homogeneous spectroscopic observational material, as described in §II. In §III we discuss the spectral classification of HD 93521 and the equivalent widths of the strongest lines in this star and in HD 55879, a star with similar spectral type. The equivalent widths of He I λ4471 and λ4387 in HD 93521 are compared with those of sixty two stars in the O4-B2 spectral range, in order to detect abnormal He abundance in the former star. The results are interpreted in §IV and conclusions are summarized in §V.

## II. OBSERVATIONS

We used the Cassegrain spectrograph attached to the 1 m telescope of the Observatorio Astronómico Nacional at Tonantzintla. The spectra on which the equivalent widths were measured and the spectral classifications made, were obtained with a Vidicon Television tube between September, 1975 and February, 1976. To include the Hα spectral region, some spectra were taken in the October-November-December, 1976 observing period; this time a SIT detector was used. In both cases the television tubes were coupled to a 500 multichannel memory device. The yielded reciprocal dispersions were approximately 1.6 Å/channel with the Vidicon tube and 2.8 Å/channel with the SIT. The wavelength ranges registered in each case were λ3900-λ4750 and λ5550-λ6800 Å, respectively. The blue spectra were obtained on the second dispersion order; to avoid contamination with the infrared radiation from the first order, a Kodak 301 filter was used.

The equivalent widths were measured directly on traces obtained from an analogical output. The data were not corrected for atmospheric extinction or instrumental response.

## III. OBSERVATIONAL RESULTS

### a) *Spectral Classification of HD 93521*

Following the spectral classification criteria given by Walborn (1971), we have reclassified HD 93521. The only previous classification we know, O9 Vp,

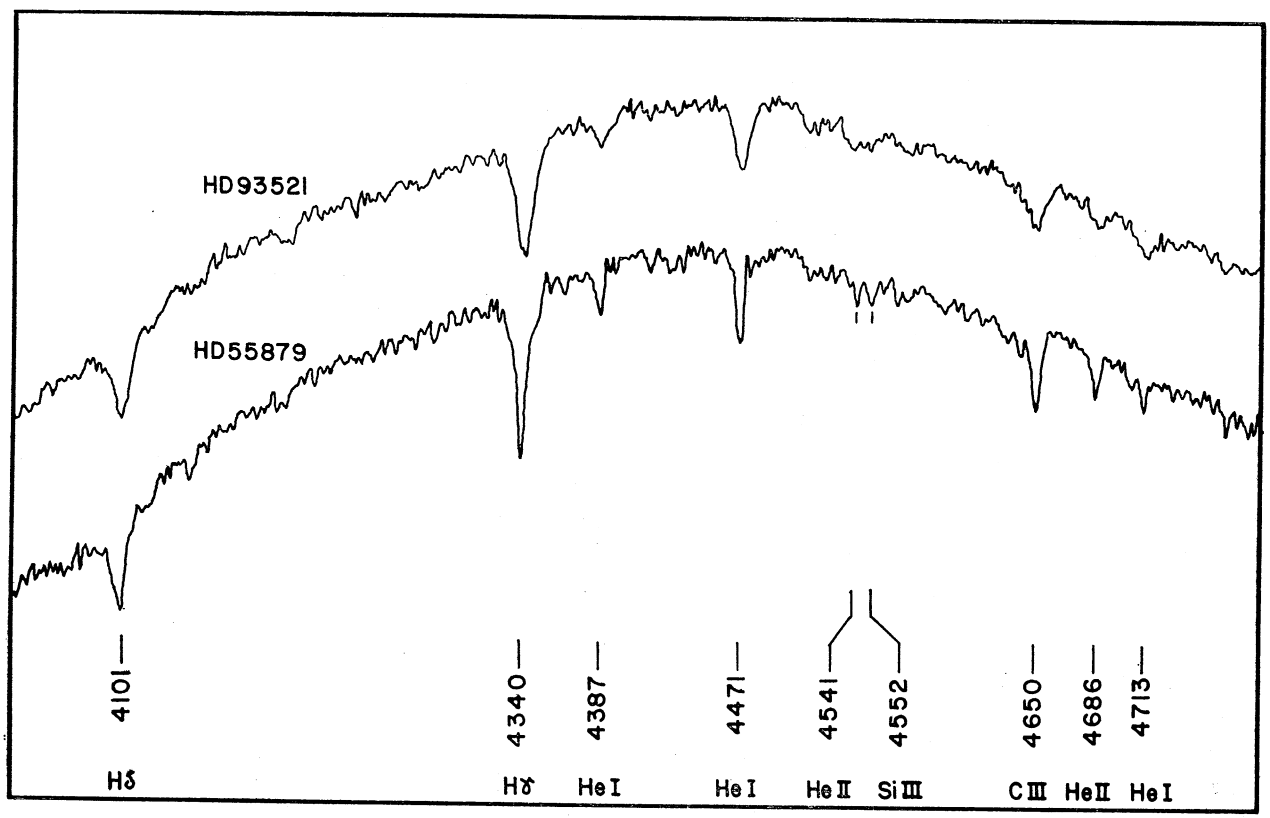


FIG. 1. Tracings of the raw spectra of HD 93521 and HD 55879. The two lines used to determine the spectral type in the O9-B0 range, λ4541 and λ4552, are marked in the spectra of HD 55879.

was given by Morgan *et al.* (1955). The procedure followed here and its reliability has been discussed by Bisiacchi *et al.* (1976).

The presence of the He II line λ4541 and probably that of Si III λ4552 locates the spectral type between O9 and B0. Unfortunately, all the lines in the star are intrinsically wide enough to blend these two lines, which makes very difficult a more precise classification. The lines in HD 93521 are very broad probably due to stellar rotation: a very high value for $v \sin i$ (400 km $s^{-1}$) has been derived by Conti and Ebbets (1977) for this star. We also obtained higher dispersion spectra (0.7 Å/channel) in an attempt to minimize instrumental-width effects; however, no noticeable improvement was achieved. Nevertheless, from the profile of the blend, we conclude that the spectral type is very nearly O9.5. The intensity of He II λ4686 as compared to those of C III λ4650 and He I λ4713 corresponds to a star of luminosity class Ib. This result is also supported by the Hα photometry by Mendoza (1976) since his value of the α(35) index for this star is in good agreement with that of a star O9.5 of luminosity class I.

Figure 1 shows the spectra of HD 93521 and HD 55879, the latter was classified O9.5 II-III by Walborn (1972) and O9.5 Ib by Bisiacchi *et al.* (1976). All through this paper HD 55879 is used as a reference star to HD 93521, because their spectra are very similar, except for the shallower and wider lines in HD 93521. The spectra of these two stars were also taken in the red region. We found them very similar to each other, except for the just mentioned differences in line widths. No Hα emission was detected in either star.

In Figure 2 the traces in the spectral range relevant to luminosity classification are shown for HD 93521 and HD 55879, and for the standard stars AE Aur (O9.5 V), 19 Cep (O9.5 Ib) and α Cam (O9.5 Ia). Note the variation of intensity of λ4686 as compared to λ4650 and λ4713.

The strength of He II λ4686 in HD 93521 is similar to that of 19 Cep, hence one is tempted to classify the star as luminosity class Ib. However, given the high rotational velocity present in this star, the possibility that the line might be filled in with emission should be contemplated. However, Hα and other hydrogen lines do not show traces of emission neither in the core nor in the wings. In addition, the equivalent width of Hγ yields a luminosity class II for this star as discussed below.

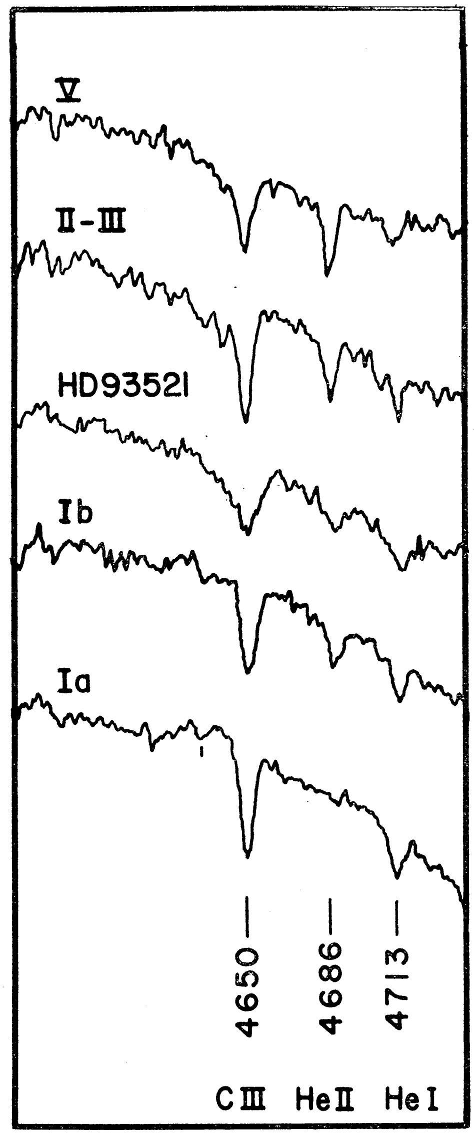


FIG. 2. Tracings of the spectra of O9.5 stars ordered by their luminosity class. From top to bottom, AE Aur, HD 55879, HD 93521, 19 Cep and α Cam. The spectral region shown is the relevant range for luminosity classification.

We conclude then, that HD 93521 is an O9.5 Ib-II star. The previous classification, O9 Vp, differs from ours mainly on the luminosity class. Our estimate of the luminosity places the star even fur-

TABLE 1

EQUIVALENT WIDTHS (mÅ)

| λ (Å) | Identification | HD 93521 W(λ) | HD 93521 σ | HD 55879 W(λ) | HD 55879 σ |
|---|---|---|---|---|---|
| 4101 | Hδ | 2243 | 150 | 2106 | 194 |
| 4340 | Hγ | 2410 | 92 | 2411 | 231 |
| 4387 | He I | 447 | 37 | 458 | 58 |
| 4471 | He I | 1124 | 95 | 905 | 139 |
| 4650 | C III | 905 | 96 | 1045 | 119 |
| 4686 | He II | 357 | 52 | 478 | 52 |

TABLE 2

EQUIVALENT WIDTHS OF λ4387 AND λ4471 (mÅ)

| HD | Sp. Type | W(4387) | W(4471) | N | HD | Sp. Type | W(4387) | W(4471) | N |
|---|---|---|---|---|---|---|---|---|---|
| 108 | O5.5 fp | ... | 312 | 1 | 46573 | O7 III ((f)) | 216 | 634 | 1 |
| 2905 | B0.7 Ia | 437 | 1079 | 1 | 46966 | O7.5 V | 316 | 794 | 2 |
| 13854 | B1 Iab | 329 | 769 | 1 | 47129 | O7.5 I (f) | 258 | 641 | 2 |
| 15558 | O5 III (f) | ... | 160 | 1 | 47839 | O7 V | 258 | 713 | 7 |
| 15570 | O4 I f+ | ... | 130 | 1 | 48099 | O6.5 V ((f)) | ... | 417 | 2 |
| 24398 | B1 Ib | 359 | 1080 | 1 | 52533 | O8.5 IV | 207 | 679 | 3 |
| 24431 | O9 V | 393 | 884 | 2 | 53975 | O7 V | 211 | 620 | 1 |
| 24760 | B0.7 III | 459 | 1270 | 1 | 55879 | O9.7 Ib | 458 | 905 | 6 |
| 24912 | O7.5 III (f) | 219 | 830 | 4 | 57061 | O9 I | 313 | 733 | 3 |
| 30614 | O9.5 Ia | 360 | 859 | 1 | 57682 | O9 V | 419 | 853 | 3 |
| 34078 | O9.5 V | 393 | 917 | 8 | 93521 | O9.5 Ib-II | 447 | 1127 | 8 |
| 34656 | O7 III (f) | 200 | 764 | 1 | 144470 | B1 V | 499 | 1078 | 2 |
| 35468 | B2 III | 723 | 1306 | 1 | 147165 | B1 III | 457 | 974 | 2 |
| 36486 | O9 III | 377 | 800 | 2 | 149438 | B0.2 V | 664 | 1096 | 1 |
| 36861 | O8 III ((f)) | 268 | 877 | 5 | 154445 | B1.5 V | 580 | 1074 | 1 |
| 36879 | O7 V | 208 | 660 | 1 | 189957 | O9.5 III | 410 | 716 | 1 |
| 36960 | B0.5 V | 484 | 1141 | 3 | 190603 | B1.5 Ia | 375 | 582 | 1 |
| 37022 | O6.5 III (f) | ... | 565 | 1 | 190919 | B0.7 Ib | 272 | 1093 | 1 |
| 37043 | O9 III | 307 | 872 | 1 | 192422 | B0.5 Ib | 450 | 1034 | 1 |
| 37128 | B0 Ia | 357 | 1007 | 1 | 192639 | O7 Ib (f) | 262 | 598 | 1 |
| 37468 | O9.5 V | 494 | 900 | 2 | 193183 | B1.5 Ib | 496 | 989 | 1 |
| 37742 | O9.7 Ib | 249 | 845 | 3 | 193322 | O8.5 V | 401 | 870 | 1 |
| 38666 | O8.5 V | 416 | 1035 | 3 | 195229 | B0.2 III | 450 | 1040 | 1 |
| 38771 | B0.5 Ia | 395 | 1044 | 1 | 195592 | O9.7 Ia | 330 | 797 | 2 |
| 41117 | B2 Ia | 408 | 705 | 2 | 201795 | B0.7 V | 824 | 1228 | 1 |
| 41161 | O8 V | 360 | 796 | 1 | 202253 | B1.5 IV | 461 | 1345 | 1 |
| 41753 | B0 V | 512 | 1073 | 1 | 203064 | O8 III ((f)) | 202 | 790 | 1 |
| 42088 | O6.5 V | ... | 588 | 1 | 204172 | B0 Ib | 405 | 1013 | 1 |
| 46150 | O5 V ((f)) | ... | 233 | | 209975 | O9.5 Ib | 368 | 836 | 2 |
| 46223 | O4 V ((f)) | ... | 155 | | 210839 | O6.5 If | ... | 453 | 1 |
| | | | | | 214680 | O9 V | 386 | 953 | 1 |
| | | | | | 214993 | B1.5 III | 589 | 1199 | 1 |
| | | | | | 216898 | O9 V | 460 | 1186 | 1 |



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ther away from the galactic plane; the consequences will be discussed in the final paragraphs of this paper.

### b) *Equivalent Widths*

Table 1 lists the equivalent widths for the strongest lines in HD 93521 and in the adopted reference star HD 55879. We give the average values as obtained from eight spectra of the former star and six of the latter. Only the spectral range comprising λ4101-λ4686 was measured. The standard deviation about the mean, σ, is also listed for each line, indicating that the probable error is about 5% or less. The strengths of the luminosity sensitive lines C III λ4650 and He II λ4686, suggest that HD 93521 is slightly more luminous than HD 55879. Using the Hγ absolute calibration given by Balona and Crampton (1974), the Hγ equivalent widths yield luminosity classes of about II for both stars. This is true even if the spectral types of these stars were B0 or O9. It should also be noted that the intensities of the He I lines are similar in the two stars; W(4471) is about 30% larger in HD 93521, but not at all as high as the value for the same line (1560 mÅ) given by Williams (1936).

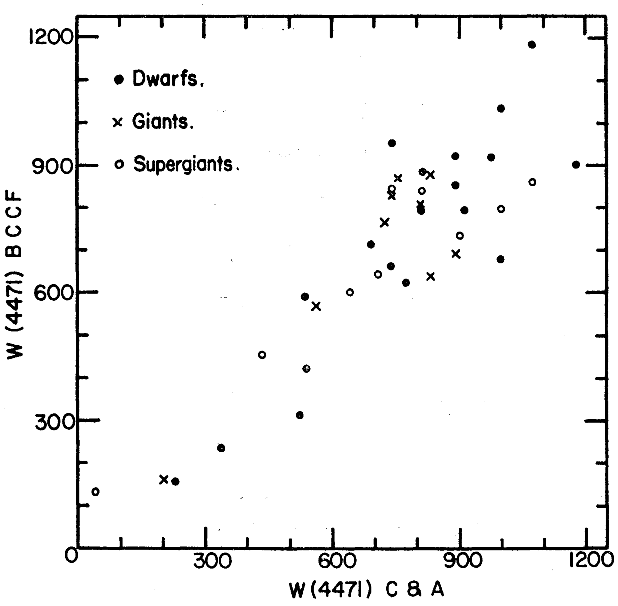


FIG. 3. Equivalent widths (in mÅ) of the He I line λ4471 as measured by Conti and Alschuler (1971), plotted versus those given in this paper.

In order to compare the equivalent widths of the He I lines in HD 93521 with those in a significant sample of OB stars, we measured λ4387 and λ4471 in other sixty-two stars. The former line was not measured in stars earlier than O7. Table 2 lists the equivalent widths, in mÅ, together with the HD number, the spectral classification and the number of spectra used in each case. The spectral classification is that given by Bisiacchi *et al.* (1976). The estimated error for a single spectrum is 15% or less in the case of W(4471) and less than about 25% for W(4387). To check for systematic errors we have compared our estimates of W(4471) with those by Conti and Alschuler (1971). The results are shown in Figure 3.

Figures 4 and 5 are, respectively, plots of W(4471) and W(4387) versus spectral type. All stars listed in Table 2 are included in these two figures. In them, black circles represent main sequence stars; crosses, luminosity classes IV, III and II; open circles, supergiants of both *a* and *b* types. HD 93521 is represented by a black triangle. Also plotted are straight lines joining theoretical values given by Auer and Mihalas (1972); in doing so we have adopted the temperature calibration for different spectral types given by Conti (1973), and his assumption that log $g = 4.0$ corresponds to main sequence stars and log $g = 3.3$ to supergiants. For completeness, the theoretical values for log $g = 4.5$ are included under the assumption that their temperatures also correspond to those of main sequence stars.

From both Figures 4 and 5 it is clear that the non-LTE models fit the data points better than the LTE ones. Moreover, the log $g = 4.0$ non-LTE model fits particulary well the observed He I equivalent widths of the luminosity class V stars. This supports Conti's preference for this value of log $g$ for O-type main sequence stars. There is a good general correlation between the observed equivalent widths in supergiants and the values of the non-LTE, log $g = 3.3$ model, but the fit is not as good as the one obtained for the main sequence stars. Anyway, the observations show that the He I lines in lower surface gravity stars are statistically weaker, as predicted by Auer and Mihalas' models.

The He I equivalent widths in HD 93521 are larger than those in stars of luminosity classes I

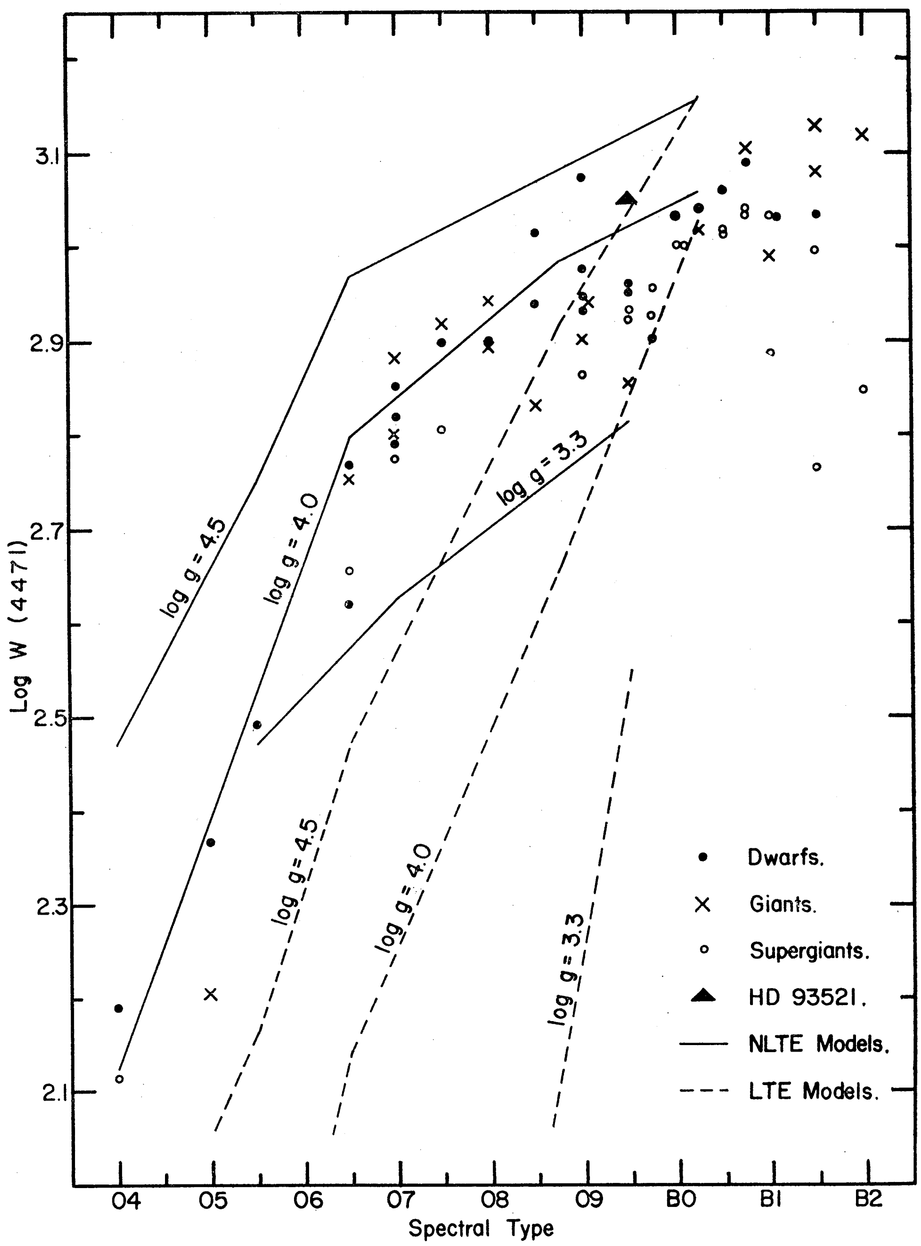


Fig. 4. Log W(4471) as a function of spectral type, for 62 stars and HD 93521. The solid and dashed lines are drawn between values of the equivalent width as calculated in models given by Auer and Mihalas (1972).

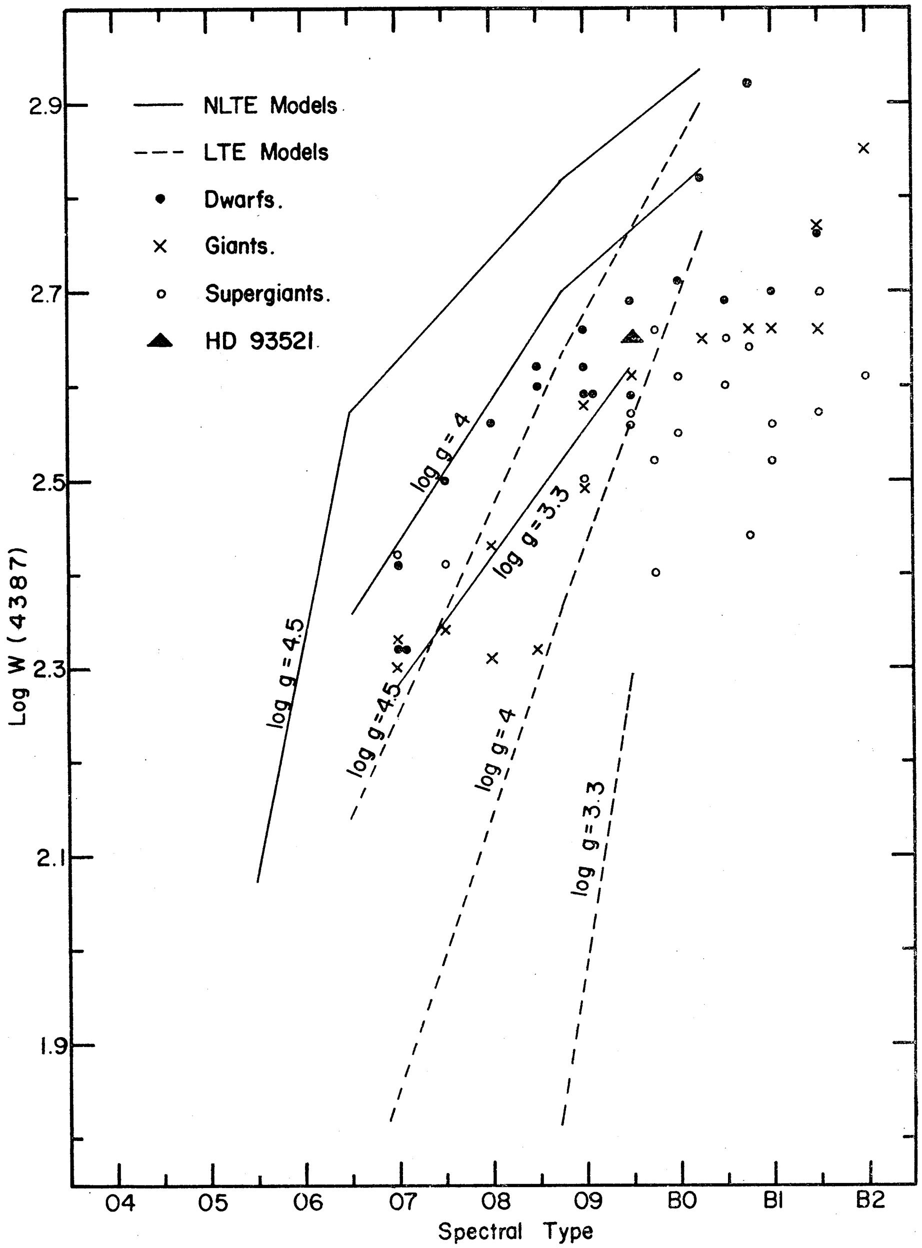


FIG. 5. Log W(4387) as a function of spectral type for 53 stars and HD 93521. The solid and dashed lines are drawn between values of the equivalent width as calculated in models given by Auer and Mihalas (1972).

and III. Although the differences are not much larger than the estimated errors, they point out towards an overabundance of He in this star, if indeed its surface gravity is as low as its spectral characteristics indicate.

## IV. DISCUSSION

In this section we would like to point out the apparent contradiction between the peculiar galactic location of HD 93521 and its spectral characteristics. This star is located at high galactic latitude, $b^{II} = 62\overset{\circ}{.}1$, in the direction of the anticenter. As has been pointed out in previous sections, its spectrum could be attributed to an O9.5 Ib-II star of Population I. Accordingly, its spectroscopic parallax (m-M = 13.04) places it about 3.6 kpc above the galactic plane. If the star were a Population I object, it must have been ejected from a site in the galactic plane. To evaluate the time since this happened we further need its galactic velocity components $u$, $v$ and $w$, which are found to be +32, +16, and +5 km $s^{-1}$, respectively. These values were obtained from the star's radial velocity component referred to the LSR, −14 km $s^{-1}$, given by Cruz-González *et al.* (1974), and from its proper motions given by Maitre (1971), reduced to the FK4 system, and corrected for errors in the precession constants of Newcomb and effects of galactic rotation (Fricke 1972).

A rough estimate of a lower limit for the time since the star was ejected is easily obtained assuming the star has been decelerated towards the galactic plane at a constant rate. We chose the maximum value for the acceleration, $9 \times 10^{-9}$ cm $s^{-2}$, given by Oort (1960) in describing the acceleration in the $z$ direction as a function of the distance to the galactic plane, in the solar neighborhood. This value is indeed an upper bound in the case of HD 93521 because the acceleration is smaller at shorter distances to the plane and because the star is located in the direction of the galactic anticenter, where the surface density of the galaxy is lower than that in the solar neighborhood. From the distance to the galactic plane, the $w$ component of the space velocity and the constant acceleration in the $z$ direction given above, the lower limit to the time elapsed since ejection is hence obtained to be $5 \times 10^7$ years. On

TABLE 3

DISTANCE TO HD 93521 AS A FUNCTION OF THE MASS

| $M/M_\odot$ | Log $g$ | $M_{bol}$ | Distance (pc) |
|---|---|---|---|
| 1.11 | 2.96 | —6.64 | 1200 |
| 0.85 | 3.09 | —6.01 | 1030 |
| 0.71 | 3.26 | —5.40 | 780 |
| 0.63 | 3.44 | —4.84 | 600 |

the other hand, the nuclear lifetime for an O9.5 V star —the latest spectral type an O9.5 I star may have had as a dwarf— is not longer than about $5 \times 10^6$ years. The contradiction between these limiting times, inferred by the assumption that the star belongs to Population I, can be solved in one of two ways: *a)* the star was formed far outside the galactic plane and with a peculiar velocity much larger than those observed for H I clouds at 21-cm; *b)* the star was ejected from the galactic plane and some presently unknown processes have slowed down the main sequence evolution of this star.

An alternative explanation would be that the star is a Population II object; that is to say, HD 93521 could be a field counterpart of the UV-bright stars in globular clusters, as those identified by several authors like Zinn (1974), Norris (1974) and Harris *et al.* (1976). These objects have been interpreted as highly evolved, double shell burning, low mass stars ($\sim 1\ M_\odot$) in evolutionary phases past the horizontal and the asymptotic giant branches, as indicated by the models calculated by Gingold (1974, 1977*a*), Sweigart *et al.* (1974) and Härm and Schwarzschild (1975). According to these theoretical tracks, the UV-bright stars reach high effective temperatures that are comparable to those of O stars. Their luminosities are lower than their Population I counterparts by an amount depending upon the stellar mass. Even more, this class of stars is generally expected to be He overabundant, since the relaxation loops they undergo due to He shell flashes may be a source of He enrichment, as pointed out by Gingold (1977*b*).

Assuming that the last hypothesis outlined above is correct, we can estimate a new distance for HD 93521. From the theoretical models, the tracks for these advanced phases of evolution are uniquely determined in the log $g$, log $T_{eff}$ plane in terms of one parameter: the stellar mass. In Table 3 we present the calculated values of log $g$ and $M_{bol}$ for a star with $T_{eff} = 33\,000°K$, as a function of its mass. On the last column we list the distance to HD 93521 obtained from the calculated $M_{bol}$, the star's visual magnitude ($m_v = 7.04$) and assuming that the bolometric correction for Population I, O-type stars apply to Population II stars of similar spectral type. This table is constructed by solving in a self-consistent manner the mass-luminosity function for the UV-bright stars, given by Paczyński (1971),

$$\frac{L}{L_\odot} = 59250\left[\frac{M}{M_\odot} - 0.522\right],$$

and the relation between surface gravity, effective temperature and stellar luminosity,

$$L/L_\odot = M/M_\odot \frac{g_\odot}{g}\left[\frac{T_{eff}}{(T_{eff})_\odot}\right]^4.$$

According to our spectral classification for HD 93521, O9.5 Ib-II, it is reasonable to assume that log $g \cong 3.3$. From Table 3 we see that this corresponds to a star with mass $M \sim 0.7\ M_\odot$ and bolometric luminosity $M_{bol} \sim -5.4$. Hence, the distance to the star is found to be about 780 pc.

Another problem remains and should be mentioned briefly. HD 93521 is a fast rotator ($v \sin i = 400$ km $s^{-1}$) with a rotational velocity as large as those observed in main sequence stars, this probably being the reason why Morgan *et al.* (1955) assigned the luminosity class V to the star. However, HD 101205, also a post-main sequence star (O7 III), has $v \sin i = 330$ km $s^{-1}$, according to Conti and Ebbets (1977). The rotational velocities for these two stars are much too large for their luminosities and are apparently in contradiction with our present day understanding of stellar evolution.

## V. CONCLUSIONS

HD 93521, being an O9.5 Ib-II star, is very likely a highly evolved, low mass star of Population II. The object is most probably in an evolutionary stage similar to that of the UV-bright stars in globular clusters or that of the nuclei of planetary nebulae. This last suggestion is strengthened by the recent work by Heap (1977) on nuclei of planetary nebulae, where she concludes that the spectra of these objects are indistinguishable from those of Population I, O-type stars. Under the population II interpretation for HD 93521, we derive for this star a distance to the galactic plane of about 700 pc, in good agreement with the previous estimate by Münch (1952), based upon the strength and structure of the interstellar absorption lines in the spectrum of this object.

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## REFERENCES

Ahmad, I. I. 1952, *Ap. J.*, **115**, 341.
Auer, L. H., and Mihalas, D. 1972, *Ap. J. Suppl.*, **24**, 193.
Balona, L., and Crampton, D. 1974, *M. N. R. A. S.*, **166**, 203.
Bisiacchi, G. F., Firmani, C., Ortega, R., and Peniche, R. 1976, *Rev. Mex. Astron. Astrof.*, **2**, 13.
Conti, P. S. 1973, *Ap. J.*, **179**, 181.
Conti, P. S., and Alschuler, W. R. 1971, *Ap. J.*, **170**, 325.
Conti, P. S., and Ebbets, D. 1977, *Ap. J.*, **213**, 438.
Cruz-González, C., Recillas-Cruz, E., Costero, R., Peimbert, M., and Torres-Peimbert, S. 1974, *Rev. Mex. Astron. Astrof.*, **1**, 211.
Fricke, W. 1972, *Ann. Rev. Astr. and Ap.*, **10**, 101.
Gingold, R. A. 1974, *Ap. J.*, **193**, 177.
Gingold, R. A. 1977*a*, *M. N. R. A. S.*, **178**, 533.
Gingold, R. A. 1977*b*, *M. N. R. A. S.*, **178**, 569.
Greenstein, J. L., and Sargent, A. I. 1974, *Ap. J. Suppl.*, **28**, 157.
Guetter, H. 1974, *Pub. A. S. P.*, **86**, 785.
Härm, R., and Schwarzschild, M. 1975, *Ap. J.*, **200**, 324.
Harris, W. E., Raane, R., and De Roux, J. 1976, *Ap. J. Suppl.*, **31**, 13.
Heap, S. R. 1977, *Ap. J.*, **215**, 864.
Maitre, V. 1971, *Ann. Obs. Besançon*, **8**, 106.
Mendoza, E. E. 1976, *Rev. Mex. Astron. Astrof.*, **2**, 33.
Morgan, W. W., Code, A. D., and Whitford, A. E. 1955, *Ap. J. Suppl.*, **2**, 41.
Münch, G. 1952, *Pub. A. S. P.*, **64**, 312.
Norris, J. 1974, *Ap. J.*, **194**, 109.
Oort, J. H. 1960, *Bull. Astr. Inst. Netherlands*, **15**, 45.
Paczyński, B. 1971, *Acta. Astr.*, **21**, 417.
Sweigart, A. V., Mengel, J. G., and Demarque, P. 1974, *Astron. and Astrophys.*, **30**, 13.
Walborn, W. R. 1971, *Ap. J. Suppl.*, **23**, 257.
Walborn, W. R. 1972, *A. J.*, **77**, 312.
Williams, E. G. 1936, *Ap. J.*, **83**, 279.
Zinn, R. 1974, *Ap. J.*, **193**, 593.