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ADDITIONAL PHOTOELECTRIC OBSERVATIONS AND ANALYSIS OF THE VARIABILITY OF THE β CEPHEI STARS 12 AND 16 LACERTAE

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RESUMEN

Se presentan observaciones fotoeléctricas hechas en 1977 en el Observatorio Astronómico de Zacatecas, de dos estrellas tipo β Cephei, 12 y 16 Lacertae. Los resultados del análisis de frecuencia basado en estos datos y en las observaciones hechas el mismo año en los Observatorios de San Pedro Mártir, Chiran y Bialków son también reportados.

ABSTRACT

We present photoelectric observations of two β Cephei variables –12 and 16 Lacertae– made in 1977 in the Observatory of Zacatecas. The results of a frequency analysis made with these data and with the data obtained in the same year at the San Pedro Mártir, Chiran and Bialków Observatories are also given.

Key words: PHOTOMETRY – STARS-β CEPHEI – STARS-VARIABLES.

I. INTRODUCTION

12 Lac = DD Lac = HR 8640 = HD 214993 ($V = 5\overset{m}{.}2$, B1.5 III) is a well-known β Cephei type variable. Its velocity and light variations with period of 4.6 hours are known since 60 years ago. A review of earlier work done on this star was recently given by Jerzykiewicz (1978), where a frequency analysis of its variability was also made. According to this investigation, six short-period sine-wave components are present in the star's velocity and light variations. As far as the light variation is concerned, these results have been confirmed by Jarzebowski *et al.* (1979, henceforth referred to as Paper I).

As to the other variable studied in the present paper, i.e., 16 Lac = EN Lac = HR 8725 = HD 216916 ($V = 5\overset{m}{.}6$, B2 IV), its 4-hour variation was discovered three decades ago by Walker (1951). A summary of the work done so far on this star can be found in Paper I, where it was also shown that the three short-period sine-wave components, previously found in the star's light variation by Fitch (1969) and by Jerzykiewicz (1976), had in 1977 amplitudes amounting to only about half of their 1965 values.

Long before the discovery of its β Cephei nature, 16 Lac has been known as a single-line spectroscopic binary

(Struve and Bobrovnikoff 1925). Recently, a comparison of the 1977 San Pedro Mártir photometry with 1965 Lowell data revealed that the star is also an eclipsing variable (Jerzykiewicz *et al.* 1978). The eclipse is a partial transit. It is only $0^{m}.037$ deep at the mid-eclipse phase, and its duration amounts to $0^{d}.37$. The light elements are the following (Jerzykiewicz 1979):

Minimum light = $JD_{\odot}$ 2439050.575 + $12^{d}.09684$ E. (1)

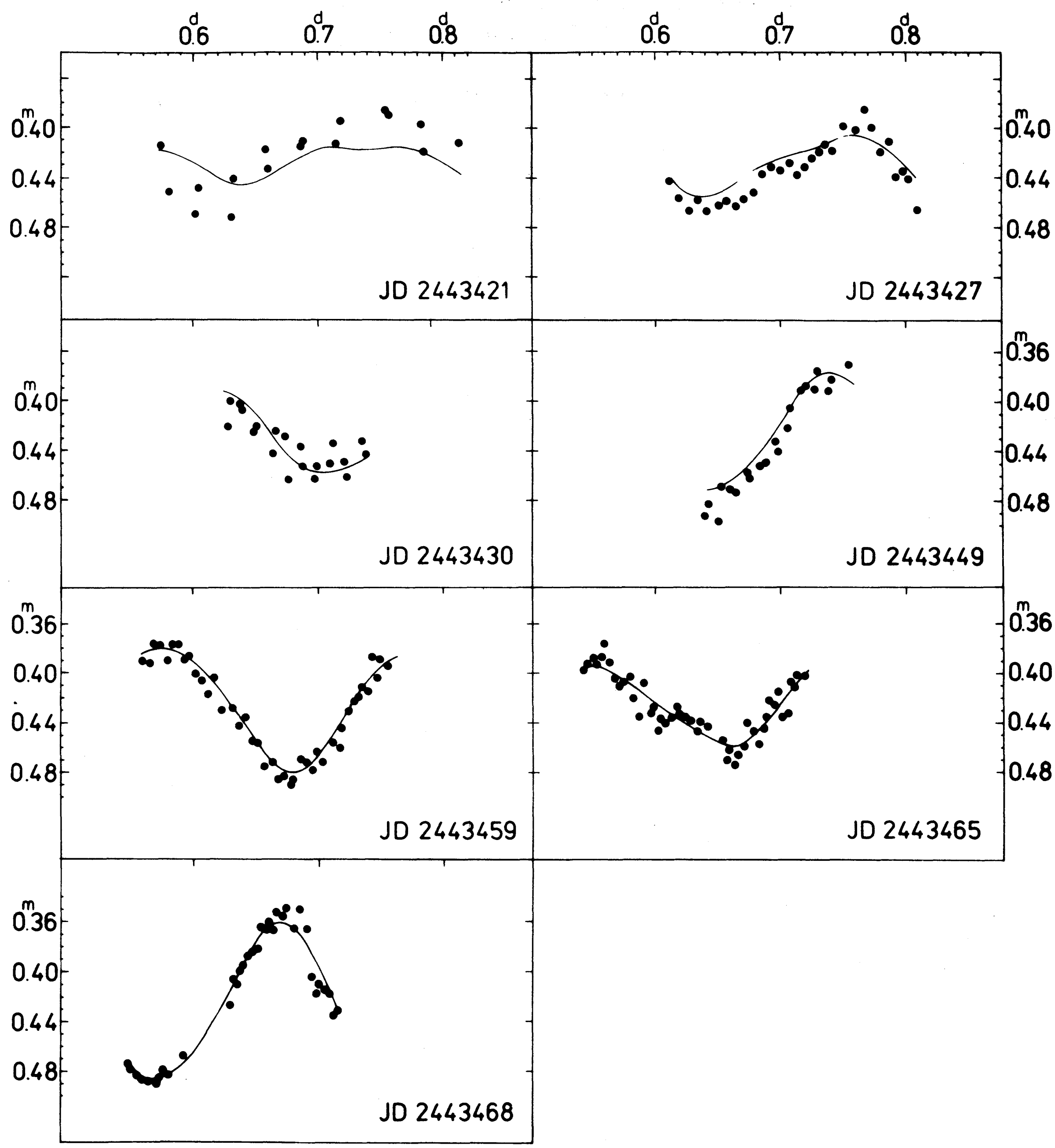

Fig. 1. The differential blue magnitudes of 12 Lac (filled circles) plotted as a function of heliocentric Julian Day. Solid lines are segments of the six-term synthetic light-curve, the parameters of which are listed in Table 3. (The observations of JD 2443423 are shown in Figure 3).

II. OBSERVATIONS

Our photometric data were obtained in the course of the 1977 observational season, carried out in four observatories, viz., Bialków (Poland), Chiran (France), San Pedro Mártir and Zacatecas (Mexico). The Bialków, Chiran and San Pedro Mártir observations have already been published in Paper I. In the present paper we report the Zacatecas photometry.

The observations were made with the 50-cm telescope of the Astronomical Observatory at Cerro de la Virgen in Zacatecas (National University of Mexico and the University of Zacatecas). The electronics consisted of a digital photometer "Pacific" model 124, and an unrefrigerated 1P21 photomultiplier tube was employed as the light-receiving unit. All measurements were made through a standard *B* filter of the *UBV* system.

12 Lac was observed on eight nights, and 16 Lac on nine nights. A single comparison star was used for each variable, viz., 10 Lac ($V = 4^{\rm m}.9$, O9.5) for 12 Lac, and 2 And ($V = 5^{\rm m}.0$, A2) for 16 Lac respectively. The atmospheric extinction coefficient was determined on several nights, by means of the Bouguer method, from observations of the comparison stars. A mean value of 0.25 mag per air mass was obtained.

We obtained differential photometry of 12 Lac with respect to 10 Lac. In the case of 16 Lac we compared against 2 And; each reported observation is the mean of two successive measurements. The data were left in the instrumental system. A total of 256 observations of 12 Lac and 201 of 16 Lac were obtained. The mean error of one observation can be estimated as equal to between $0^{\rm m}.003$ and $0^{\rm m}.005$. This relatively large value (in spite of the observatory's elevation of 2714 m above sea level) was caused mainly by imperfect telescope tracking.

The observations of 12 Lac are listed in Table 1 and also plotted as a function of heliocentric Julian Day in Figures 1 and 3, while those of 16 Lac are listed in Table 2 and displayed in Figures 2 and 3.

There were two nights on which the stars were ob-

TABLE 1

THE BLUE-MAGNITUDE OBSERVATIONS OF 12 LAC

$JD_\odot$	Δm
2443421+	
$^{\rm d}.5736$	$^{\rm m}.414$
.5806	.452
.6024	.469
.6048	.449
.6313	.473
.6324	.441
.6590	.418
.6604	.433
.6868	.415
.6882	.411
.7146	.414
.7181	.395
.7542	.386
.7563	.389
.7826	.397
.7854	.419
.8125	.412
2443423+	
$^{\rm d}.5924$	$^{\rm m}.472$
.5944	.478
.6035	.477
.6056	.455
.6146	.462
.6167	.455
.6319	.416
.6340	.424
.6410	.406
.6424	.410
.6542	.375
.6563	.395
.6660	.349
.6694	.350
.6792	.359
.6806	.347
.6924	.355
.6938	.364

$JD_\odot$	Δm
$^{\rm d}.7097$	$^{\rm m}.364$
.7153	.399
.7160	.388
.7215	.404
.7354	.421
.7424	.453
.7444	.459
.7521	.463
.7535	.452
.7597	.480
.7611	.463
.7688	.484
.7715	.470
.7792	.482
.7806	.481
.7958	.475
.7979	.472
.8056	.444
.8076	.466
.8153	.469
.8167	.448
.8299	.426
.8313	.406
.8382	.407
.8410	.398
.8486	.378
2443427+	
$^{\rm d}.6118$	$^{\rm m}.443$
.6194	.457
.6271	.467
.6340	.458
.6417	.467
.6507	.462
.6569	.459
.6646	.463
.6715	.457
.6792	.452
.6854	.437

$JD_\odot$	Δm
$^{\rm d}.6924$	$^{\rm m}.431$
.7000	.434
.7076	.428
.7146	.437
.7201	.431
.7257	.424
.7313	.419
.7368	.413
.7424	.419
.7507	.398
.7604	.401
.7667	.384
.7736	.399
.7806	.419
.7875	.411
.7924	.439
.7979	.435
.8028	.441
.8090	.465
2443430+	
$^{\rm d}.6278$	$^{\rm m}.421$
.6299	.400
.6382	.402
.6396	.407
.6486	.424
.6507	.420
.6639	.442
.6653	.424
.6736	.428
.6757	.464
.6854	.437
.6875	.452
.6965	.463
.6986	.452
.7090	.450
.7111	.434
.7208	.448
.7229	.461

$JD_\odot$	Δm
$^{\rm d}.7354$	$^{\rm m}.433$
.7375	.443
2443449+	
$^{\rm d}.6389$	$^{\rm m}.492$
.6417	.483
.6500	.497
.6521	.469
.6604	.471
.6618	.472
.6743	.458
.6757	.462
.6847	.451
.6868	.450
.6958	.432
.6979	.440
.7056	.421
.7076	.405
.7181	.391
.7194	.388
.7271	.390
.7292	.375
.7375	.392
.7403	.382
.7542	.370
2443459+	
$^{\rm d}.5593$	$^{\rm m}.390$
.5660	.392
.5694	.375
.5736	.376
.5799	.389
.5847	.376
.5889	.375
.5944	.388
.5972	.387
.6028	.400
.6076	.406

$JD_\odot$	Δm
$^{\rm d}.6132$	$^{\rm m}.416$
.6174	.404
.6229	.429
.6313	.428
.6368	.442
.6424	.435
.6479	.454
.6507	.455
.6569	.474
.6639	.471
.6688	.485
.6722	.482
.6778	.490
.6813	.484
.6868	.470
.6903	.472
.6958	.478
.6986	.463
.7028	.472
.7111	.455
.7167	.460
.7184	.444
.7243	.430
.7285	.422
.7326	.419
.7354	.411
.7396	.414
.7431	.386
.7472	.403
.7493	.388
.7542	.393
2443465+	
$^{\rm d}.5431$	$^{\rm m}.397$
.5451	.392
.5507	.387
.5535	.392
.5563	.387
.5583	.375

$JD_\odot$	Δm
$^{\rm d}.5625$	$^{\rm m}.391$
.5674	.404
.5729	.408
.5750	.407
.5792	.402
.5826	.420
.5868	.435
.5903	.407
.5958	.432
.5979	.427
.6021	.446
.6042	.437
.6083	.440
.6125	.436
.6174	.427
.6194	.432
.6236	.435
.6271	.437
.6326	.447
.6354	.439
.6403	.442
.6528	.454
.6569	.470
.6590	.462
.6632	.473
.6660	.466
.6708	.459
.6729	.440
.6785	.447
.6819	.458
.6861	.444
.6882	.435
.6917	.422
.6938	.424
.6979	.414
.7007	.436
.7056	.432
.7083	.407
.7111	.411
.7132	.401

$JD_\odot$	Δm
$^{\rm d}.7181$	$^{\rm m}.401$
2443468+	
$^{\rm d}.5486$	$^{\rm m}.475$
.5507	.479
.5549	.484
.5583	.486
.5632	.487
.5701	.489
.5715	.486
.5757	.479
.5792	.483
.5910	.467
.6285	.427
.6319	.407
.6340	.410
.6375	.399
.6396	.395
.6444	.386
.6465	.384
.6507	.382
.6549	.364
.6583	.366
.6604	.360
.6639	.367
.6667	.353
.6722	.356
.6743	.349
.6806	.365
.6854	.350
.6903	.366
.6938	.404
.6979	.418
.7000	.410
.7049	.414
.7076	.415
.7111	.435
.7153	.431

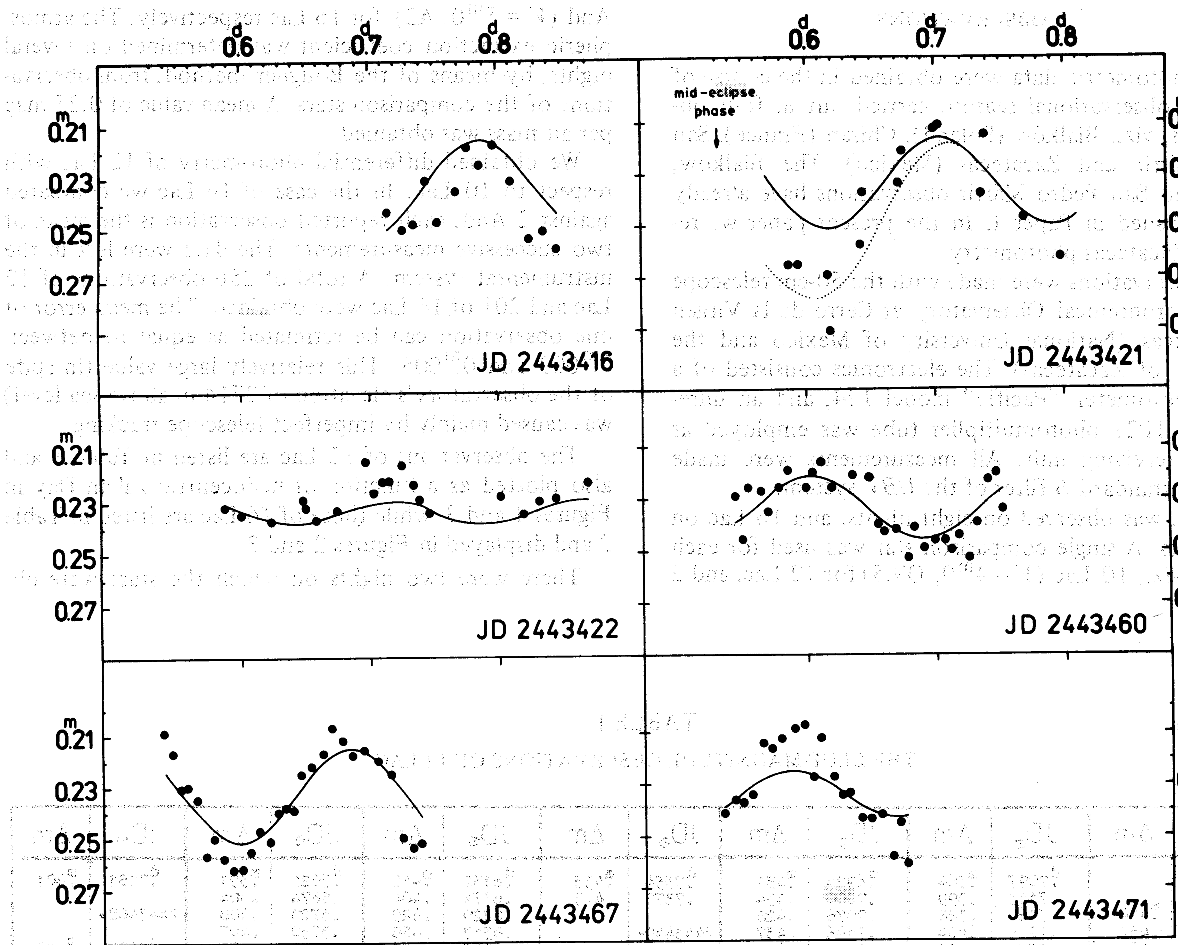

Fig. 2. The observations of 16 Lac plotted as a function of heliocentric Julian Day (JD 2443432 and JD 2443451 are not shown; JD 2443424 is in Figure 3). Solid lines are segments of the three-component synthetic light-curve, the parameters of which are given in Table 4. The dotted line on JD 2443421 includes the effect of the eclipse according to the solution provided by Jerzykiewicz (1979).

served simultaneously in Zacatecas and in Bialków; 12 Lac on JD 2443423 and 16 Lac on JD 2443424. In Figure 3 we show these data.

III. FREQUENCY ANALYSIS

A frequency analysis of the 1977 Bialków, Chiran and San Pedro Mártir observations of 12 and 16 Lac was carried out in Paper I. The procedure was the same as that previously applied by Jerzykiewicz (1976, 1978) and by Jerzykiewicz and Wenzel (1977) to similar series of observations of multiperiodic variable stars. It consisted in first deriving approximate frequencies by means of the least-squares periodogram technique (cf. Lomb 1976), and then using them as initial values in a nonlinear least-squares solution of observational equations of the following form:

$$\Delta m = A_0 + \sum_{i=1}^{n} A_i \cos\left[2\pi f_i(t - T) + \varphi_i\right] . \quad (2)$$

In other words, the data were approximated by a synthetic light-curve composed of n sine-wave terms with frequencies f_i, amplitudes A_i, and initial phases Φ_i. (The periods $P_i = 1/f_i$). No *a priori* relations were assumed to exist between these parameters.

In the present investigation we combined the Zacatecas data with those of Paper I and solved eqs. (2), taking the frequencies derived in Paper I as the initial values.

TABLE 2

THE BLUE-MAGNITUDE OBSERVATIONS OF 16 LAC

JD⊙	Δm	JD⊙	Δm	JD⊙	Δm	JD⊙	Δm	JD⊙	Δm	JD⊙	Δm	JD⊙	Δm
2443416+		2443422+		^{d}.6806	^{m}.246	^{d}.6979	^{m}.201	^{d}.6813	^{m}.258	^{d}.7066	^{m}.246	^{d}.6940	^{m}.216
				.6896	.247	.7056	.212	.6833	.260	.7122	.250	.7048	.220
^{d}.7146	^{m}.243	^{d}.5875	^{m}.232	.6910	.240	.7076	.222	.6917	.270	.7180	.244	.7153	.225
.7264	.250	.5996	.235	.6986	.233	.7153	.215	.7010	.258	.7253	.253	.7243	.250
.7354	.248	.6240	.237	.7181	.241	.7174	.237	.7104	.251	.7320	.239	.7323	.254
.7451	.231	.6479	.229	.7271	.230	.7257	.225	.7125	.254	.7400	.223	.7392	.252
.7535	.247	.6500	.232	.7354	.229	.7271	.231	.7225	.238	.7465	.220		
.7680	.235	.6583	.237	.7368	.227	.7340	.226	.7313	.260	.7514	.234	2443471+	
.7767	.218	.6750	.233	.7437	.231	.7361	.254	.7326	.252				
.7864	.225	.6965	.214	.7521	.231	.7444	.239	.7417	.257	2443467+		^{d}.5340	^{m}.241
.7972	.217	.7034	.226	.7535	.238	.7465	.239					.5420	.236
.8104	.231	.7111	.222	.7621	.237	.7549	.251	2443460+		^{d}.5389	^{m}.209	.5486	.237
.8250	.254	.7166	.222	.7701	.233	.7563	.240			.5455	.217	.5555	.234
.8357	.251	.7243	.216	.7715	.231	.7660	.253	^{d}.5441	^{m}.229	.5517	.231	.5646	.214
.8465	.258	.7340	.223	.7812	.228	.7674	.248	.5500	.246	.5580	.230	.5711	.216
		.7390	.229	.7917	.219	.7774	.245	.5542	.226	.5635	.235	.5785	.212
2443421+		.7475	.234	.8014	.233			.5639	.227	.5702	.257	.5885	.208
		.8014	.228	.8097	.245	2443451+		.5700	.235	.5771	.250	.5961	.207
^{d}.5868	^{m}.265	.8201	.235	.8354	.253			.5778	.226	.5840	.258	.6028	.227
.5944	.265	.8325	.230	.8417	.250	^{d}.5639	^{m}.246	.5847	.219	.5917	.262	.6090	.212
.6181	.269	.8448	.229			.5757	.225	.5986	.226	.5978	.262	.6191	.227
.6194	.291	.8695	.223	2443432+		.5861	.243	.6045	.220	.6050	.255	.6243	.234
.6431	.257					.5975	.221	.6111	.216	.6121	.247	.6316	.233
.6444	.237	2443424+		^{d}.6340	^{m}.199	.6090	.232	.6184	.226	.6201	.251	.6400	.243
.6729	.234			.6361	.216	.6111	.230	.6350	.221	.6264	.240	.6478	.243
.6750	.221	^{d}.5799	^{m}.221	.6507	.210	.6271	.247	.6480	.222	.6326	.238	.6559	.242
.7014	.212	.5885	.230	.6597	.182	.6292	.251	.6552	.240	.6392	.239	.6635	.258
.7035	.211	.5979	.215	.6625	.198	.6385	.243	.6600	.243	.6455	.225	.6700	.245
.7375	.238	.6076	.223	.6736	.185	.6472	.239	.6684	.242	.6531	.222	.6753	.261
.7389	.215	.6326	.220	.6750	.185	.6493	.229	.6771	.253	.6614	.217		
.7694	.247	.6486	.227	.6840	.201	.6576	.235	.6836	.241	.6694	.207		
.7979	.262	.6500	.234	.6854	.196	.6604	.243	.6903	.249	.6775	.212		
		.6593	.236	.6958	.194	.6718	.246	.6986	.246	.6850	.218		

For 12 Lac the frequency analysis of Paper I was based on 429 observations taken over an interval of 43.5 days. Adding the Zacatecas data increased the number of observations to 685 and extended the time interval to 68.4 days. The mean magnitude difference of the Zacatecas observations amounted to $0^{m}_{.}426$; this value was subtracted from the present data in order to reduce them to a common zero point with the data of Paper I. Eqs. (2) were then solved for n = 6. Since $f_4 = (f_1 + f_3)/2$ and $f_5 = f_1 + f_4$ (Jerzykiewcz 1978), these relations were incorporated into the solution. The results are listed in Table 3, and the six-term synthetic light-curve is compared with the observations in Figures 1 and 3.

In the case of 16 Lac, three Zacatecas nights were omitted in the present analysis. The reasons were the following. As can be seen from eq. (1) an eclipse occured on JD 2443421. Several observations on this night were taken during the egress, so that the light-curve is actually a combination of the intrinsic variation and the effect of the eclipse (cf. Figure 2). As to the other two dates, viz. JD 2443432 and 2443451, the mean brightness of the

TABLE 3

THE PARAMETERS OF THE SYNTHETIC LIGHT-CURVE OF 12 LAC BASED ON THE 1977 OBSERVATIONS*†

i	f_i (c/d)	P_i	A_i	φ_i (rad)
1	5.1791	$0^{d}_{.}193082$	$0^{m}_{.}0402$	5.25
	1	4	6	3
2	5.0663	0. 197382	0. 0169	4.16
	3	13	6	7
3	5.4910	0. 182115	0. 0104	3.80
	5	15	6	.11
4	5.3351	0. 187438	0. 0078	5.84
	5	16	5	9
5	10.5142	0. 095109	0. 0049	2.89
	5	5	5	.12
6	4.2403	0. 23583	0. 0036	0.07
	14	8	5	.34

* Underneath each parameter we give the mean error without the leading zeroes.

† Number of observations = 685, standard deviation = $0^{m}_{.}0091$, $A_o = 0^{m}_{.}0006 \pm 0.0004$, $T = 2443400^{d}_{.}0$.

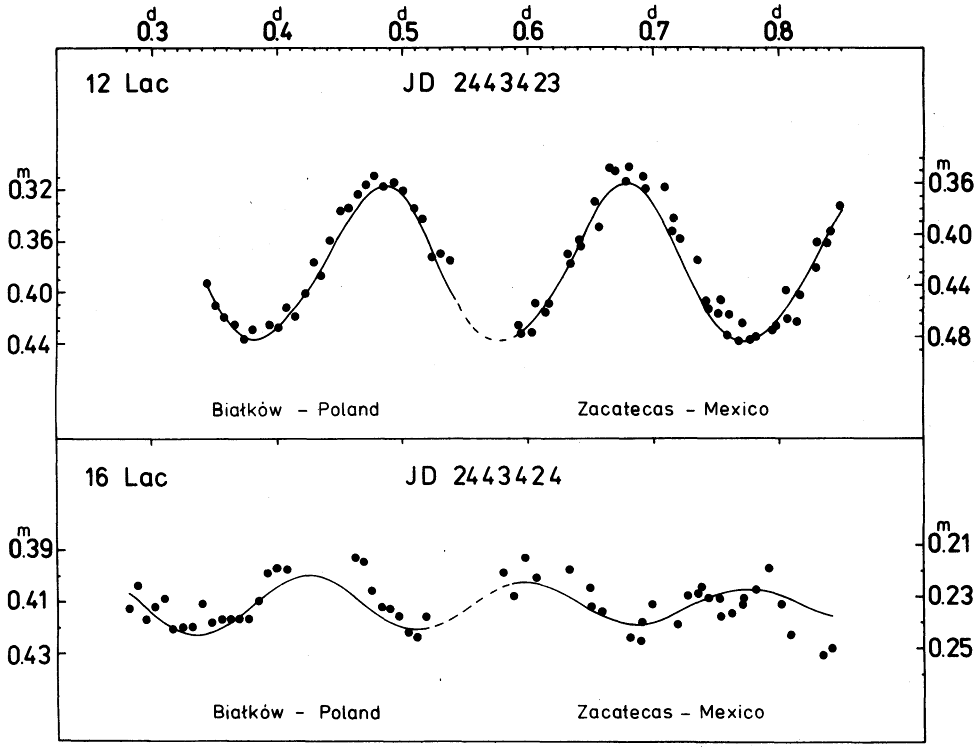

Fig. 3. In the upper panel are plotted the observations of 12 Lac obtained on the same date in Bialków and in Zacatecas. In the lower panel – the same for 16 Lac. (Time difference between the two observatories amounted to nearly eight hours). Solid lines are synthetic light-curves of Table 3 and 4, respectively. The Bialków observations are from Paper I.

star differed on those nights by about $0^m.01$ from the general mean value of $0^m.234$. (We are unable to provide an explanation for these deviations).

The observations of 16 Lac made on the remaining six nights, combined with 444 data points from Paper I (spanning an interval of 43.5 days), increased this number to 581 and extended the time interval to 71.4 days. The corresponding solution of eqs. (2) with n = 3 is given in Table 4. A comparison of the three-term synthetic light-curve with the observations can be seen in Figures 2 and 3.

IV. DISCUSSION

a) *12 Lac*

The six-term solution of Table 3 does not differ significantly from the corresponding solution of Paper I. The periods and amplitudes of all components agree to within about one standard deviation. The overall standard deviation increased from $0^m.0064$ to $0^m.0091$; this is due to the higher accuracy of the San Pedro Mártir and Chiran observations as compared with the ones from Zacatecas. However –as can be seen from the figures– the synthetic curve fits the data satisfactorily.

TABLE 4

THE PARAMETERS OF THE SYNTHETIC LIGHT-CURVE OF 16 LAC BASED ON THE 1977 OBSERVATIONS*

i	f_i (c/d)	P_i	A_i	φ_i (rad)
1	5.9117	$0^d.169155$	$0^m.0094$	1.84
	4	11	4	8
2	4.8560	0.170765	0.0054	1.78
	7	20	4	.15
3	5.5040	0.181688	0.0061	6.15
	6	19	4	.13

* Number of observations = 581, standard deviation = $0^m.0058$, $A_0 = -0^m.0004 \pm 0^m.0002$, T = $2443400^d.0$.

b) *16 Lac*

Adding the Zacatecas observations did not change the three-term solution significantly, so that the results of the present analysis agree with those of Paper I to within one standard deviation, except the primary component amplitude, which differs by two standard deviations. The present value of A_1 amounts to $0\overset{m}{.}0094$; the Paper I value amounted to $0\overset{m}{.}0083$.

On JD 2443471 the observed amplitude appears to be greater than the computed one. On other nights the agreement is satisfactory. This applies also to JD 2443421, the night on which the eclipse occurred, provided that light variation due to the eclipse is taken into account.

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