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THE ROTATION CURVE OF OUR GALAXY; HOW WELL DO WE KNOW IT?

(Invited Paper)

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RESUMEN

Se ha hecho un breve comentario histórico conducente a la formulación de la rotación de la Galaxia así como un recordatorio de las fórmulas de la rotación diferencial galáctica. Se ha señalado la necesidad de una curva global de rotación en la era de radioastronomía. Sólo por medio de la curva de rotación es posible obtener la distancia cinemática de H I, de nubes moleculares y de algunas regiones H II.

Se han señalado las desviaciones observadas de la curva suave de rotación; en particular, que la curva muestra "ondas", fenómeno que en el presente se acepta ser común en galaxias espirales; los máximos y mínimos corresponden respectivamente a los brazos e interbrazos. La interpretación de estas ondas como efectos de poblaciones, propuesta anteriormente por este autor, se enfatiza una vez más. Observaciones recientes de regiones H II y de nubes de CO sugieren que el Sol se encuentra cerca del mínimo de una onda. Asimismo, otra irregularidad, la probable diferencia de las curvas de rotación norte-sur, se discute también brevemente.

Basada en una suposición plausible de que la estructura espiral puede representarse por una par de espirales logarítmicas colocadas simétricamente, se muestra que si las ondas existen –e independientemente de la causa de éstas– la curva de rotación en nuestra Galaxia y en otras, será una función de la dirección del centro galáctico. Al contrario de las galaxias externas, no es posible deducir la velocidad de rotación en todas las direcciones debido a la excéntrica posición que ocupa el Sol. Una curva promedio puede obtenerse sobre todas las direcciones que abarca un ángulo central de 180°. Pero desde el Sol se obtiene información sobre la ley de la rotación en el mejor caso dentro de un ángulo central de $\simeq 120°$.

Finalmente se enfatiza que la curva de rotación en general se refiere esencialmente al sistema en rotación más rápida o sea la Población I, cuya masa no es mayor que un 10% de la masa de la Galaxia. La curva de rotación no es por lo tanto única.

ABSTRACT

Following an historical sketch of the relevant circumstances leading to the formulation of the rotation of the Galaxy, the differential rotation formulae are recalled. The necessity of obtaining an overall rotation curve at the advent of radioastronomy is stressed; only through the knowledge of such a curve can the kinematic distances of H I profiles, H II regions and molecular clouds be obtained.

The existence of the deviations from a smooth rotation curve are pointed out; in particular it is shown that the curve exhibits "waves", a phenomenon at present known to be rather common in spiral galaxies. Maxima and minima correspond to arm and interarm regions, respectively. The interpretation of these waves as population effects suggested earlier by this author is emphasized once again. Recent observations of H II regions and CO clouds suggest that the Sun is located close to the minimun of a wave. Another irregularity, the presumed difference in the north and south rotation curves, is also briefly discussed.

Based on a plausible assumption that the spiral structure can be represented by a pair of symmetrically located logarithmic spirals, it is shown that if waves do indeed exist –irrespective of the cause of such waves– the rotation curve in our Galaxy and in others will be a function of direction from the galactic center. Unlike external galaxies, from the location of the Sun we are not able to obtain the rotation velocity in all directions.

An average rotation curve where the waves are smoothed out can be obtained from the mean over directions within a central angle of 180°. However, from our eccentric position in the Galaxy we can obtain information on the rotation law at best within a central angle of 120°.

Finally it is emphasized that the rotation curve discussed usually is that of the fastest rotating system, the Population I, which contains not more than 10% of the total mass of the Galaxy. The rotation curve is therefore not unique.

Key Words: GALAXIES-MILKY WAY – GALAXIES-STRUCTURE – GALAXIES-INTERNAL MOTIONS

I. INTRODUCTION

That our Galaxy rotates around a center at a distance of 10 kpc from the sun, is well known at present. But it was not until 1927 that a differential rotation was shown to be compatible with stellar motions known at the time. Curiously enough evidence for the rotation of external galaxies was obtained a decade earlier.

Slipher (1914) at the Lowell Observatory secured spectra of the brightest nuclear region of the Galaxy M104 with an exposure of 35 hours; the absorption lines which were inclined to the direction of dispersion were correctly interpreted as due to the rotation of the central regions. No inclination of spectral lines was present along the minor axis. Shortly after, Pease at Mt. Wilson (1916) was able to extend observations halfway through M104 with an exposure of 79 hours!

Unfortunately, in our Galaxy the discovery and the study of the rotation cannot be carried out directly as in other galaxies. Being located within the Galaxy we do not have the advantage of a perspective. However, as early as 1912 the British astronomer Turner (1912) suggested that the phenomenon of two-star-streams towards the direction of the galactic center and away from it –or its equivalent ellipsoidal distribution of the residual velocities– would be a natural consequence if the stars were describing elliptical orbits around a distant point, the center of the Galaxy, and that we were observing some of them approaching and others receding from us. Also proper motion data available at the time showed an overall systematic residual in the peculiar proper motions of stars. This could arise, according to Charlier (1913), if the observed stars were rotating about a distant center. As it happens all too often with a new idea, the suggestion of a rotation stayed latent; aside from this, the data were scanty and could not provide a good check on the proposed phenomenon. However, as observations kept increasing, it became more and more evident that the different statistical findings could not be explained piecemeal and that the cause of these was one and the same and it lay in the overall dynamics of the Galaxy. A unified explanation was needed. The real breakthrough is due to Bertil Lindblad's foresight. We shall dwell on his epoch making proposition in what follows but first I like to recall the main observational facts which have led to Lindblad's formulation of the rotation of the Galaxy and in particular of its subsystems. These are :

1. The asymmetric drift of stellar peculiar motions.

2. The two-star-streams (ellipsoidal distribution) which was known earlier, as mentioned above.

The first of these is beautifully illustrated by Strömberg (1924). Figure 1 is adapted from his graph. It shows clearly that objects grouped according to physical properties such as RR Lyrae stars, Cepheids, globular clusters are clearly distinguished from one another kinematically. The motion of their standard of rest, their

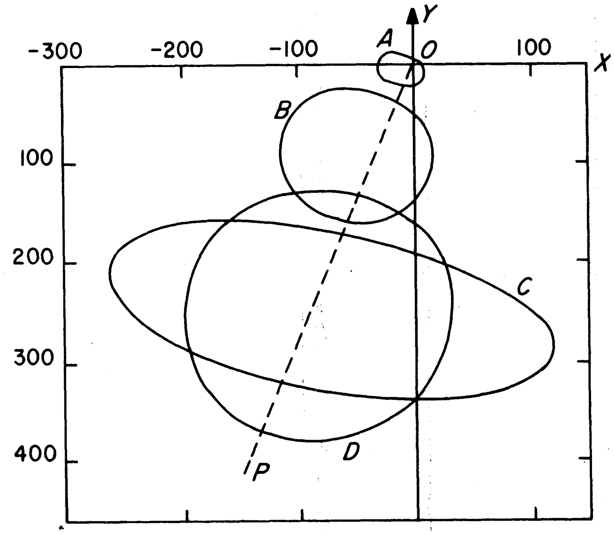


Fig. 1. Graph showing the "asymmetrical drift" of the centroic motion of the star groups in the galactic plane as observed fron the sun and the dispersion of velocities, around that motion adapted from Strömberg (1924). Note that the velocity plan has the orientation of the old galactic coordinate system. Th coordinates are velocities in km s^{-1}.

centroid, with respect to the local standard of rest i: larger, the larger the peculiar velocities and that the velocity vector of the group motion is in a directior perpendicular to the direction of the galactic center. It i: to be regretted that the impact of Strömberg's observa tions on the formulation of galactic rotation is not duly recognized by modern writers on the subject.

The asymmetry of stellar motions as described by Strömberg led Lindblad (1925*a*, 1925*b*) to seek ar overall dynamical explanation. He suggested that th galactic system was composed of several subsystem coexisting but differing as to their rotational motion anc their overall distribution within the galaxy (see Figur 2). The flattened subsystems consisting of the Milk

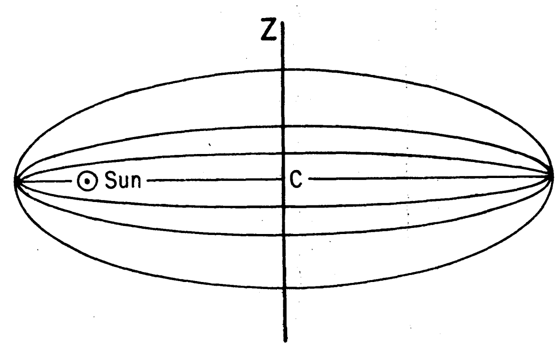


Fig. 2. The scheme of subsystems in our Galaxy proposed b Lindblad to explain the asymmetrical drift of stellar motions a observed from the sun.

/ay stars and clouds (mostly OB stars, interstellar louds, H II regions, Cepheids) rotated fastest at a given istance from the center, while the system of globular lusters rotated slowest. Thus the globular clusters, RR ,yrae stars stayed behind the sun in their motion around he galactic center. Further, the dispersion of the esidual velocities was correlated with rotational veloci-y. Lindblad thus explained in an unified manner the bservationally obtained overall character of group notions. The "high velocity stars" studied by Oort 1926) found an immediate place within the scheme roposed by Lindblad; these stars simply belonged to a roup rotating more slowly with respect to the flat ubsystem. Today we call these subsystems populations ollowing Baade's formulation (Baade 1944).

A most important step was immediately taken by)ort (1927, 1928). Oort worked out the kinematics of a ifferentially rotating subsystem, the flattest subsystem, o which the majority of the stars and objects known to hat time belonged, as seen from the local standard of est. The formulae derived by Oort showed a "double /ave" dependence of radial velocities and proper mo-ions on galactic longitude. Oort showed that OB stars nd Cepheids gave excellent agreement with his formulae f differential rotation. It is worth to mention the xtensive material on the radial velocities of O and B tars and their analysis by Plaskett and Pearce (1936) nd of Cepheids by Joy (1939); these brought definite upport to Oort's formulae and differential rotation was efinitely accepted to be operative in the Galaxy.

I shall stop here the historical account and will iscuss the problem as of today.

II. REVIEW OF DIFFERENTIAL GALACTIC ROTATION FORMULAE

Let us recall Oort's formulae in radial velocity and roper motion for the sake of completeness. These ɔrmulae obtained on the basis of pure rotation are pplicable to objects close to the sun such that if r and R re the distances of the object from the sun and from ıe galactic center respectively, r/R should be small and ıat terms of second order or beyond, are negligible (say or $r \simeq 1$ or 1.5 kpc). The formulae read as follows:

$$v_r = rA \sin 2\ell$$

$$4.74\ \mu = A \cos 2\ell + B \quad ,$$

ere v_r (km s^{-1}) and μ (arcsec year^{-1}) are respectively, he radial velocities and proper motions with respect to he local standard of rest, ℓ the galactic longitude of an bject and A and B are constants, referred to as Oort's onstants. Note that the equation in μ does not depend n the distance of the object; this is an advantage. Oort's ırther showed that A and B yield important dynamical uantities for the solar neighborhood:

$$A - B = \frac{\theta_0}{R_0} = \omega_0$$

where θ_0 (km s^{-1}) is the linear rotational velocity of the local standard of rest, R_0(kpc) the distance of the sun from the galactic center, and ω_0(km s^{-1} kpc^{-1}) the angular rotation at the sun; further,

$$-(A+B) = \left(\frac{d\theta}{dR}\right)_0 \quad ,$$

is the variation of rotational velocity with R at the sun. The presently adopted values are $A = 15$ km s^{-1} kpc^{-1} and $B = -10$ km s^{-1} kpc^{-1}. Although these are local values of the rotation they are very important in kinematical and dynamical studies of our Galaxy. To date it is my belief –probably shared by others– that we are not quite certain as to what are the best values of A and B. We shall return to discuss this point and show the different values obtained by numerous workers in the field.

III. ROTATION CURVE AND NEUTRAL HYDROGEN

Discussion of the motions in the Galaxy in the large in terms of a "rotation curve" was introduced in the early fifties to help estimate distances of the H I clouds corresponding to the different maxima of the 21-cm emission line profiles obtained for the first time with radiotelescopes. I now recall some pertinent and simple formulation. It is true that Oort's formula can yield the distance if the radial velocity of an object is known but it cannot be applied for the large distances spanned by the radio observations. The problem to estimate kinematic distances was ably tackled by Oort and coworkers (Van der Hulst *et al.* 1954). We shall describe their procedure in what follows.

All radio reductions are based on the exact formula of galactic rotation in radial velocity

$$V_{LSR} = R_0\,[\omega(R) - \omega(R_0)] \sin \ell \qquad (3)$$

where ω (R) and ω (R_0) are the angular velocities at a point R and at the solar distance, R_0, respectively; ℓ as before is the galactic longitude of a specified direction. Radio astronomy has taken full advantage of this relation. To start with it is assumed that the dominant motion is one of rotation (circular motion) and random motions are negligible. From observations of peak velocities of the different maxima in the line profiles one derives the V_{LSR} of an H I cloud. Further $\omega(R_0)$ can be obtained from the Oort constants as seen before, R_0 is presumably a known quantity usually adopted as 10 kpc; one then needs to know ω(R) so as to obtain R from the formula (3) and then by simple trigonometry, a value of r for that cloud is obtained.

The determination of a reliable curve giving ω –or θ– as a function of R, that is, a complete "rotation curve", has been the subject of numerous investigations both by radio data particularly by the H 21-cm-line, and by optical means. How far we have succeded in arriving at this goal is what I shall discuss below.

The rotation law for the inner regions of the galaxy where optical data are not available is obtained from radio observations. Until lately this information was entirely hinged on the 21-cm-H I line. The procedure is now well known: one assumes that along a direction ℓ the distance from the center of the galaxy where this maximum occurs is minimum compared with the distance of all other peaks of the profile. The rotation curve derived from radio recombination lines of H^+ are consistent with the 21-cm data.

We note that rotation velocities obtained from the external peak velocities show dips attributed to the absence of gas (or molecules) in the interarm region where the line of sight is not tangent to the region of highest gas density of the arm. In such a case the real rotation curve of the spiral structure will be the upper envelope of the wavy curve. Recently a rotation curve interior to the sun is obtained by Burton and Gordon (1978) from CO velocities. In their Figure 4 the CO velocities are plotted together with H I 21-cm velocities obtained earlier by Simonson and Mader (1973). The agreement in all the details of maxima and minima, is rather good. The two regions of minimum rotation occur at $R \simeq 6.75$ kpc and $R \simeq 9$ kpc respectively.

Beyond the solar circle radio data cannot give a rotation curve as the observed velocities at a given direction vary monotonically with no maximum or minimum. In this anti-center region the rotation law must be obtained by optical means.

Meanwhile gathering all available velocity data Schmidt (1957, 1965) has constructed a model of mass distribution of the Galaxy. The circular velocities derived from this model, as a function of R, from 1 kpc on the 50 kpc from the galactic center, is referred to as the Schmidt rotation curve. The Schmidt curve is taken as a basis for comparison of all later work covering limiting intervals of R. I like to emphasize now and make further comments later that the so-called rotation curve is that of the flattest component of the galaxy, the Pop. I extreme and not representative of intermediate or Pop. II systems.

In the past decade or so the rotation curve from optical data has been extended roughly to 3-4 kpc around the sun allowing thus a comparison and an overlap with radio data. Kraft and Schmidt (1963) used classical Cepheids to obtain an ω versus R relation. Miller (1968) has provided a portion of the rotation curve using radial velocities of H II regions and the distances of their ionizing stars. Georgelin, Georgelin and collaborators have compiled a comprehensive list of velocities from the Hα emission line, 233 at present, following the method of Fabry-Pérot interferometry developed by Courtès (1960). Their data have yielded information on the velocity curve in both center ai anti-center regions (see in particular Georgelin 197: Here again the distance are those of the stars ionizi: the respective H II regions.

IV. DEVIATIONS FROM A SMOOTH ROTATION CURVI

As velocity data both from optical and radio regio: accumulated particularly in the past decade, it becan increasingly evident that the rotation law deviates from smooth and simple one. Below I list two of the mo relevant findings which will justify the doubt implicit the title of this paper: that we do not know well enou; the rotation curve of our Galaxy.

a) It was pointed out by Kerr (1964) that tl rotation curve observed from the north (roughly ℓ = (to 90°) is higher than the southern curve (270°-360 (see Figure 3). Kerr showed moreover that the asyı

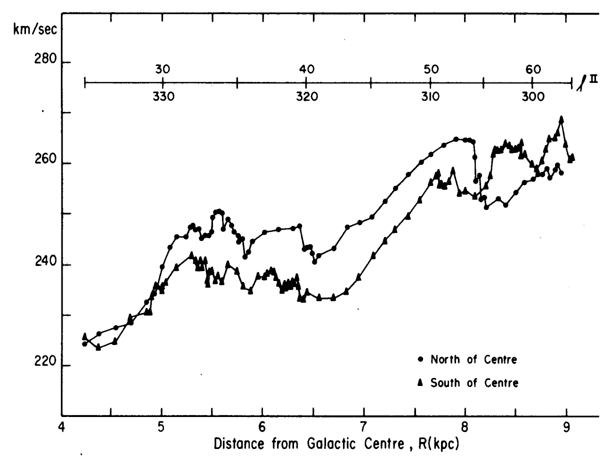


Fig. 3. The north-south asymmetry of the rotation curve with the solar circle obtained by the 21-cm H-line (Kerr 1964).

metry between the two hemispheres would be remove if one postulated an expansion velocity from the cent of 7 km s^{-1}. A similar asymmetry in the anticent direction is suggested by optical data. However, H velocities at higher galactic latitudes on both sides of tl galactic plane do not seem to show such asymmetry. might be that the phenomenon is in some way related t the spiral structure. Are we observing, then, at high latitudes a smooth component of our galaxy (di population). The problem has to be pursued by furth observations to be certain whether the difference in tl rotation curve between the north and south is a loc phenomenon or not, or if it is at all physically real.

b) The rotation curve of the Galaxy is wavy. Tl existence of waves (undulations as they are termed ;

present) in the rotation curve of external galaxies was called to attention back in 1965. Our Galaxy could be no exception. In a series of papers (Pişmiş 1965, 1966, 1975) I showed that 75% of the rotation curves of galaxies given by Burbidge *et al.* (for a complete listing of their data see Burbidge *et al.* 1975) showed waves which could not be attributed to chance irregularities. A sample curve showing waves, that of NGC 3521 is shown in Figure 4. It was argued that for a composite system like our galaxy consisting of different and co-existing kinematical groups (subsystems) lower rotational velocity between the spiral arms would be expected. In the spiral arms the observed velocity would result from the average of the overwhelming Population I component (fast rotating) and a population resembling the disk population (rotating more slowly); whereas in the interarm we would be observing essentially the disk population, clouds or stars, and hence a slower rotation.

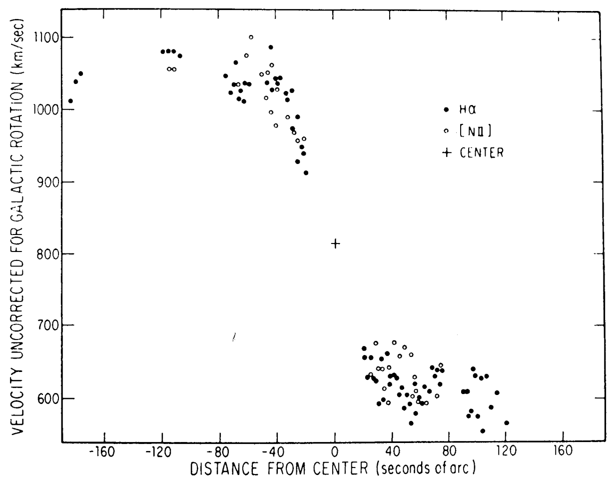


Fig. 4. Rotation curve obtained by Burbidge *et al.* (1964) for the spiral NGC 3521 showing waves.

Indeed the maxima and minima do occur at the arm and interarm regions respectively. A good example is the galaxy NGC 2998 for which a reliable rotation curve is reproduced in Figure 5 (Rubin *et al.* 1978). As a crucial test for my suggestion the dispersion of velocities should be analyzed. If my suggestion is correct the dispersion of the interarm will be larger than in the spiral arms themselves. That a slower rotation will be compensated by a larger dispersion of velocities is entirely compatible and is a direct consequence of a steady state theory for the Galaxy.

The suggestion that the waves in rotation curves are physically significant features had to wait about a decade until Rubin *et al.* (1978) confirmed that undulations are rather the rule than the exception.

Advocates of the density wave theory attribute

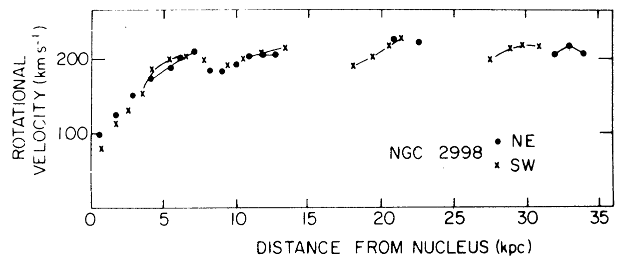


Fig. 5. Rotation curve for the spiral NGC 2998 obtained by Rubin *et al.* (1978) where the undulations are clearly manifested.

undulations in the rotation curves of galaxies to the streaming motions along the spiral features. Undulations are shown also by the stellar component of galaxies (see the rotation curve of NGC 2903 by Simkin (1975) reproduced in Figure 6.

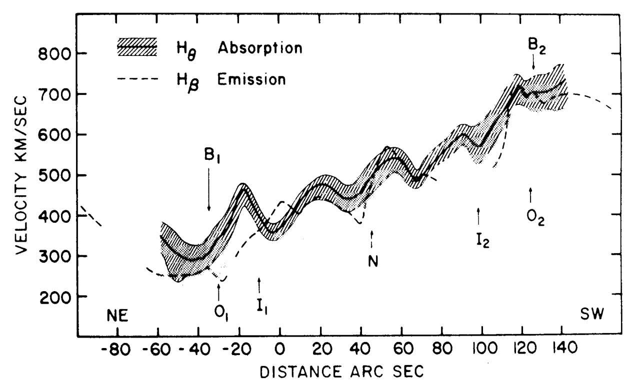


Fig. 6. Rotation curve for the spiral NGC 2903 by Simkin (1975) where both stars and gas show waves.

One can now safely state that undulations do exist in our Galaxy. Within the solar circle radio data have clearly shown the existence of waves. I believe that although part of the dip is presumably the result of paucity in the interarm as to neutral H, it is reasonable to expect, as argued above, that some H I is present in there with a lower rotational velocity. Unfortunately outside the solar circle the velocity mapping of both optical and radio data are affected by the uncertainty in the distances (photometric distances mostly) which may cause confusion and hence undulations may be ill-defined. This is true also of optical data within the solar circle. More and better data are needed.

We like to call further attention here to the results from 21-cm H line data supporting the physical reality of the waves. There exist now complete velocity fields for a sufficiently large number of galaxies. Figure 7 shows a montage made by Bosma (1978). The wavy

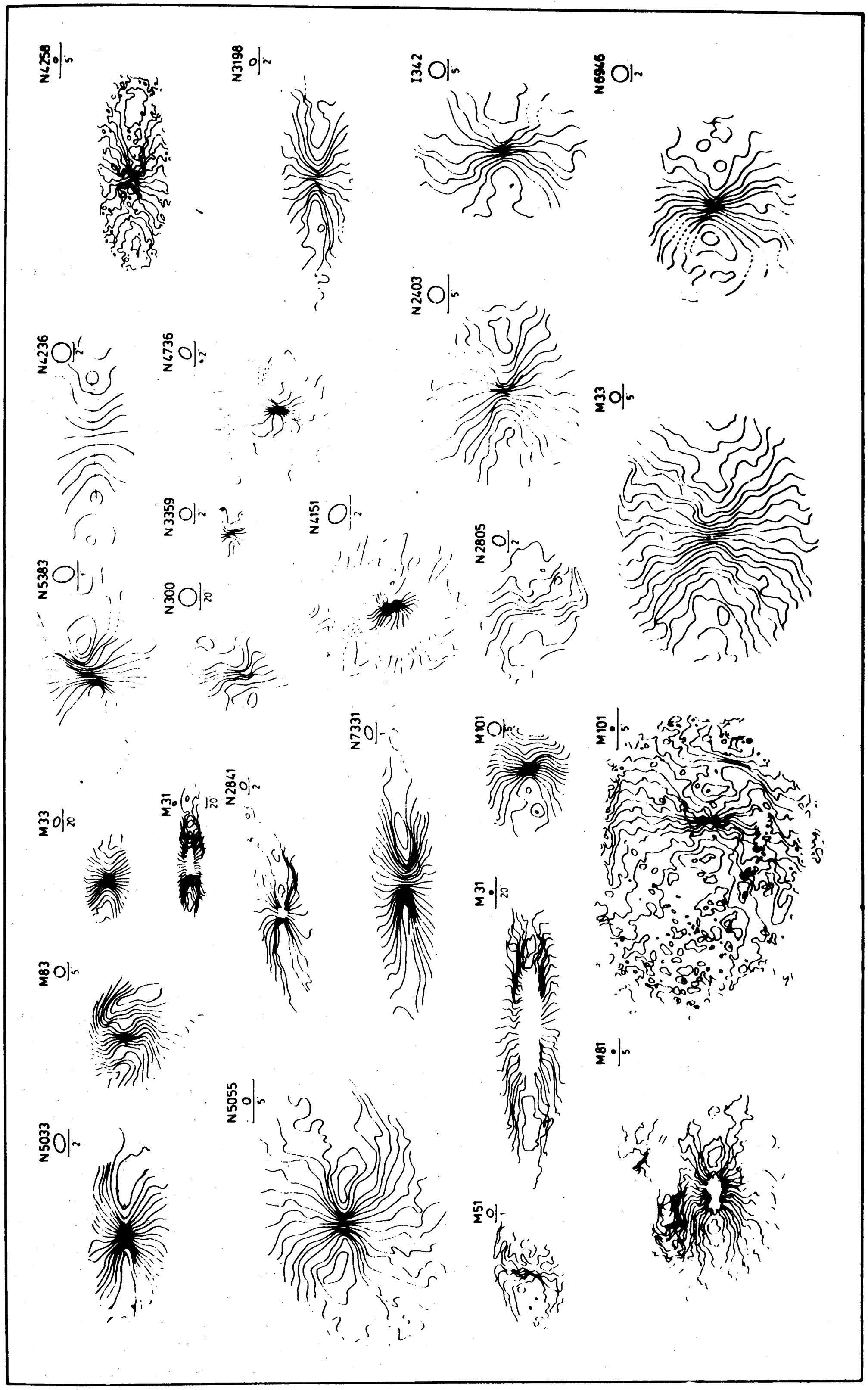
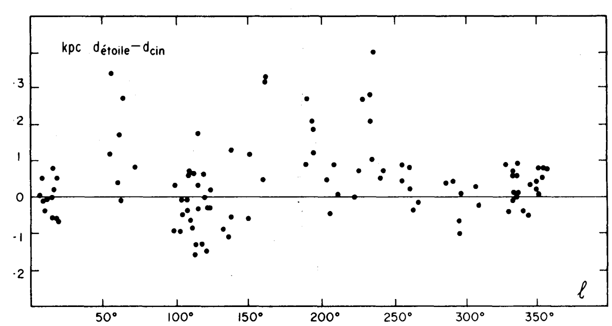


Fig. 7. The velocity field of galaxies by the 21-cm H-line a montage by Bosma (1978). The velocity undulations over the whole galaxy appear to be more the rule than the exception.

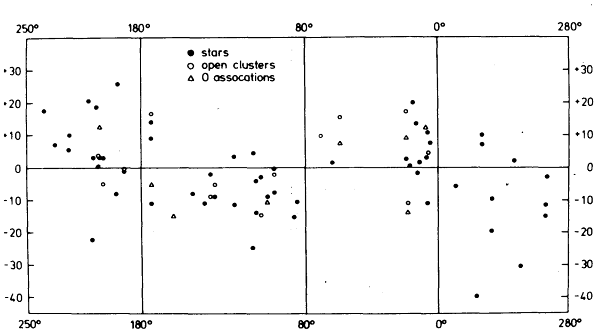
sovelocity contours are clearly shown by the majority f these galaxies.

V. KINEMATIC VERSUS PHOTOMETRIC DISTANCES OF H II REGIONS

Georgelin, Georgelin and collaborators have obtained adial velocities of some 230 H II regions in all. To map ut these in the galactic plane required a knowledge of he distances of the nebulae from the sun. The estimaion of the distances they carried out both kinematically ssuming circular rotation for these objects and using the econd order differential rotation formula with contants determined by their data; and wherever *UBV* nagnitudes existed the distance of the ionizing star of he nebula was also estimated. There emerged a curious ffect: the difference in the distances $r_{star} - r_{H\ II}$ howed a variation with galactic longitude. Their curve howing this variation is reproduced in Figure 8. Minn ınd Greenberg (1973) have challenged this result Figure 9); combining all known velocities of H II



ig. 8. Photometric distance of the ionizing star minus the inematic distance of the H II region plotted against galactic ngitude. A variation of this difference with galactic longitude ems to exist in the data.



ig. 9. Velocity of H II region minus the velocity of the ionizing ar(s) of the region is plotted against galactic longitude (Minn d Greenberg 1973) using both optical and radio data.

regions both from optical and radio ranges, ionizing stars and clusters, they argue that it is the difference in the velocities (ionizing star velocity minus H II region) which shows a trend varying with ℓ. With more data Crampton and Georgelin (1975) refute the conclusions of Minn and Greenberg. Thus the disagreement of quadrants second and third (Figure 10) persisted, namely that $r_{kin} > r_{star}$ in the second quadrant and $r_{kin} < r_{star}$ in the third quadrant. Georgelin and Crampton make use of a rotation curve different in the north and south quadrants of the anti-center region removing thus the discrepancy. However no physical justification for a difference in the rotation law in the second and third quadrants is given by the authors. I like to stress here that the presumed flatness of the rotation curve –the extended maximum– beyond the sun, although probably true, does not remove the discrepancy of the

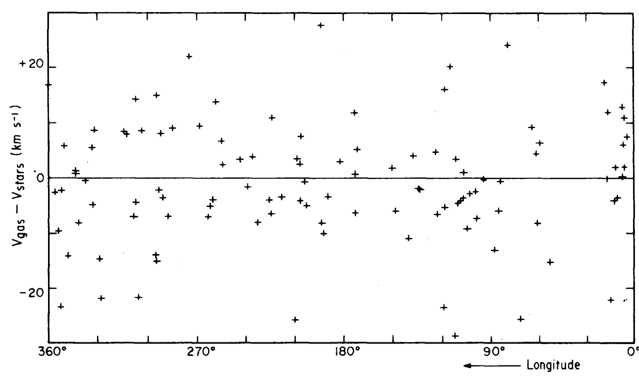


Fig. 10. The velocity of the gas minus the velocity of the ionizing star plotted against galactic longitude (Crampton and Georgelin 1975) using more and better data where no variation with ℓ of the velocity differences is noticeable although the dispersion is rather large.

kinematic and the photometric distances. Such a flat curve would affect both quadrants of the anti-center region equally. Below I shall show that the rotation curve not only will exhibit waves but that the distance from center of the maxima (or minima) of the waves will depend on the polar angle due to the composite nature of the structure of the Galaxy (or of other spirals) and the spiral pattern of the Population I component. We may state therefore that the rotation curve will be dependent on direction from the center of the galaxy!

VI. ROTATION CURVE FUNCTION OF DIRECTION: THE NORTH-SOUTH ASYMMETRY

It may seem absurd and unreasonable at first glance to doubt the symmetry of a rotation curve. Therefore I should at once explain the circumstances leading to my assertion, that the "rotation curve" may depend on the direction from the galactic center. Below I list some basic premises suggested by observation on which our arguments will hinge.

a) Rotation curves of the Galaxy and galaxies are not smooth. This point was discussed earlier in this paper. The maxima of the curve correspond to the spiral arms and the minima to the interarm.

b) The spiral structure in our Galaxy as well as in other well developed spirals exhibits bi-symmetry. We may characterize the morphology of the extreme Population I component by two intertwining similar spirals starting from the ends of a diameter. In such a model the bi-symmetry is assured. This double spiral is co-existent with a smooth, rotationally symmetrical substratum, the disk-halo population say Population II for short.

c) It is well known that the substratum rotates slower than the Population I extreme, namely the spiral arms. Within the spiral arms observations should give the combined average rotation of the substratum and the coexisting Pop I extreme, whereas in the interarm we shall be observing the average velocity of essentially the "Population II" component.

To illustrate the consequences of this situation we take a simple numerical example; it is assumed that at a given point in the galaxy, in this case at the solar vicinity, the rotational velocity of the pure Population I is 260 km s^{-1} with a velocity dispersion around this mean of 10 km s^{-1}. At the same location the rotational velocity of the substratum is taken as 220 km s^{-1} and the dispersion as 40 km s^{-1}. Moreover I have assumed that the mass of the spiral material is four fold of the substratum at the arms. This is consistent (and is probably a lower limit) with the total luminosity per unit surface of the arms as compared to the neighboring interarm regions in galaxies.

With these assumptions I have calculated the maximum and minimum rotational velocities representative of the arm and interarm regions around the sun as well as the dispersion of velocities. The results are shown in Table 1 reproduced from an earlier paper (Pişmiş 1975).

A crucial test is then to search if indeed at the minima the dispersion is larger than at the maxima of the rotational velocity.

We now return to envisage a galaxy with two intertwined spirals. To fix ideas let us take a logarithmic spiral which is believed to represent satisfactorily the overall spiral structure of a galaxy. We have, then, in polar coordinates the equation of the spirals as follows:

$$R = a\,e^{b\lambda} \text{ (spiral 1), and } R = a\,e^{b(\lambda + \pi)} \text{ (spiral 2);}$$

where R, the radius vector, is the galactocentric distance, λ the angular coordinate reckoned from an arbitrary direction, while *a* and *b* are constants. Figure 11 shows a pair of such spirals. Since there is bi-symmetry

$$R = ae^{b\lambda} = ae^{b(\lambda + \pi)}$$

Be the cause what it may, I repeat that at present it is quite certain that the rotation curve of a spiral is wavy the maxima and minima occurring roughly at the arm and interarm regions respectively. Accepting these premises it follows immediately that the rotation curve in the second and third quadrants of the Galaxy (also in the first and fourth quadrants) will be different from one another; that in general the form of the rotation curve will depend on λ. In any direction λ, the radius vector R of a spiral detail, say within an arm, will be the same only at λ and $\lambda + \pi$. Only in this case will the waves coincide if the curve is folded by an angle π. This is true for any λ. The waves of the velocity curve versus R will be different in any other $\lambda +$ a where a $\neq \pi$.

To illustrate the effect of this situation on the kinematic distance we take any two directions, say $\lambda = \pm 30°$, on each side of the direction sun-center. The direction to the center marks, thus, the origin of the central angle, λ. It is clear that if at $\lambda = -30°$ a region is within a spiral arm at a distance R from the galactic center, at the same distance in the direction $\lambda = +30°$ a region will no more be within a spiral arm, instead it will be in the interarm. Applying the same average rotation curve to both regions we would be underestimating the

TABLE 1

ESTIMATED VALUES FOR ROTATIONAL VELOCITY AND DISPERSION AT ARM AND INTERARM REGIONS

	Pop. I extreme	Pop. II	Arm	Interarm
θ_r	260 km s^{-1}	220 km s^{-1}	250 km s^{-1}	220 km s^{-1}
Relative density	4	1	5	1
$\langle \pi^2 \rangle^{1/2}$	10 km s^{-1}	40 km s^{-1}	20 km s^{-1}	40 km s^{-1}

$\langle \pi^2 \rangle^{1/2}$ is the dispersion in the radial direction.

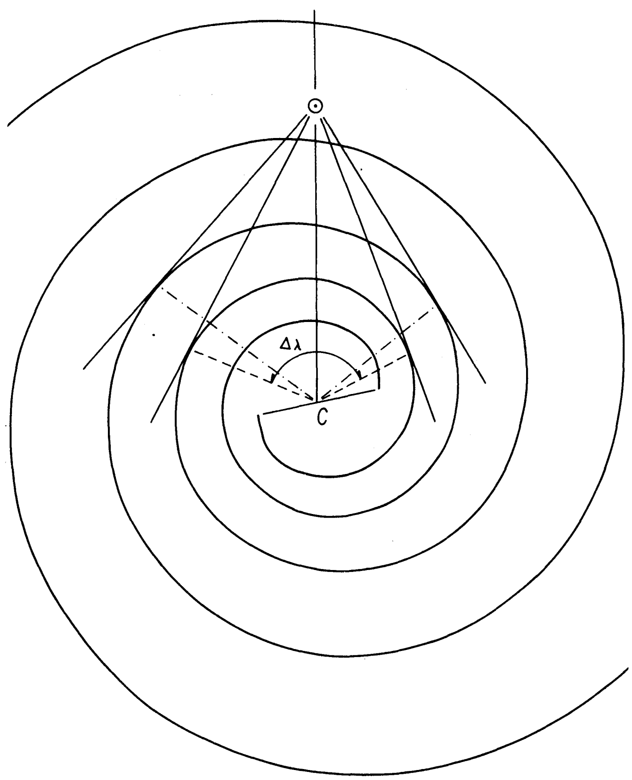


Fig. 11. Model of our galaxy with two logarithmic intertwined spirals. Tangents drawn from the sun to the spirals determine the central angle Δλ within which observation of the rotational velocity is possible.

distance of the region at −30° in the second quadrant, and overestimating the distance of the region at $\lambda = +30°$ (third quadrant).

As observed from the sun, the maxima and minima of the observed radial velocity in a given direction ℓ will always lie within an angle λ less than π. Thus, observations with known distance will not cover a range of λ equal to 180° or π/2 on each side of the central direction.

In external galaxies rotation curves are usually obtained (optically at least) from observations of radial velocities along the "major axis" which is very close to the real line of nodes. The curves on the two sides of the center of the galaxy are then folded over and thus the rotation as a function of distance from the center, obtained. In such a case, in the model galaxy we have described above, the waves will coincide with one another.

In Kraft and Schmidt's rotation curve ω(R), Figure 12 (Kraft and Schmidt 1963) waves are detectable. This is easy to understand as the maximum distance reached by their Cepheids is 5 kpc. According to these authors their data is complete to 1.5 kpc but, however incomplete, there are a few Cepheids as distant as 5 kpc. We take 4 kpc as an effective limit of the extent of the study. This implies that the Cepheids will be contained within a circle with a radius of 4 kpc centered at the sun. The angle subtended at the galactic center by the two tangents to this circle will be of the order of 80° within which the Cepheids will lie. We shall denote such a limiting central angle by Δλ. Δλ in the case of Cepheids being much smaller than 180° the rotation curve averaged over all Cepheids studied by Kraft and Schmidt

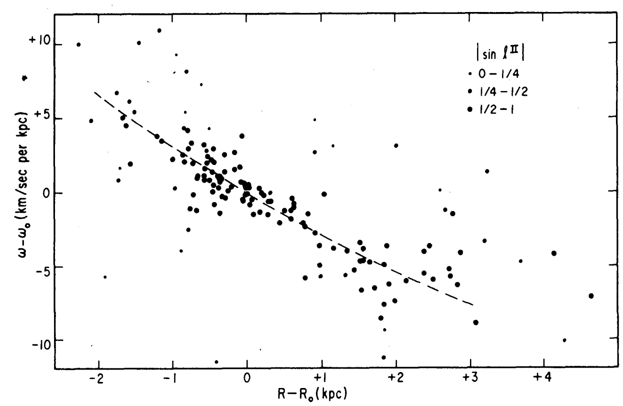


Fig. 12. Rotation curve (angular rotation ω versus distance) of our galaxy obtained from Cepheid variables by Kraft and Schmidt (1963). Note the slight wave shown by the observations.

will not conceal the waves. Although there would be a certain relative displacement of the waves in the average rotation, complete cancellation of the waves will occur only when the angle Δλ is 180°, a value which cannot be attained by observations from the sun. Indeed the maximum angle Δλ formed at the galactic center within which all possible observations from the sun will be contained does not exceed 120° as can be easily seen from Figure 11.

Thus both in the center and anti-center regions it is expected that some sort of residual wavy variation of the rotation curve will be observable. The average rotation curve in one quadrant will then be different from the next quadrant. The partial overlap of the successively displaced waves as λ varies will diminish the prominence of the waves in amplitude and will increase the dispersion of the individual observation around the average curve. If the random motions and errors in the velocities permit, average curves for the northern and southern hemispheres of the Galaxy are expected to show the maxima (or minima) of the waves occurring at different distances from the center.

I recommend therefore that we look into the north-south asymmetry in the light of the arguments brought forth above and the conclusions reached regarding the limitation imposed by our eccentric position in the Galaxy.

VIII. WAVY ROTATION CURVE AND THE VALUES OF OORT'S CONSTANTS

In the solar neighborhood the rotation curve may seem to be better known; optical observations e.g., radial velocities and proper motion, as well as distances are certainly better determined. Oort's constants A and B, as we have seen above yield the velocity of rotation of the LSR and its derivative at that point. Yet it is fair to state that these important dynamical quantities are not so well determined as expected and leave much to be desired.

To support this doubt, I list in the following tables values of the constants A and B obtained by various authors. Table 2 is the simultaneous determination of A and B using proper motions, while Table 3 gives values of A from radial velocities of different classes of objects. It is a curious fact that the early determinations of A have yielded large values, as large as 21 km s^{-1} kpc^{-1} and that the values have decreased through the years! (Table 4). At the Hamburg Assembly of the IAU after long discussion the adopted values were A = 15 km s^{-1} kpc^{-1} and B = –10 km s^{-1} kpc^{-1} respectively. It is interesting to go through the different values of A and B to be aware of how different they are. Aside from dynamical, there may be other causes such as the uneven galactic distribution of the objects, systematic differences in the distance estimates by different authors, systematic errors in the velocities and the insufficient number of objects used. I shall now discuss an essential point not taken into account in the determination of A and B. We take up A first. We know that

$$A = -1/2\ R_0\ (d\omega/dR)_0\ .$$

A, obtained from observations, is not strictly a measure of the derivative of ω with respect to R but it is of $\Delta\omega/\Delta R$. If the variation of the rotation curve at the solar vicinity is fast, $\Delta\omega/\Delta R$ will vary with the interval ΔR. For a good determination of A, a large number of stars is necessary and this condition requires that more distant objects be included in the solution. Also, since A, the amplitude of the double wave, is the unknown solved by least squares solution, r (and hence R) should cover a larger interval (assuring a sufficiently larger number of

TABLE 2

VALUES OF OORT'S CONSTANTS FROM PROPER MOTIONS

A (km s^{-1} kpc^{-1})	B (km s^{-1} kpc^{-1})	Object	Number	Reference
20	–7	stars	5300	Morgan and Oort 1951
15	– 10	...	...	Decree (!) 1964, IAU
11.5	– 7	Cepheids	45	Wielen 1974
16.9	– 4.8	Gal. cluster	30	Buscombe 1972
14.8	– 11.3	all stars	163811	Dickvoss and Vegt 1967
16.1	– 9.0	all stars	166179	Asteriadis 1977
26.5	– 37.0	O-B 2	599	" "

TABLE 3

VALUES OF OORT'S CONSTANT A FROM RADIAL VELOCITIES

A (km s^{-1} kpc^{-1})	Object	Number	References
17.7	B stars	79	Petrie, Cuttle and Andrews 1956
17.5	B stars	314	Feast and Thackeray 1958
19.5	Cepheids	76	Stibbs 1956
17.5	Cepheids	37	Gascoigne and Eggen 1967
17.4	Cepheids	51	Walraven *et al.* 1958
15	Cepheids	150	Kraft and Schmidt 1963
15.5	Cepheids	109	Crézé 1970
15	Gal. cluster	36	Johnson and Svolopoulos 1963
14.2	Gal. cluster	143	Taff and Littleton 1972
14	Supergiant	669	Humphreys 1970
16.8	O9-B9.5	590	Balona and Feast 1974
13	O stars	252	Cruz-González and Arellano 1978

TABLE 4

EARLY DETERMINATIONS OF THE OORT CONSTANTS A AND B

Constants A and B ($km\ s^{-1}\ kpc^{-1}$)	Object	References
A = 31.7 ± 3.7	Supergiants, Cepheids, O stars	Oort 1927
A = 19; B = − 24	RV and proper motions	Oort 1928
A = 21	Cepheids	Joy 1939
A = 13	O, B stars	Pişmiş 1945
A = 13	Proper motions	Weaver 1955

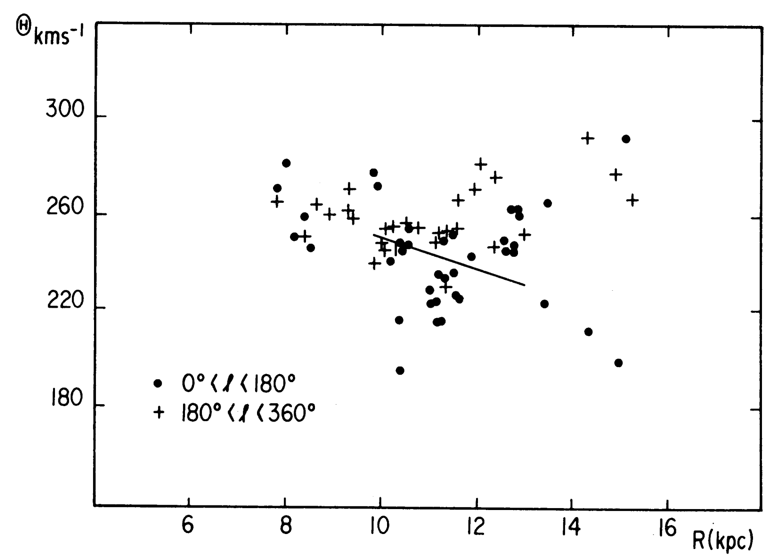
stars for a satisfactory determination of A). All these imply that ΔR may be large and different from solution to solution. Second order terms will help but they were rarely taken into account in the past.

For a smoothly varying function ω(R) [or θ(R)] the approximation $\Delta\omega/\Delta R$ for $d\,\omega/dR$ will not affect appreciably the results, but the ω(R) function is probably not so at the solar vicinity as will be shown now.

We have emphasized on several occasions that we expect the galactic rotation curve to be wavy. Within the solar circle waves may be due partly to the paucity of the interarm regions of neutral H;therefore, I only will discuss the anti-center region where no such spurious effect exists. The same arguments must hold for the region within the solar circle.

Optical radial velocities by Georgelin *et al.* (1973) with Fabry-Pérot interferometry are used to construct a rotation curve in the solar vicinity given in Figure 13. The distance of all regions are "photometric" ones of the ionizing stars of the nebulae. Recently Blitz (1979) also gave a segment of a rotation curve for the anti-center region using his CO velocities. In Figure 14 I have plotted Blitz's data together with the rotation velocities obtained from the spectroscopic radial velocities of H II regions by Miller (1968). Blitz concludes from his data that the rotation curve remains flat to a distance of 14-15 kpc from the center. In general this is true; however the graph also shows clearly that the average curve bends upward; we are here witnessing a wave where the sun in in the trough of it. Optical velocities also follow this trend if we leave out the odd-behaving Perseus arm. I expected to distinguish the waves in the two quadrants but the material does not allow a clear distinction between the behavior of the curve in second and third quadrants. However using data of O type stars compiled by Cruz-González *et al.* (1974) we find a slight indication that the rotation curve in the north and south hemispheres are different (see Figure 15).

The implications of this local wave are interesting. Let us consider the tangent of the average curve at the sun (Figure 14). The quantity A–B is this gradient, which



ig. 13. The rotation velocity Θ_{LSR} plotted against distance from galactic center. The data are Fabry-Pérot radial velocities of H II egions by Georgelin *et al.* (1973). The straight line segment is the gradient of the rotation curve based on A = 15 and B = −10.

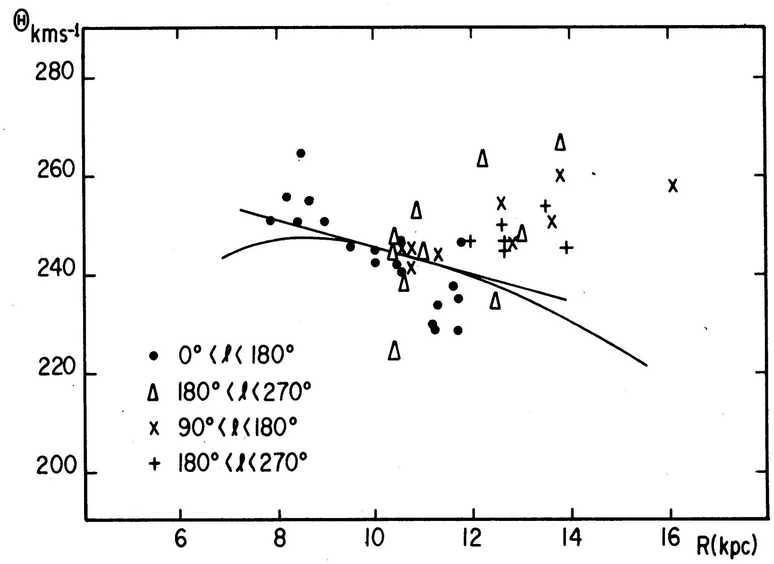


Fig. 14. The rotation velocity Θ_{LSR} versus distance for CO clouds of the anti-center region obtained by Blitz (1979). Designations (x) and (+) refer to the nothern and southern hemispheres respectively. Plotted also are the data from H II regions determined spectroscopically by Miller (1968). The points and triangles refer to the northern and southern hemispheres, respectively. The full line is the Schmidt rotation curve. Note that the scatter of the individual velocity points is about half as large as those in Figure 13.

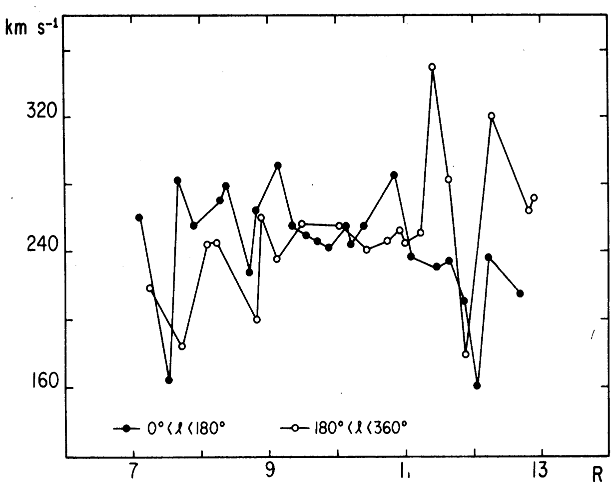


Fig. 15. Rotation velocities using O-type stars from the Catalogue of Cruz-González *et al.* (1974); the points are averages within intervals of the distance from the galactic center. Different symbols are used to designate velocities in the northern and southern hemispheres.

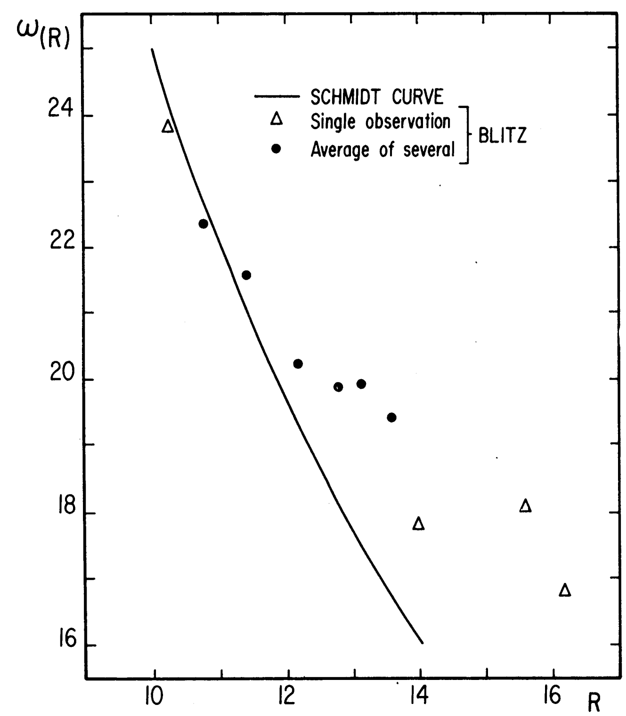


Fig. 16. The velocity data of the CO clouds obtained by Blitz (1979) plotted as a rotation curve giving the angular velocity versus distance from the center.

is negative. If we take now farther stars as well, the gradient is much less negative. The same data are plotted in the ω(R) versus R diagram (Figure 16) where the gradient will be only function of A, R_0 being constant at the sun. The figure shows clearly that at the anti-center, the gradient of the angular rotation shown by CO is smaller than that given by the Schmidt curve. This implies a smaller value for A.

In a general way we can thus explain the diminution of A through the years. Probably the earlier data used to determine A were based on nearby stars and therefore with relatively smaller R. Here the upward bending section of the rotation curve did not enter into the determination whereas at larger distances spanned by later determinations, A was affected by the upgoing branch, and hence the average direction of the gradient was pulled upward (less negative) giving thus an A which was smaller.

IX. IMPORTANCE OF A ROTATION CURVE

A rotation curve not only is an important relation for dynamical studies, and in the computation of a potential function in our Galaxy but it is a much used implement to estimate distances of gas clouds be they H I, molecular clouds or H II regions for which the ionizing source is too faint and not observable due to extinction. Thus a reliable mapping of such regions to obtain the overall structure of the Galaxy is possible only through the knowledge of a rotation curve. Therefore the importance of a well-determined rotation law cannot be over-emphasized. This statement applies also to the values of A and B which fix a point on the curve in the immediate vicinity of the sun.

All what we have discussed so far refers essentially to regions close to the galactic plane. Although the observed curve includes the implicit effect of some disk population objects, the overwhelming effect is that of the extreme Population I. To discuss objects at higher z (for larger b) one should multiply the right hand side of Oort's radial velocity formula by $\cos^2 b$ and the exact formula by cos b. For z distances larger than about 100 pc this correction may not be sufficient as the correction is only geometrical; the procedure assumes that the rotation curve is the same at higher galactic latitudes as in the galactic plane. For objects not belonging to the extreme Population I the velocity field is not dominated by pure rotation. Not only is the dispersion of velocities important but also the rotation is slower.

X. THE ROTATION CURVE OF THE GALAXY IS NOT UNIQUE

We should keep in mind that the mass of the extreme Population I is about 10% of the total mass of the Galaxy. The rotation curve we have discussed above is that of 10-15% of the galactic mass. The disk population has lower velocity of rotation at the sun. It may well be that the rotation of the disk population, slower than that of Population I, follows a curve resembling the curve obtained from objects of extreme Population I reduced by a factor less than 1. Based on my interpretation of the undulations the rotation curve of the Pop. I will be the upper envelope of the conventional curve while that of the substratum, the disc population, would be the lower envelope. It would be desirable to determine rotation curves of planetary nebulae and other disc objects and if possible of Pop. II objects such as RR Lyrae stars.

XI. EPILOGUE

In concluding we may state that at present we have become aware that rotation curves of our and other galaxies are more complicated than previously thought. Refinement of observations and confrontation of these with theory will be necessary to disentangle the rotation characteristics of the different kinematical components of our and other galaxies and be able to determine the rotation curve for the different populations.

This is Contribution No. 1 of Instituto de Astronomía, UNAM.

REFERENCES

Asteriadis, G. 1977, *Astr. and Ap.,* **56** 25.
Baade, W. 1944, *Ap. J.,* **100**, 137.
Balona, L.A. and Feast, M.W. 1974, *M.N.R.A.S.,* **167**, 621.
Blitz, L. 1979, *Ap. J. Letters*, **231**, L115.
Bosma, A. 1978,. Ph. D. Dissertation, University of Groningen.
Burbidge, E.M. and Burbidge, G. R. 1975, *Galaxies and the Universe,* eds. A Sandage, M. Sandage and J. Kristian (Chicago: The University of Chicago Press), p. 81.
Burbidge, E.M., Burbidge. G.R., Crampin, D.J., and Rubin, V. C. 1964, *Ap. J.,* **139**, 1058.
Burton, W.B. and Gordon, M.A. 1978, *Astr. and Ap.,* **63**, L7.
Buscombe, W. 1972, *Observatory,* **92**, 141.
Charlier, C.L. 1913, *Lund Medd. Ser. II,* No. 9, p. 78.
Courtès, G. 1960, *Ann. d'Ap.,* **23**, 115.
Crampton, D. and Georgelin, Y. M. 1975, *Astr. and Ap.,* **40**, 317.
Crézé, M. 1970, *Astr and Ap.,* **9**, 410.
Cruz-González, C. and Arellano-Ferro, A. 1978, *Rev. Mexicana Astron. Astrof.,* **2**,339.
Cruz-González, C., Recillas-Cruz, E., Costero, R., Peimbert, M., Torres-Peimbert, S. 1974, *Rev. Mexicana Astron. Astrof.,* **1**, 211.
Dieckvoss, W. and Vegt, Ch. de 1967, *Astr. Nachr.,* **290**, 125.
Feast, M.W. and Thackeray, A.D. 1958, *M.N.R.A.S.,* **118**, 125.
Gascoigne, S.C.B. and Eggen, O.J. 1967, *M.N.R.A.S.,* **117**, 430.
Georgelin, Y.M. 1975, Thesis, University of Marseille.
Georgelin, Y.P. and Georgelin, Y.M. 1970, *Astr. and Ap.,* **6**, 349.
Georgelin, Y.M., Georgelin, Y.P., and Roux, S. 1973, *Astr. and Ap.,* **25**, 337.
Humphreys, R.M. 1970, *A.J.,* **75**, 602.
Johnson, H. L. and Svolopoulos, S.N. 1963, *Ap. J.,* **134**, 868.
Joy, A.H. 1939, Ap. J., **89**, 356.
Kerr, F.J. 1964 in *The Galaxy and the Magellanic Clouds, IAU Symposium No. 20,* eds. F.J. Kerr and A.W. Rodgers (Canberra: Australian Academy of Sciences), p. 81.
Kraft, R. and Schmidt, M. 1963, *Ap. J.,* **137**, 249.

Lindblad, B. 1925*a*, *Uppsala Astr. Obs, Medd.*, No. **3**.
Lindblad, B. 1952*b*, *Archiv für Mathematik*, 19A, No. **21**.
Miller, J.S. 1968, *Ap. J.*, **151**, 473.
Minn, Y.K. and Greenberg, J.M. 1973, *Astr. and Ap.*, **24**, 393.
Morgan, H. R. and Oort, J.H. 1951, *Bull. Astr. Inst. Netherlands*, **11**, 379.
Oort, J.H. 1926, *Kapteyn Astr. Lab. Groningen Publ.*, No. **40**.
Oort, J.H. 1927, *Bull. Astr. Inst. Netherlands*, **3**, 275.
Oort, J.H. 1928, *Bull. Astr. Inst. Netherlands*, **4** 269.
Pease, F.G. 1916, *Proc. Nat. Acad. Sci. USA*, **2**, 517.
Petrie, R.M., Cuttle, P.M., and Andrews, D.H. 1956, *A.J.*, **61**, 289.
Pişmiş, P. 1965, *Bol. Obs. Tonantzintla y Tacubaya*, **4**, 8.
Pişmiş, P. 1966, in *Non Stable Phenomena in Galaxies, IUA Symposium No. 29*, Ed. Ambartsumian (Yerevan: Academy of Sciences of Armenian SSR), p. 429.
Pişmiş, P. 1974, *in Galaxies and Relativistic Astrophysics, Proceedings First European Astr. Meeting*, eds. B. Barbanis and J.D. Hadjidemetriou (Berlin: Springer) p. 133.
Pişmiş P. and Prieto, A. 1945, *Ap. J.*, **101**, 314.
Plaskett, J.S. and Pearce, J.A. 1936, *Pub. D.A.O.*, **3**, 241.
Rubin, V.C., Ford, W.K., and Thonnard, N. 1978 *Ap. J.*, **225**, L10.
Schmidt, M. 1957, *Bull. Astr. Inst. Netherlands*, **13**, 15.
Schmidt, M. 1965, in *Galactic Structure*, eds. A. Blaauw and M. Schmidt, (Chicago: The University of Chicago Press), p. 513.
Simkin, S.M. 1975, *Ap. J.*, **195**, 293.
Simonson, S.C. and Mader, G.L. 1973, *Astr. and Ap.*, **27**, 337.
Slipher, V.M. 1914, *Lowell Obs. Bull.*, **11**, No. 12.
Stibbs, D.W.N. 1956, *M.N.R.A.S.*, **116**, 453.
Strömberg, G. 1924, *Ap. J.*, **59**, 228.
Taff, L.G. and Littleton, J.E. 1972, *Astr. and Ap.*,**21**, 443.
Turner, H.H. 1912, *M.N.R.A.S.*, **72**, 387.
Van de Hulst, H.C., Muller, C.A., Oort, J.H. 1954, *Bull. Astr. Inst. Netherlands*, **12**, 117.
Walraven, Th., Muller, A.B., and Oosterhoft, P. Th. 1958, *Bull. Astr. Inst. Netherlands*, **14**, 81.
Weaver, H.F. 1955, *A.J.*, **60**, 211.
Wielen, R. 1974, *Astr. and Ap. Suppl.*, **15**, 1.

DISCUSSION

Fuenmayor: ¿Podría hacer un comentario acerca de la coplanaridad de los brazos espirales de la galaxia?

Pişmiş: La desviación de coplanaridad no debe ser muy marcada. Pero quiero señalar que los resultados obtenidos por el grupo de Maryland muestran indicios que alrededor del Sol un brazo espiral está arriba y otro abajo del plano promedio de la Galaxia. Este resultado está en buen acuerdo con un corolario del mecanismo de formación de los brazos espirales en nuestra y en otras galaxias que he propuesto en 1964.

Serrano: ¿Cree Ud. que la variación de velocidades con longitud para nubes de hidrógeno de alta velocidad, no asociadas con la Corriente Magallánica (y concentradas en el plano) indicaría una expansión general del tipo que propuso Kerr?

Pişmiş: Antes de interpretar el fenómeno que menciona usted, hay que tener la seguridad que la búsqueda de las nubes de alta velocidad está hecha sistemáticamente en todo el cielo. Sólo cuando esta condición esté satisfecha podremos saber si el fenómeno es local o es un fenómeno proveniente de la dinámica en gran escala de la galaxia.

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