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A DERIVATION OF THE STELLAR LUMINOSITY FUNCTION IN THE DIRECTION OF THE SOUTH GALACTIC POLE BY A STATISTICAL METHOD

Steven L. Kipp

Centro de Investigación de Astronomía,
Venezuela

RESUMEN

Se desarrolla un método para calcular la distribución de probabilidad de magnitudes absolutas de estrellas en catálogos de movimiento propio. Este método utiliza una distribución elipsoidal de velocidades para predecir distribuciones de velocidad tangencial. Las distribuciones de velocidad tangencial se transforman en distribuciones de magnitudes absolutas. Las distribuciones de magnitudes absolutas para cada estrella pueden ser sumadas con ciertas restricciones, para producir una función de luminosidad. Se aplicó el método a estrellas en el área del polo sur galáctico y obtuvimos una función idéntica a la función de luminosidad general de 1968 de Luyten.

ABSTRACT

A method is developed to calculate absolute magnitude probability distributions for stars from a proper motion survey. This method uses ellipsoidal velocity distributions to predict tangential velocity distributions. The tangential velocity distributions are transformed into absolute magnitude distributions. The absolute magnitude distributions for the stars in a proper motion survey may be summed to produce a luminosity function. This method was applied to stars in the region of the south galactic pole and the resulting luminosity function is statistically identical to Luyten's 1968 luminosity function.

Key words: LUMINOSITY FUNCTION – GALAXIES-STELLAR STATISTICS

I. INTRODUCTION

The stellar luminosity function is a distribution function giving the number of stars per unit absolute magnitude per unit volume in the solar neighborhood. Knowledge of the luminosity function yields information on the stellar mass density in the vicinity of the sun. The standard determination of the stellar luminosity function is by Luyten (1968). However some recent luminosity function determinations disagree with Luyten's results. Wanner (1972) and Muzzio (1973) found fewer stars and Weistrop (1972) and Sanduleak (1976) found many more stars than did Luyten. Weistrop (1976) and others (Faber *et al.* 1976) have now resolved much of the discrepancy between her luminosity function and that of Luyten. In addition very recent determinations of the luminosity function by Robertson (1979) and Chiu (1980) support the Luyten luminosity function. In this paper we will outline another method for the determination of the luminosity function and present the results of applying this method to nearly 7000 stars in the direction of the south galactic pole.

II. METHOD

Our method, like that of Luyten, is a kinematic method based on a knowledge of the tangential velocities of stars in the vicinity of the sun. Luyten obtains such kinematic information from his absolute magnitude versus reduced proper motion regression. However our method obtains such kinematic information directly from velocity ellipsoid determinations. This is both an advantage and a disadvantage since, although velocity ellipsoidal determinations potentially incorporate proper motion, radial velocity and parallax information, they have not been made for intrinsically faint stars. Like Luyten's method, our method is designed to determine the luminosity function from proper motion survey data.

For a given star in a proper motion survey we have a position (α, δ), proper motion μ, θ, and magnitude m. Assume for simplicity that we also know the parameters (solar motion, $u_\odot$, $v_\odot$, $w_\odot$ and dispersions σ_u^2, σ_v^2, σ_w^2) of the velocity ellipsoid ϕ(u, v, w) which characterize this star. We convert the ellipsoidal distribution ϕ(u, v, w) into a tangential velocity distribution Ψ(T) by rotating the u, v, w velocity system to a tangential, radial velocity system T_δ, T_α, RV, integrating out the radial velocity dependence and finally converting the T_α, T_δ system to a polar system T,θ (Trumpler and Weaver 1952). The general form of Ψ(T) is

$$\Psi(T)\, dT = C_1 T \exp[-C_2(AT^2 + BT + C)]\, dT \quad ,$$

where the constants C_1, C_2, A, B, and C contain the velocity ellipsoidal parameters, the parameters of the rotation matrix, and functions of θ. Since θ is given for each star of the proper motion survey we have dropped the functional dependence on θ. To convert $\Psi(T)$ into an absolute magnitude distribution $\chi(M)$, we proceed as follows: We guess a series of absolute magnitudes M_i which imply a series of tangential velocities T_i through

$$4.74\,\mu 10^{0.2}\,(m - M_i + 5)$$

Note that μ and m are fixed for each star of the proper motion survey. The probability that our star has a tangential velocity T_i is $\Psi(T_i)$ and hence this yields the probability $\chi(M_i)$ that the star has an absolute magnitude M_i. Thus we create an absolute magnitude distribution for each star in the proper motion survey. Estimates for the absolute magnitudes of individual stars may be obtained by using the most probable or expectation value of the absolute magnitude distribution. A luminosity function may be synthesized from all the stars of a proper motion survey by summing those elements of the absolute magnitude distributions for which the distance modulus $(m\text{-}M_i)$ indicates that the element is within the distance limit of the luminosity function.

There are two major complications of the process described above. First, the velocity ellipsoid parameters change with absolute magnitude (Delhaye 1965). Thus a new set of ellipsoid parameters must be selected for each choice of M_i. We adopt relationships describing this variation from the work of Delhaye and other more recent determinations of velocity ellipsoid parameters.

The second complication is that the derived distribution function $\Psi(T)$ describes an unbiased group of stars with respect to velocity. Consequently it is necessary to numerically generate from $\Psi(T)$ a function $\Psi(T)$ which describes the velocity distribution of a proper motion survey with given limits in magnitude and proper motion.

III. TESTING THE METHOD

Our method was tested by applying it to a sample of 160 stars within 8 parsecs. This sample was chosen since it is reasonably complete kinematically. Absolute magnitude distributions were calculated for all the stars in this sample and the expectation values of these distributions were compared with the trigonometrically determined absolute magnitudes. The residual distribution and its statistics are shown in Figure 1. The absolute magnitude distributions for the 160 stars were summed to form a luminosity function (defined in this instance as the number of stars per unit absolute magnitude within 8 parsecs). Also, the expectation values of the absolute magnitude distributions were collected into a histogram to form a luminosity function. Both of these estimates of the luminosity function are compared with the "true" luminosity function, the histogram of the trigonometrically determined absolute magnitudes, in Figure 2. The true luminosity function is well fit by botl estimates, however the estimate obtained from the sun of the absolute magnitude distributions is a smoothec estimate of the true function. There is a small bu systematic excess for the faintest absolute magnitudes.

Note that in this test the sample is constrained to b

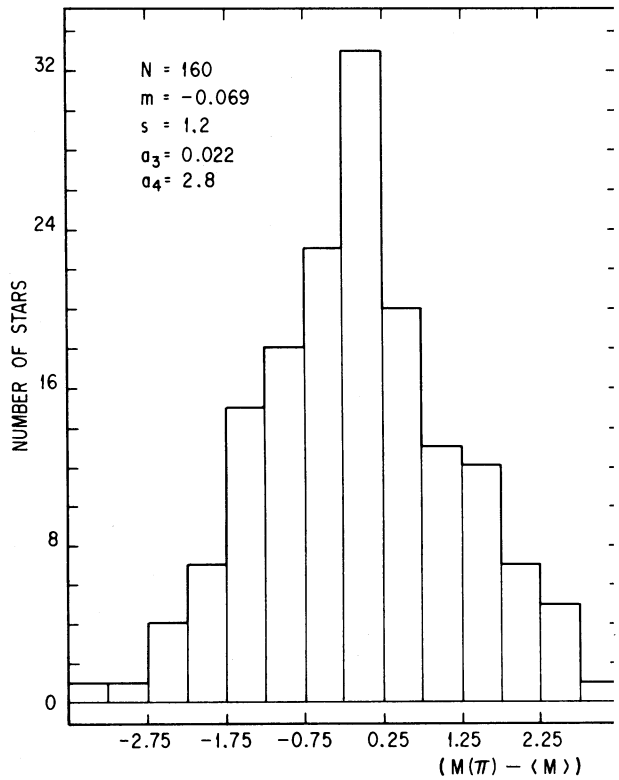


Fig. 1. Distribution of residuals for 160 stars within 8 parsecs Statistics for the distribution are included.

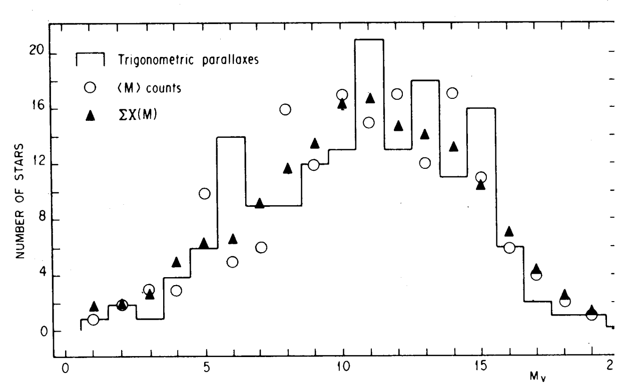


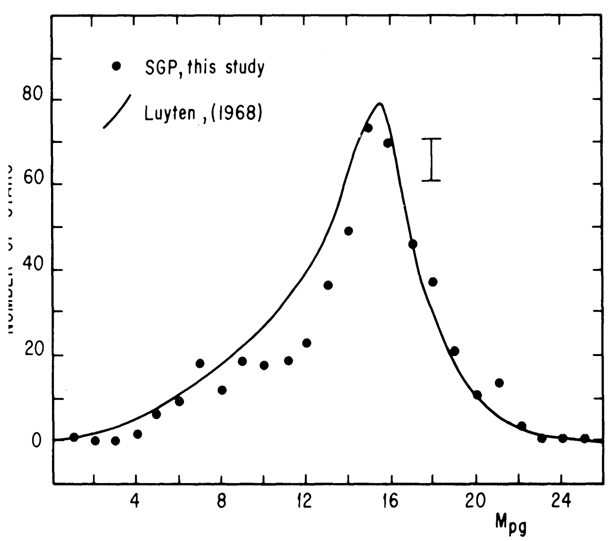
Fig. 2. Luminosity function for 160 stars within 8 parsec compiled by different techniques.

‚vithin a certain distance contrary to the way in which he metod would normally be applied.

IV. LUMINOSITY FUNCTION IN THE DIRECTION OF THE SOUTH GALACTIC POLE

We have applied our method to the nearly 7000 stars ı the proper motion survey catalogue *The South ;alactic Pole* by Luyten and La Bonte (1973). This atalogue covers roughly the area $22^h\,00^m$ to $3^h\,40^m$ ınd $-32°\,40'$ to $3°\,20'$ which is about 7.1% of the sky. 'he catalogue contains stars as faint as $m_{pg} = 21.2$ and ›roper motions to $0''.179$ year^{-1} but it is not complete o these limits and the general completeness is not ıniform. Therefore it was decided to use the entire atalogue in the determination of the luminosity func- ion since limiting our selection of stars to a complete et would greatly reduce the total number of stars and ve wish to minimize statistical fluctuations in the final uminosity function.

The final results for the south galactic pole lumi- ıosity function are shown in Figure 3 compared with ,uyten's 1968 function. (Of course our results for 7.1% ıf the sky have been scaled up to the entire sky and ıence our original numbers were about 1/14 of those hown.) Luyten's results came from an analysis of ›roper motion data from the north celestial polar cap nd various other fields south to $-30°$ declination. Nevertheless there is very little difference between our esults and Luyten's. In addition, our results show a naximum for the luminosity function at $M_{pg} = 15.3$ while Luyten and La Bonte estimated the maximum for the south galactic pole area would be 15.4. The estimated internal error for our luminosity function determination was obtained numerically.



'ig. 3. Stellar luminosity function, number of stars within 10 ›arsecs per unit magnitude, for the South Galactic Pole ompared with Luyten's 1968 results. Typical internal error is hown.

Among the nearly 7000 stars in the Luyten-La Bonte catalogue, 250 were found with published parallaxes at least one sigma above zero. Comparing the trigonome- trically determined absolute magnitudes with the expec- tation values of the absolute magnitude distributions, we obtain the residual plot of Figure 4. Some of the large negative residuals which form a slight tail are produced by stars for which either proper motion or parallax is in serious error.

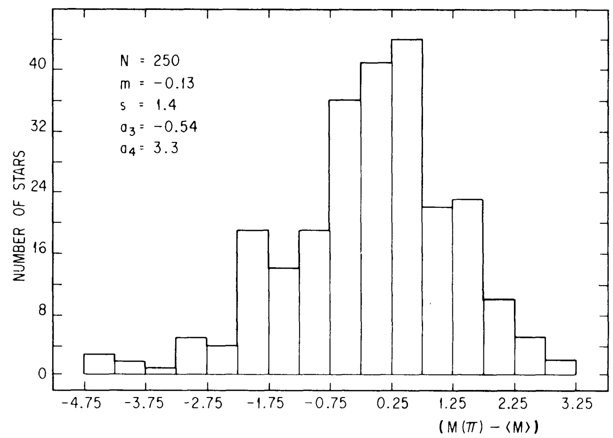


Fig. 4. Distribution of residuals for 250 South Galactic Pole stars with trigonometric parallaxes. Statistics for the distribution are included.

V. DISCUSSION

There are two major problems associated with our method. The first and potentially most serious is the lack of information about the velocity ellipsoids of stars much fainter than 12th absolute photographic magni- tude. For the results quoted here the ellipsoid pa- rameters of stars fainter than 12th magnitude were assumed to be identical to the parameters of 12th magnitude stars. Serious deviation from this assumption could result in a radically different luminosity function. We investigated numerically the effect on the luminosity function of varying the ellipsoidal parameters of faint stars. Specifically, we tried assigning to intrinsically faint stars the ellipsoidal parameters of A stars to investigate the effect of a low velocity population of faint stars as proposed by Sanduleak (1976). The result was a greatly enhanced luminosity function qualitatively similar to Sanduleak's prediction. Of course, since truly low-veloc- ity stars would not be in the proper motion survey, our results are not exactly what Sanduleak predicts.

An additional problem for our method is the asymmetry of the v distribution due to the presence of high-velocity stars. We fit the v distribution with a Gaussian distribution. This undoubtedly produces systematic errors in predicted absolute magnitudes and in the resulting luminosity function, however systematic effects are not seen in our results and numerical experimentation indicates they will be small.

Our method is based on several approximations and assumptions, the most fundamental of which is the implicit one of homogeneity and isotropy of stellar kinematics in the solar neighborhood. Thus our results should be carefully qualified. However, in general, the method in insensitive to changes in input parameters and to exact numerical techniques. Thus the similarity of our results to Luyten's supports the correctness of Luyten's luminosity function.

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DISCUSSION

Serrano: Is the decline in the luminosity function for m < 16 real?
Kipp: Luyten claims that the Palomar Proper Motion Survey is complete past the maximum of his 1968 luminosity function. If this is correct, and I believe it is, it would also apply to my results.
Anon: Can you say anything about differences between the luminosity function of the halo and the disk?
Kipp: The method does not distinguish stars of different populations.

Steven L. Kipp: Centro de Investigación de Astronomía "Francisco J. Duarte", Apartado Postal 264, Mérida 5101-A Venezuela.