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BLOCK ADJUSTMENT IN PHOTOGRAPHIC ASTROMETRY

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RESUMEN

Se ha desarrollado y verificado un nuevo método para el ajuste de bloques basado en la conversión de coordenadas cartesianas tridimensionales y su consiguiente rotación alrededor de 3 ejes. El material medido consiste de 22 pares de placas tomadas con prisma objetivo. Como sistema de referencia se usaron los catálogos SAO, Yale,Perth 70 y FK4. El número de estrellas de referencia varía desde más de 1000 como en el caso del Catálogo Yale, a solamente 9 como en el caso del FK4. El número de enlaces entre pares de placas varía desde un mínimo de 43 a un máximo de 344. Los resultados demuestran que el sistema funciona bien aun cuando el número de objetos de referencia es considerablemente menor que el número de campos. El sistema de referencia más recomendable para zonas australes parece ser el del catálogo de Perth 70.

ABSTRACT

A new method for block adjustment, based on a conversion to three dimensional cartesian coordinates and subsequent rotation around three axes has been tested. The measured material consists of 22 plate pairs taken with objective prism. As reference systems, the SAO Catalogue, the Yale Catalogue, the Perth 70 Meridian Circle Catalogue, and the FK4 have been used. The number of reference stars varied from well over a thousand in the case of the Yale Catalogue to only nine in the case of the FK4. The number of links between plates varies from a minimum of 43 to a maximum of 344. The results show that the system works very well, even when the number of reference objects is considerable less than the number of fields. The most recommendable reference system for southern fields seems to be the Perth 70 Catalogue.

Key words: STARS-POSITIONS – ASTROMETRY

I. INTRODUCTION

The basic problem of photographic astrometry is the conversion of rectangular coordinates measured on a plate into spherical coordinates on the sky. The process normally used is described by the symbolic scheme

$$(\alpha, \delta) \rightarrow \begin{pmatrix}\text{projection}\\ \text{geometry}\end{pmatrix} \rightarrow (\xi, \eta) \rightarrow (\text{polynomials}) \rightarrow (x, y)$$

It means that right ascension (α) and declination (δ) for a given equinox and the epoch of the plate are taken from a reference catalogue and converted into theoretical plane rectangular coordinates (ξ,η) with the help of an adequate projection geometry, which may be the tangential projection or the concentric projection or any other one, depending on the optical system used. These then are compared with the measured coordinates (x, y) and the relation between the two sets of coordinates is established, usually in the form of polynomials. These may have the form:

$$\xi = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2 + \ldots (1)$$

$$\eta = q_{00} + q_{10}x + q_{01}y + q_{20}x^2 + q_{11}xy + q_{02}y^2 + \ldots (2)$$

The number of coefficients to be carried depends principally on the number N of reference objects available. It is always preferred to include less coefficients than N, and to determine these coefficients by least-squares methods, but this may at times not be possible because of lack of reference stars. Once the coefficients have been determined, the scheme outlined above may be used in reverse, calculating α and δ from x and y for any measured object.

Additional conditions may be introduced when one is dealing with several partially overlapping plates. An example is shown in Figure 1. Naturally, one can reduce each plate separately using the method outlined above. However, in this case one may find systematic differences between the plates in the area in common. It is tempting to avoid this problem in the following manner: One reduces for instance first plate No. 1, using only the reference stars which appear on it. For the reduction of plate No. 2 one may use not only its legitimate reference stars, but also those for which a position is already known from plate No. 1. For the third plate even more "artificial" reference stars will be available, and so on. The question arises whether the same final results are obtained, if one starts out with a different plate, and goes around the area in a different order. The answer is that the results are not necessarily the same. What we

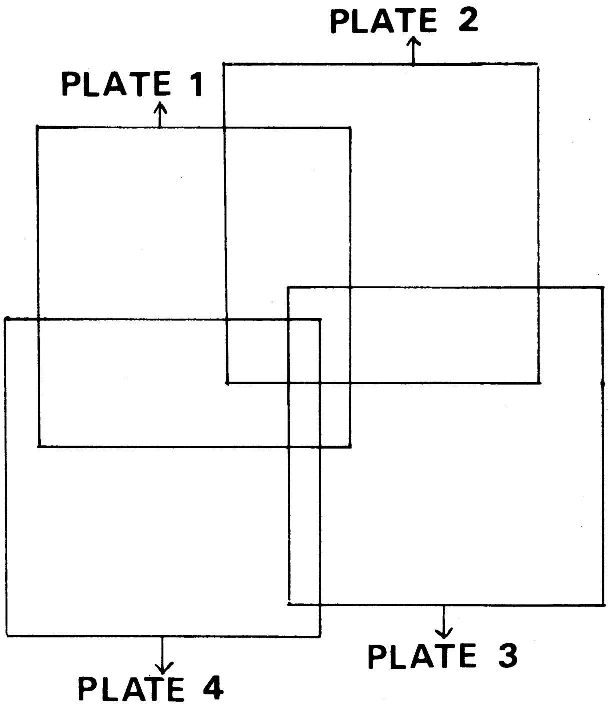

Fig. 1. Example of four overlapping plates.

need is a system which reduces all plates simultaneously, imposing on the solution for each plate not only the condition of best fit with the reference catalogue, but also best fit with neighbouring plates. This is what we call a block adjustment.

Block adjustment procedures are extensively used in photogrammetry. The first astronomical application is due to Eichhorn (1960). Similar approaches were discussed during the Conference on Photographic Astrometric Technique held at Tampa, Florida in 1968 (Eichhorn 1971). Methods covering the entire sky were proposed by Vegt and Ebner (1972). While block adjustment programs are under way at different observatories, so far there are practically no empirical results available in the literature. It is the purpose of this paper to present a new method and to apply it to practical cases.

There are other reasons that those mentioned above which also make the use of block adjustment methods for extensive position catalogues very desirable. The only source of accurate fundamental position is the FK4, or soon its successor, the FK5. It contains a sky coverage of about one star per 25 square degrees, far too little for even a large-field astrograph. The block adjustment method permits to cover a large part of the sky in one single process and thus can work with only a few reference stars.

II. COORDINATE TRANSFORMATION

No simple relation exists between the coordinates on different plates taken with different plate centers. However, following a suggestion by Eichhorn (1971) a simple relation can be established through a transformation. We may consider stellar positions as points on a sphere with unit radius, with the three-dimensional coordinates ξ, η, and ζ obtained from

$$\xi = \sin\alpha \quad \cos\delta, \tag{3}$$

$$\eta = \cos\alpha \quad \cos\delta, \tag{4}$$

$$\zeta = \sin\delta \tag{5}$$

The measured coordinates are transformed to cartesian coordinates u, v, and w on the unit sphere by

$$r = \frac{(x^2 + y^2)^{1/2}}{F}, \quad \tan r = \frac{(x^2 + y^2)^{1/2}}{F}, \tag{6}$$

$$u = \frac{x \sin r}{F\,r}, \qquad u = \frac{x \cos r}{F}, \tag{7}$$

$$v = \frac{y \sin r}{F\,r}, \qquad v = \frac{y \cos r}{F}, \tag{8}$$

$$w = \cos r \quad , \tag{9}$$

where F is the focal length of the telescope. The equations to the left are for concentric projection, and those to the right for tangential projection. It is also implied that in the case of the concentric projection the origin of these coordinates coincides with the geometrical center of the plate, while in the case of the tangential projection it coincides with the tangential point.

The system u, v, w can be transformed into the system ξ, η, ζ by

$$\xi = a_{11}u + a_{12}v + a_{13}w \quad , \tag{10}$$

$$\eta = a_{21}u + a_{22}v + a_{23}w \quad , \tag{11}$$

$$\zeta = a_{31}u + a_{32}v + a_{33}w \quad . \tag{12}$$

The coefficients a_{ij} form an orthogonal rotation matrix and are functions of the three angles A, B and C. For the matrix we may choose the form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where

$$
\begin{aligned}
a_{11} &= -\cos A \cos C - \sin A \sin B \sin C \\
a_{12} &= \sin A \cos C - \cos A \sin B \sin C \\
a_{13} &= \cos B \sin C \\
a_{21} &= \cos A \sin C - \sin A \sin B \cos C \\
a_{22} &= \sin A \sin C - \cos A \sin B \cos C \\
a_{23} &= \cos B \cos C \\
a_{31} &= \sin A \cos B \\
a_{32} &= \cos A \cos B \\
a_{33} &= \sin B .
\end{aligned}
$$

Comparing the last column with equations (3) to (5) we note that A and B are the Right Ascension and Declination of the plate center. The angle C is the rotation of the system x, y with respect to the equatorial system.

III. BLOCK ADJUSTMENT

When dealing with N plates, we have to determine N matrices a_{mij}, where m is the number of the plate, while i and j refer to line and column as before.

We now may write two types of equations. When the reference star i is measured on plate m, then, in the absence of errors of any type.

$$a_{m11} u_{mi} + a_{m12} v_{mi} + a_{m13} w_{mi} = \xi_i \quad , \qquad (13)$$

$$a_{m21} u_{mi} + a_{m22} v_{mi} + a_{m23} w_{mi} = \eta_i \quad , \qquad (14)$$

$$a_{m31} u_{mi} + a_{m32} v_{mi} + a_{m33} w_{mi} = \zeta_i \quad . \qquad (15)$$

On the other hand, for a star which is measured on plates m and n we can write

$$a_{m11} u_{mi} + a_{m12} v_{mi} + a_{m13} w_{mi} - a_{n11} u_{ni} - a_{n12} v_{ni} - a_{n13} w_{ni} = 0 \quad , \qquad (16)$$

$$a_{m21} u_{mi} + a_{m22} v_{mi} + a_{m23} w_{mi} - a_{n21} u_{ni} - a_{n22} v_{ni} - a_{n23} w_{ni} = 0 \quad , \qquad (17)$$

$$a_{m31} u_{mi} + a_{m32} v_{mi} + a_{m33} w_{mi} - a_{n31} u_{ni} - a_{n32} v_{ni} - a_{n33} w_{ni} = 0 \quad . \qquad (18)$$

These last equations simply express that the coordinates measured on both plates should lead to the same position on the sky.

The minimum condition for a solution of the system is three stars per plate, and at least three stars in common with the reference catalogue. In practice much more data will be available, particularly stars in common to two plates. Thus least squares methods will normally be applied. Even then the system can be solved with as little as three references stars. The system consists of 3N simultaneous linear equations with three different right hand sides.

IV. FIELD DISTORTION

If there is no distortion of the field of any kind, and no accidental errors in the data, then the matrices determined for every plate should fulfill the conditions which apply to an orthogonal rotation matrix, namely that the squares of its elements should sum up to unity for any line or column. This, however, will normally not be the case for a number of reasons. We shall analyze several of them.

1. If there are reasons to believe that there is no distortion present in the data, neither due to the optical system, nor to atmospheric refraction, aberration, etc., then we are dealing with the effect of accidental errors only. In such a case one has to find for each plate the angles A, B and C such that the elements calculated from the matrix given in paragraph 2 will fit best the empirically determined elements.
2. If the focal length F used in equations (6) to (8) is in error, the empirical matrix will appear multiplied by a scale factor different from unity.
3. If there are scale factors in either x or y or in both, the different columns will yield different sums of the squares of the elements. Such distortions are, for instance, produced by differential refraction or by differential aberration.
4. If no relations at all are imposed between the elements, the empirical matrix will absorb even more complicated distortion terms.
5. A plate tilt can be allowed for by adding absolute terms to the equations (13) to (18). Higher order terms can be allowed for by adding higher order terms in u and v. Such approach, naturally, increases enormously the size of the system of equations.

V. EMPIRICAL RESULTS

The block adjustment method described in the previous chapters was tested in different ways, using plate material on hand. In all cases the plate material consists of pairs of objetive prism plates taken with the Curtis Schmidt telescope on Cerro Tololo equipped with a four-degree prism. The advantage of using objective prism plates for astrometric purposes was described previously by Stock (1978).

First we used six successive fields with partial overlap. The reference system was in part the Smithsonian Astrophysical Observatory Catalogue (SAO), in part the Yale Catalogue (Hoffleit 1967). Table 1 shows the scale factors for u, v and w obtained for the six fields. The w-fac-

TABLE 1

EMPIRICAL SCALE FACTORS

Field	f_u	f_v	f_w
1	1.00008479	1.00209255	0.99999877
2	1.00004107	1.00213100	0.99999878
3	1.00019842	1.00205075	0.99999865
4	1.00018503	1.00203129	0.99999867
5	1.00025504	1.00221539	0.99999839
6	1.00023625	1.00221533	0.99999842

tors show that the focal length used needs only a very small correction due to the objective prism.

Subsequently we combined 22 successive fields, using as reference system the Yale Catalogue, the Smithsonian Astrophysical Observatory Catalogue, the Perth 70 Catalogue, (Hφg and von der Heide 1976) and the FK4. In the case of the Yale Catalogue the reference system consisted of more than 1000 stars, in the case of the FK4 of only 9 stars. The number of links between plates varied from 43 to 344 stars. The most adequate reference system seems to be the Perth 70 Catalogue.

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DISCUSSION

Valdés: ¿Hay alguna condición fotogramétrica, para obtener el traslapo en los modelos?
Stock: La única condición por modelo es el mínimo de tres enlaces, ya sea con modelos vecinos, o con el modelo cero (catálogo de referencia).
Rayo: ¿Con qué instrumento han medido sus placas?
Stock: Con un estereo-comparador PSK 2 de la firma Zeiss.
Peimbert: ¿Cuál es el número máximo práctico de placas para obtener un ajuste aceptable?
Stock: Depende del catálogo de referencia. Las pruebas han demostrado que errores razonables de medición, o de datos de catálogo, afectan solamente los coeficientes de la placa donde ocurren y los de las placas vecinas. Por eso, se combinan varias placas para obtener un número suficiente de estrellas de comparación. Como ejemplo: aproximadamente 20 placas con traslapo de 20% para usar el FK4.

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