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THE TENSOR VIRIAL THEOREM APPLIED TO AN INHOMOGENEOUS SPHEROIDAL STELLAR SYSTEM

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RESUMEN

Se ha aplicado el teorema virial tensorial a un sistema inicialmente oblato y sin colisiones, para estudiar el cambio de sus configuraciones seculares. Se ha encontrado que el sistema muestra una oscilación entre una forma oblata y prolata. Asímismo se encuentra que el cuadrado del período de oscilación es inversamente proporcional a la densidad promedio del sistema.

ABSTRACT

We have applied the tensor virial theorem to an initially oblate stellar system without collisions in order to study the variation of its configuration in time. We find that the system undergoes an oscillation between an oblate and a prolate form. We also find that the square of the oscillation period is inversely proportional to the average mass-density of the system.

key words: GALAXIES-ELLIPTICAL – GALAXIES-OSCILLATION – TENSOR VIRIAL THEOREM

I. INTRODUCTION

Models for the formation and evolution of elliptical ıd disk galaxies by the collapse of a protogalaxy have ɜen proposed and investigated by several authors notably y Larson (1969, 1974, 1975 and 1976), and by Gott ıd Thuan (1976). Particularly in the discussion of a ıodel for the formation of an elliptical galaxy Larson 1975) shows that a completely inviscid collapse does not ad to a realistic model but, if a plausible turbulent scosity is assumed to exist, a significant outward trans- ɜr of angular momentum during the collapse will take lace leading thus to configurations which closely resem- le elliptical galaxies.

In the present paper we undertake a discussion of a ɜry simplified model of a spheroidal galaxy. We assume ıat such a galaxy has already reached the state of an ssembly of stars and propose to follow its early evolu- on from there on.

We envision an E galaxy to be an isolated system of ravitating mass points in which collisions are either neg- gible or are such that the distribution function in phase- ɔace is invariant in time. The system is assumed to be ıitially an oblate spheroid with initial parameters to be ɔecified later. We discuss in what follows the applica- on of the tensor virial theorem to study the evolution f such a spheroid. Chandrasekhar (1972) has discussed ıe problem for a system with a homogeneous distribu- on of mass points. In the present paper we give a gener- ization of the problem to the heterogeneous spheroid nd work out its application to a concrete case.

II. FORMALISM

The tensor virial theorem can be expressed as follows by the use of the Boltzmann equation:

$$1/2 \frac{d^2 I_{jk}}{dt^2} = \int_V \rho \langle v_j v_k \rangle dV - 1/2 \int_V \left(x_j \rho \frac{\partial \Phi}{\partial x_k} + x_k \rho \frac{\partial \Phi}{\partial x_j} \right) dV \quad , \tag{1}$$

where

$$I_{jk} = \int_V \rho x_j x_k \, dV \quad ;$$

ρ is the density of mass points, and Φ, the potential function. The integrations are over the volume occupied by the system.

For a system where the principal axes are along the coordinate axes the only non-zero components of equation (1) are along the diagonals of the tensor such as

$$1/2 \frac{d^2 I_{kk}}{dt^2} = \int_V \rho \langle v_k^2 \rangle \, dV - \int_V \rho x_k \frac{\partial \Phi}{\partial x_k} \, dV$$

Adopting the following notations:

$$K_{kk} = 1/2 \int_V \rho \langle v_k^2 \rangle \, dV \quad ;$$

$$W_{kk} = - \int_V \rho x_k \frac{\partial \Phi}{\partial x_k} \, dV \quad .$$

Equation (1) takes the form

$$1/2 \frac{d^2 I_{kk}}{dt^2} = 2K_{kk} + W_{kk} \quad . \qquad (2)$$

The moment of inertia of the system with respect to the center of mass of the system is clearly

$$I = \int_V \rho(x^2 + y^2 + z^2) \, dV = I_{xx} + I_{yy} + I_{zz} \quad ,$$

and the total kinetic energy of the system with respect to the center of mass is:

$$K = 1/2 \int_V \rho \langle v_x^2 + v_y^2 + v_z^2 \rangle \, dV = K_{xx} + K_{yy} + K_{zz} \quad .$$

It can also be shown that the potential energy is

$$W = W_{xx} + W_{yy} + W_{zz} \quad .$$

Summation over the diagonal components gives the well known virial theorem:

$$1/2 \frac{d^2 I}{dt^2} = 2K + W \quad .$$

We now have the three non-zero components of the tensorial virial as follows:

$$1/2 \frac{d^2 I_{xx}}{dt^2} = 2K_{xx} + W_{xx} \quad ,$$

$$1/2 \frac{d^2 I_{yy}}{dt^2} = 2K_{yy} + W_{yy} \quad ,$$

$$1/2 \frac{d^2 I_{zz}}{dt^2} = 2K_{zz} + W_{zz} \quad .$$

If the system has rotational symmetry around the axis, the first two equations reduce to one which may be taken to be either in the x or the y direction. From now on we chose the x-axis to be this direction. Thus we have the equations:

$$1/2 \frac{d^2 I_{xx}}{dt^2} = 2K_{xx} + W_{xx} \quad ,$$

$$1/2 \frac{d^2 I_{zz}}{dt^2} = 2K_{zz} + W_{zz} \quad ,$$

and (4

$$K = 2K_{xx} + K_{zz} \quad ,$$

$$W = 2W_{xx} + W_{zz} \quad .$$

We now assume that the total energy is conserved throughout the evolution. This means that $K + W = E =$ constant. This can be also written as follows

$$2K_{xx} + K_{zz} = E - 2W_{xx} - W_{zz} \quad . \qquad (5$$

III. INITIAL KINEMATIC CONDITIONS

At present it appears quite well established that th flattened shape of E galaxies is not wholly supported b rotation and that dispersion of velocities plays a majo role in determining the morphology of such systems This circumstance has to be taken into account in con structing models for E galaxies. In our approach usin the Tensor Virial Theorem the dispersion of the velocitie does not enter in the formalism in an explicit manner. I is implicit in the $^{I}K_{xx}$ and $^{I}K_{zz}$ tensors.

We adopt the concept of centroids of motion to des cribe the kinematics of our model. $^{I}K_{xx}$ and $^{I}K_{zz}$ wil include contributions from systematic as well as randon motions; for $^{I}K_{xx}$ the motion of the centroid (systemati motion) has two components, namely, one due to rota tion and another due to motion in the radial directio (expansion or contraction). $^{I}K_{zz}$ also contains randon motions –dispersion of velocities– around the centroid In $^{I}K_{zz}$ the motion of the centroid is clearly the compo nent of the radial motion in the z direction (expansio or contraction) only. It also contains the dispersion o the velocities in the z direction.

We assume that $^{I}K_{xx}$ is related to $^{I}K_{zz}$; the simples relation will be adopted, namely, that $^{I}K_{xx} = {}^{I}K_{zz}$ Any other relation may also be considered but at presen we only deal with the case of equality as a first approx imation since there are no compelling observationa reasons to the contrary. It should be emphasized tha the assumption of the equality of the kinetic energy com ponents *does not imply isotropy of the dispersion c* *velocities*. We further like to point out that the virtue of the Tensor Virial approach is indeed the circumstance that one does not need to postulate a dependence of the dispersion on position in the system. It is the overall value of the combined velocity components that is expressed by $\mathrm{I}\,K_{xx}$ and $\mathrm{I}\,K_{zz}$. Our model therefore does not exclude the anisotropy in the random motion shown to exist in elliptical galaxies.

Incidentally, it may be interesting to estimate numerically the variation of the dispersion of velocities as a function of direction and distance in our model; but that will await a future treatment.

We can then write the following

$$3K_{xx} = E - 2W_{xx} - W_{zz} \quad ,$$

and the system of equations now takes the form

$$1/2\,\frac{d^2 I_{xx}}{dt^2} = 2/3\,E - 1/3\,W_{xx} - 2/3\,W_{zz} \quad ,$$
$$1/2\,\frac{d^2 I_{zz}}{dt^2} = 2/3\,E - 4/3\,W_{xx} + 1/3\,W_{zz} \quad . \tag{6}$$

IV. THE DENSITY LAW

To proceed further we assume that the equidensity surfaces of the system can be represented by concentric spheroids and that the density law remains invariant during the evolution.

We define the non-homogeneity of the system by adopting the density law given by Schmidt (1965) for the Galaxy; however, any other law, say a polytropic density distribution, could also be assumed in which case it can be shown that the main results of our treatment would remain unchanged. At present we do not have sufficient information on the density law in spheroidal galaxies to adopt a "better" distribution of mass than that of Schmidt.

Schmidt's density variation with the distance R from the center is as follows:

$$\rho(R) = qR^{-1} + p_1 R \quad , \; R \leqslant R_{sph} \;\; \text{region 1} \quad , \tag{7}$$

$$\rho(R) = sR^{-n} \quad , \qquad R \geqslant R_{sph} \;\; \text{region 2} \quad .$$

R_{sph} refers to the semi-major axis of the spheroid of Schmidt's model; the values of the parameters p_1, q and s depend on the shape of the configuration. Further, let z_{sph} represent the semi-minor axis of the spheroid. R_{sph} and z_{sph} are convenient parameters to use in following the evolution of the system, and we shall adopt them as such.

V. DIFFERENTIAL EQUATIONS IN R_{sph} AND z_{sph}

We now make use of the equations derived by Roberts (1962) to evaluate I_{xx}, I_{zz}, W_{xx} and W_{zz}. We consider two values for n, namely, n = 4 and n = 6.

For n = 4 we have:

$$I_{xx} = 2/3\,K\alpha\,R_{sph}^2 \quad ,$$

$$I_{zz} = 2/3\,K\alpha\,z_{sph}^2 \quad ,$$

$$W_{xx} = -\frac{4K^2 G\beta}{(15)^2 R_{sph}} \cdot \frac{A_1^*}{\sqrt{1-e^2}} \quad ,$$

$$W_{zz} = -\frac{4K^2 G\beta}{(15)^2 R_{sph}} \cdot \sqrt{1-e^2}\;\; A_3^* \quad ;$$

K is the total mass of the system, G the gravitational constant, and

$$\alpha = 4/15\,\zeta - 1/6 \quad ,$$

$$\beta = 8/3\,\zeta^{-3} - 15/2\,\zeta^{-2} + 391/42 \quad ,$$

$$\zeta = \frac{a_1}{R_{sph}} \quad ,$$

a_1 is the extent of the system in the R direction; we also have

$$A_1^* = \frac{\sqrt{1-e^2}}{e^3}\;\text{arc sin e} - \frac{(1-e^2)}{e^2} \quad ,$$

$$A_3^* = \frac{2}{e^2} - \frac{2\sqrt{1-e^2}}{e^3}\;\text{arc sin e} \quad .$$

The eccentricity of the system is clearly

$$e = \sqrt{1 - \left(\frac{z_{sph}}{R_{sph}}\right)^2} \quad .$$

For n = 6 we have

$$I_{xx} = 1/3\,K_1\,R_{sph}^2 \quad ,$$

$$I_{zz} = 1/3\,K_1\,z_{sph}^2 \quad ,$$

$$W_{xx} = -\frac{2559\,K_1^2 G}{8575\,R_{sph}} \cdot \frac{A_1^*}{\sqrt{1-e^2}} \quad ,$$

$$W_{zz} = -\frac{2559\,K_1^2 G}{8575\,R_{sph}} \cdot \sqrt{1-e^2}\;\; A_3^* \quad .$$

where K_1 denotes now the total mass of the system.

We note that for $n = 4$, when the radius of the envelope tends to infinity, I_{xx} and I_{zz} are infinite since $\alpha \to \infty$. But the invariance of the density law can still be used by assuming a homologous evolution; in this case the variable $\zeta = a_1/R_{sph}$ will be a constant throughout the evolution of the system and hence α = constant. However, for the general case the equations should be treated without the restrictive assumption of homologous evolution.

The differential equations giving R_{sph} (t) and z_{sph} (t) for the two values of n are now:

(a) $n = 4$ (8)

$$\ddot{R}_{sph} = \frac{1}{R_{sph}} \left\{ -\dot{R}_{sph}^2 + \frac{1}{K_\alpha}\left[E + \frac{2K^2 \, G\beta}{(15)^2 \, R_{sph}} \psi(e)\right]\right\},$$

$$\ddot{z}_{sph} = \frac{1}{z_{sph}} \left\{ -\dot{z}_{sph}^2 + \frac{1}{K_\alpha}\left[E + \frac{2K^2 \, G\beta}{(15)^2 \, R_{sph}} \phi(e)\right]\right\}.$$

(b) $n = 6$

$$\ddot{R}_{sph} = \frac{1}{R_{sph}} \left\{ -\dot{R}_{sph}^2 + \frac{1}{K_1}\left[2E + \frac{2559 \, K_1^2 \, G}{8575 \, R_{sph}} \psi(e)\right]\right\},$$

$$\ddot{z}_{sph} = \frac{1}{z_{sph}} \left\{ -\dot{z}_{sph}^2 + \frac{1}{K_1}\left[2E + \frac{2559 \, K_1^2 \, G}{8575 \, R_{sph}} \phi(e)\right]\right\};$$

where

$$\psi(e) = \frac{A_1^*}{\sqrt{1-e^2}} + 2\sqrt{1-e^2}\, A_3^* ,$$

$$\phi(e) = \frac{4A_1^*}{\sqrt{1-e^2}} - \sqrt{1-e^2}\, A_3^* .$$

Since $e = \sqrt{1 - \left(\frac{z_{sph}}{R_{xph}}\right)^2}$ we have a pair of simultaneous equations which have to be solved for R_{sph} (t) and z_{sph} (t).

Preliminary computations showed that the spheroid evolves gradually into a sphere. From there on the coefficients of the differential equations have to be altered since the expression for the eccentricity becomes imaginary and this is physically unacceptable. This circumstance implies that the configuration becomes prolate. We shall give in what follows the pertinent relations for the prolate case and for the sake of completeness also the relations for the oblate case with similar notations; the subindices o and p will indicate oblate and prolate configuration respectively.

$$\gamma_o(e_o) = \frac{A_{1o}^*}{\sqrt{1-e_o^2}} , \quad \gamma_p(e_p) = \sqrt{1-e_p^2}\; A_{1p}^* ,$$

$$\delta_o(e_o) = \sqrt{1-e_o^2}\; A_{3o}^* , \quad \delta_p(e_p) = \frac{A_{3p}^*}{\sqrt{1-e_p^2}} .$$

We have (from Chandrasekhar 1969)

$$A_{1o}^* = \frac{\sqrt{1-e_o^2}}{e_o^3} \arcsin e_o - \frac{(1-e_o^2)}{e_o^2} ,$$

$$A_{3o}^* = \frac{2}{e_o^2} - \frac{2\sqrt{1-e_o^2}}{e_o^3} \arcsin e_o ,$$

$$A_{1p}^* = \frac{1}{e_p^2} - \frac{(1-e_p^2)}{2e_p^3} \ln \frac{1+e_p}{1-e_p} ,$$

$$A_{3p}^* = \frac{1-e_p^2}{e_p^3} \ln \frac{1+e_p}{1-e_p} - 2\frac{(1-e_p^2)}{e_p^2} ,$$

e_o and e_p are the eccentricities such that

$$e_o = \sqrt{1 - \left(\frac{z_{sph}}{R_{sph}}\right)^2} ; \quad e_p = \sqrt{1 - \left(\frac{R_{sph}}{z_{sph}}\right)^2} .$$

For either the oblate or the prolate case, the differential equations are:

$$1/2 \frac{d^2 I_{11}}{dt^2} = 2/3\, E - 1/3\, W_{11} - 2/3\, W_{33} \qquad (10)$$

$$1/2 \frac{d^2 I_{33}}{dt^2} = 2/3\, E - 4/3\, W_{11} + 1/3\, W_{33}$$

which are the same as in (6). The subindices 11 and 33 are used here in place of xx and zz respectively.

The explicit expressions for the I's and the W's are given below.

1) when ρ(envelope) $\propto R^{-4}$

$$\left.\begin{aligned} I_{11} = I_{22} &= 2/3\, K_\alpha\, R_{sph}^2 \\ I_{33} &= 2/3\, K_\alpha\, z_{sph}^2 \end{aligned}\right\} \text{ oblate and prolate,}$$

$$\left.\begin{aligned} W_{11} = W_{22} &= \chi\, \gamma_o(e_o) \\ W_{33} &= \chi\, \delta_o(e_o) \end{aligned}\right\} \text{ oblate;}$$

$$\left.\begin{aligned} W_{11} = W_{22} &= \chi\, \gamma_p(e_p) \\ W_{33} &= \chi\, \delta_p(e_p) \end{aligned}\right\} \text{ prolate.}$$

$$\text{where } \chi = -\frac{4 K^2 G\beta}{(15)^2 R_{sph}}$$

2) For ρ(envelope) $\propto R^{-6}$ we have

$$\left.\begin{aligned} I_{11} = I_{22} &= 1/3\, K_1\, R_{sph}^2 \\ I_{33} &= 1/3\, K_1\, z_{sph}^2 \end{aligned}\right\} \text{ oblate and prolate,}$$

$$\left.\begin{aligned} W_{11} = W_{22} &= \psi\, \gamma_o(e_o) \\ W_{33} &= \psi\, \delta_o(e_o) \end{aligned}\right\} \text{ oblate,}$$

$$\left.\begin{aligned} W_{11} = W_{22} &= \psi\gamma_p(e_p) \\ W_{33} &= \psi\delta_p(e_p) \end{aligned}\right\} \text{ prolate;}$$

where

$$\psi = -\frac{2559\, K_1^2\, G}{8575\, R_{sph}} \quad .$$

VI. SOLUTION OF THE EQUATIONS

One can generalize the solution of the differential equations (6) by introducing the functions f(t) and g(t) such that

$$R_{sph}(t) = f(t)\, R_{sph}(0) \quad ,$$

and

$$z_{sph}(t) = g(t)\, z_{sph}(0) \quad .$$

where $R_{sph}(0)$ and $z_{sph}(0)$ refer to the initial dimensions of the spheroid of region 1. The solution of the equations using the dimensionless functions f(t) and g(t) are then applicable to configurations with varying dimensions provided they have the same initial eccentricity.

The simultaneous equations (8) and/or (9) can be solved numerically making use of an algorithm due to Fehlberg (1972). This is the Runge-Kutta-Nyström algorithm of the sixth order.

We have considered such solutions only for n = 6, ρ(envelope) $\alpha\ R^{-6}$, and for the case where the configuration deviates only slightly from sphericity and is close to an equilibrium state. For a spherical system in steady state the virial theorem states that E = 1/2 W; for our configuration which is oblate at the start of the evolution, we can use the expression $E = 1/2\ W_0$, where W_0 is the initial potential energy.

We further assume the total energy, E, to be constant, the initial velocities of contraction or expansion to be zero, and the eccentricity e = 0.3; with this eccentricity the initial ratio of the dimensions is

$$\frac{z_{sph}}{R_{sph}} \simeq 0.954$$

our assumption on the near-sphericity of the initial configuration is thus justifiable.

With these initial conditions and using the Fehlberg algorithm we have obtained the numerical values of the functions f(t) and g(t). Table 1 lists some sample values, while Figure 1 gives the plot of f(t) and g(t) as a function of the parameter $3 \times 10^{-12}\ t\ \rho_*^{1/2}$. Here t is given in years and ρ_* is given by the following expression:

$$\rho_* \equiv \frac{K_1}{R_{sph}^3(0)} \quad \text{and is}$$

4/3 π times the mean density of the system (in $M_\odot$ kpc^{-3}).

TABLE 1

COMPUTED VALUES OF f(t) AND g(t) ASSUMING e = 0.3

t[a]	f(t)	g(t)	g(t) f²(t)
0	1	1	1
3.698	0.998	1.004	0.999988
7.397	0.993	1.015	1.0008
11.095	0.986	1.030	1.0013
14.794	0.978	1.046	1.0004
18.493	0.972	1.058	0.99958
22.191	0.969	1.063	0.99811
25.890	0.970	1.062	0.99923
29.588	0.974	1.053	0.99895
33.287	0.981	1.039	0.99989
36.985	0.989	1.023	1.00061
40.684	0.995	1.009	0.99893
44.382	0.999	1.001	0.99899
48.081	0.999	1.0007	0.99869

a. in units of 0.977×10^8 y.

In Figure 2 we plot the variation of the eccentricity as a function of time, or strictly as a function of 3 ×

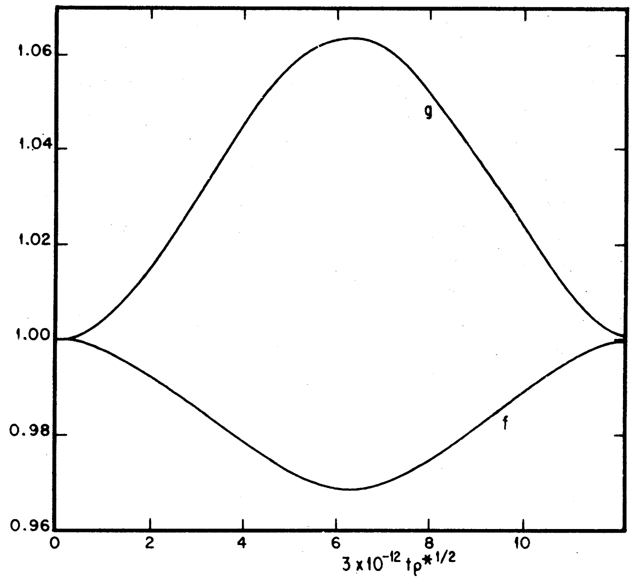

Fig. 1. The variation of the functions f(t) and g(t) in time. The abscissa is the quantity $3 \times 10^{-12}\ t\ \rho_*^{1/2}$; ρ_* is equal to $4/3\ \pi\rho$, where ρ is the average density of the configuration.

$10^{-12} t\ \rho_*^{1/2}$, as in Figure 1. One can see from both figures that the configuration varies between an oblate and prolate form. Several test solutions with varying initial eccentricities have indicated that the period of oscillation does not depend on the initial value of the eccentricity, if the latter is small.

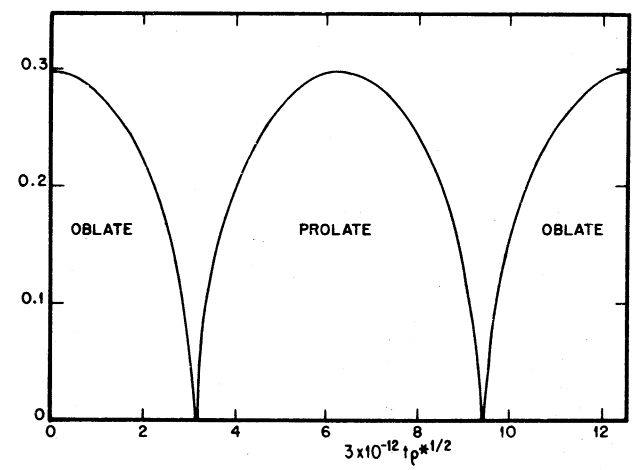

Fig. 2. The variation of the eccentricity, e, in time. The abscissa is the quantity $3 \times 10^{-12}\ t\ \rho_*^{1/2}$.

It is clear that the configuration undergoes an oscillation; starting from an oblate form it evolves gradually into a sphere and then proceeds towards a prolate spheroid to return again to a spherical form and finally to complete its first cycle by going over to the initial oblate spheroidal state until a violent relaxation takes over.

The period of oscillation corresponds to 12.5 units in the abscissae of Figures 1 and 2. One can have an idea of the time scale of the variations by applying the results of the solutions to specific cases. In Table 2 we have listed the period of oscillation, P, in years for a total mass of the system of 10^{11} solar masses and for the different initial values of R_0, namely 30, 40 and 50 kpc (e = 0.3).

TABLE 2

PERIOD OF OSCILLATION FOR DIFFERENT DIMENSION OF THE SYSTEM

R_0 (kpc)	K_1 (M⊙)	ρ_*[a] (M⊙ kpc^{-3})	P (y)
30	10^{11}	3.7×10^6	2.16×10^9
40	10^{11}	1.5×10^6	3.33×10^9
50	10^{11}	8.0×10^5	5.23×10^9

$\rho_* = 4/3\ \pi\ \rho$, where ρ is the average density of the system.

The procedure discussed above is also reasonably applicable when e = 0.5. The corresponding ellipticity is 0.87 For e = 0.5 the time of oscillation is shorter by 17%, compared to the case of e = 0.3.

VII. DISCUSSION

As was to be expected there exists a relation between the period of oscillation and the mean density of matter in the system. Figure 3 is a plot of log P versus log ρ_*. The slope of the straight line is nearly $-1/2$. Thus we have that $P^2\rho$ = constant. This relation is well known for adiabatic pulsations of Cepheid variables. What appears to be new –so far as our information goes– is the variation in time of the form of the spheroid from an oblate to a prolate one and back.

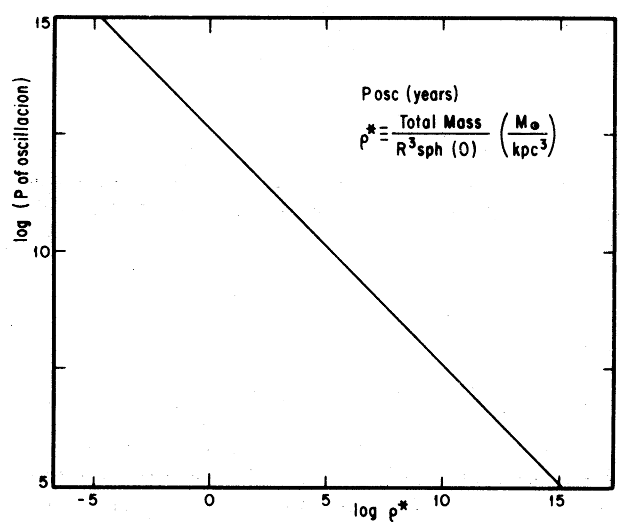

Fig. 3. The relationship between the period of oscillation, P, and the average density.

The lifetime in the oblate stage as well as in the prolate one is the same and should be slightly less than the interval between two successive spherical forms since as they approach a spherical form they are indistinguishable from it. Thus for $R_0 = 50$ kpc it is at most equal to 2×10^9 years. This is rather short compared to the age of a galaxy. Also according to our results the oblate and prolate configurations should be equally probable. It is difficult to distinguish a prolate system from an edge-on oblate one without further data than optical inspection.

It is interesting to have an estimate of the average value of the crossing time of a particle in our model. As we mentioned earlier the system of mass points is initially close to an equilibrium state: $E \approx 1/2\, W_0$ (for $e = 0.3$). An average value of the dispersion of velocities to assure an equilibrium state for the spherical case with dimensions $R = 50$ kpc —neglecting the very tenuous envelope— and for the assumed density law, is ~ 30-40 km s^{-1}. The crossing time will then be $t_c \approx R_{sph}/\sigma \sim 10^9$ years. The oscillation period is thus of the order of a few crossing times. Our treatment is thus valid within one oscillation period.

In the likely case that the onset of oscillation is due to the tidal perturbations of neighboring galaxies, the formalism presented above also enables one to calculate the evolutionary steps that the perturbed system goes through. It is worth mentioning that Bertola and Galletta (1978) have shown that five galaxies, namely NGC 1947, 5128, 5363 and those associated with Cyg A and PKS 1934-63, show prolate structure. At present we lack a systematic search to know the relative frequency of prolate and oblate systems.

The orthodox belief has it that galaxies are formed through contraction of a more extended gaseous configuration. Our results show that a system as we have envisaged does not undergo contraction. In our treatment encounters are assumed either absent or are such that the density in phase space remains invariant. It appears therefore that collisions between the particles of the system (viscosity) are needed for the system to contract and flatten if no external forces are acting. Recent numerical simulations (Brahic 1975) confirm the theoretical findings of Poincaré (1911) that a rotating system of gravitating mass points will flatten as a result of inelastic collisions. Thus the existence of subsystems or populations with differing mass distribution and kinematical properties can reasonably be explained by the process of gradual contraction if encounters (at least a fraction of them) are inelastic. The apparently smooth and monotonic distribution of the stars in E galaxies is generally believed to be the result of relaxation. But it is quite probable that the monotonic (or nearly so) distributions may have been a primeval property of the cloud, before the stars were formed as such.

Be the reason what it may, E galaxies at present seem to fulfill the initial conditions for our model to be applicable to them. In fact the only well established cases of prolate spheroids are E galaxies. They may therefore be in a transient stage. The same can be said as to the transient nature of oblate E galaxies.

The formation of a prolate configuration may have some bearing on the existence of "tri-axial" regions within elliptical galaxies and in the central regions of spirals. The tri-axial figures referred to may not be distinguishable observationally from a prolate one, or there may exist subsystems of varying eccentricities within the system as a whole. It is interesting to point out that a recent study of the geometry of NGC 3379 by Nieto and Vidal (1984) gives clear evidence to the existence of subsystems in elliptical galaxies. To attempt a model with less restrictive initial conditions might bring one closer to understanding the existence of prolate regions within an oblate system and viceversa. Some of the hypotheses to be tried are: i) a different density law which may be allowed to vary in time, ii) a larger initial eccentricity ($e > 0.5$), iii) existence of viscosity or encounters in an evolving system. The models discussed by Larson with viscosity can be reconsidered in the light of our approach.

It is desirable to determine observationally the rotation axis of the "tri-axial" region (or regions) of a galaxy with respect to the remaining parts; if the postulated tri-axial region has a prolate form, its axis of symmetry is expected to be its rotation axis and therefore that of the whole system. As yet the orientation and the axes of symmetry of elliptical galaxies are not satisfactorily determined.

Finally we like to mention that the formalism of the Tensor Virial Theorem can also be applied to study the oscillation of a globular star cluster undergoing a perturbation due to the galactic potential field or to cluster-cluster approaches.

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