ON THE VACUUM DEFINITION FOR NON-INERTIAL OBSERVERS

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RESUMEN. Se analiza la definición de vacío conforme para observadores acelerados uniformemente que siguen las trayectorias de los 9 vectores de Killing conformes en el espacio plano de Minkowski.

Se introduce la noción de fluido de referencia físico para caracterizar las trayectorias y para utilizar de manera general el método de diagonalización instantánea del Hamiltoniano. El formalismo propuesto permite sugerir la existencia de algunas hipersuperficies privilegiadas sobre las cuales existe una *buena* definición del estado de vacío. Se hace la distinción con el estado de vacío *verdadero* y se destaca la operatividad del estado $|0\rangle$ definido de esta manera para calcular $\langle T_{\mu\nu}\rangle$, el valor de expectación de vacío del tensor energía-momento que será la fuente de las ecuaciones de Einstein.

ABSTRACT. The conformal vacuum definition for non-inertial observers following the trajectories of the 9 conformal Killing vectors in Minkowski space-time is analyzed with the method of Hamiltonian diagonalization. The notion of physical fluid of reference is used to characterize the trajectories and to deduce the diagonalization condition. The existence of some privileged hypersurfaces on which a *good* vacuum definition can be made is suggested to be the first step towards a general theory of non-trivial vacua.

I. INTRODUCTION

The formulation of a theory containing naturally all the interactions present in nature has not been possible yet. The most serious difficulties are met when the unification of general relativity and quantum mechanics is faced. However, a semiclassical theory has been developed as an approach to a more general, full quantum theory, in which gravity is treated as an external, non-quantized field.

This semiclassical limit provides a reasonable frame to study other quantum fields in the presence of a strong gravitational field up to regions characterized by the Planck scale ($l_p \sim 10^{-32}$ cm, $t_p \sim 10^{-43}$ seg, $E_p \sim 10^{27}$ eV). Particle creation has been predicted when there

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is a dynamical gravitational background field (Parker 1969, Hawking 1975). This is an interesting phenomenon that has originated various cosmological and astrophysical applications (Grib and Mamaev 1970, 1972, Zel'dovich 1970, Zel'dovich and Starobinsky 1972). It might have eliminated the initial annisotropies in the early universe (Parker 1969). It could also help to avoid the classical Hawking and Penrose's hypothesis on the inevitability of singularities and, therefore, to propose cosmological models without singularities (Hawking and Penruse 1970, Hartle and Hu 1980).

However there are some difficulties which arise in the quantization of the matter fields even in flat space-time. Plane waves are undoubtedly a good solution of the scalar field equation. However a non-Cartesian space-time coordenalization will lead to a different *particle model* (Fulling 1977) or, equivalently, to different vacuum states. The Rindler-Minkowski coordinate transformation has been extensibly studied. Thermal radiation will be detected by a (uniformly accelerated) *Rindler observer* in the usual Minkowski vacuum state (Unruh 1976). Other non-inertial vacuum definitions have been studied by Brown *et al.* (1982).

Is the vacuum state observer-dependent?

In the Heisenberg picture the field is assumed to be always in the same quantum state, let us say $|0_v\rangle$; however the ideal particle detectors that have been designed (Unruh 1976, Birrell and Davies 1982) are not *objective* since they measure the modes defined with respect to its carrier's proper time. Therefore, they are not appropriate to measure *real* particles.

It has been said that local quantities such as $\langle\Psi|T_{\mu\nu}(x)|\Psi\rangle$, may lead to a more objective knowledge of the *real* state of the field. The energy-momentum tensor $T_{\mu\nu}(x)$ provides, in fact, a detailed, observer-independent physical interpretation of the quantum field theory at point x. However, as the only quantity which makes sense in a quantum theory, is $\langle T_{\mu\nu}(x)\rangle$, the problem of the vacuum definition is not avoided in this way. The state $|\Psi\rangle$ in which $\langle\Psi|T_{\mu\nu}|\Psi\rangle$ is evaluated must be specified anyway.

In section II we introduce the notion of physical fluid of reference, which will be associated in section III with each non-inertial observer in order to formulate a quantum field theory containing the observer in a natural way. This approach does not solve the problem of finding the *real* vacuum state. We are just able to point out some properties of the different possible vacuum states. However, the method might be generalized to curved space-time where all the symmetries of Minkowski space are lost and the problem becomes more complicated: the natural vacuum state in an initial static region of space-time will, in general, be a many-particle state in a final, also static, region when the universe has undergone an evolution

The formalism that will be introduced in sections II and III will be used to analyse, in a forthcoming paper, the thermal radiation of black holes (Hawking 1975) and the cosmological particle creation in universes with Robertson-Walker metrics in what intends to be a general theory of non-trivial vacuum states. The method of Hamiltonian diagonalization is reexamined and used to test the conformal vacuum definition.

II. PHYSICAL FLUID OF REFERENCE

Some of the notions introduced by Lichnerowicz (1955) and Cattaneo (1961) will be summarized in this section. They will be used in the next section to solve the field equation.

To choose a Galilean reference system in flat Minkowski space is equivalent to choose an arbitrary time direction. At each instant, from a given origin on this temporal axis, the normal 3-space becomes completely determined and represents the physical space at that instant. The physical state is easier to be visualized if it is full of infinite ideal particles rigidly tight together, called *reference particles*. The particles time directions are the lines parallel to the time axis.

We consider the space-time a globally hyperbolic Riemannian V_4 manifold as in general relativity. The metric tensor has signature (-, +, +, +). A physical reference fluid S is a congruence of lines i.e., a family of curves each associated to a point within a domain of the space-time manifold. These lines are, in fact, the time trajectories of the particles which constitute the reference fluid. We may assign three spatial coordinates (x^1, x^2, x^3) to each one of these particles which remain constant for the same particle. In a system of physically admissible local coordinates (x_0, x_1, x_2, x_3) (such that the lines x^0 = var. are oriented on the time direction and the hypersurfaces x^0 = cons. on the orthogonal 3-space) the history of the particles coincides with the coordinate line x^0 = var.

2. Locally associated space and time

Let (x^μ) be a physically admissible local coordinate system and S the corresponding reference fluid. The unitary future-directed vector $\gamma(x)$ tangent to the line x^0 = var. and normalized as

$$g_{\mu\nu}\gamma^\mu(x)\,\gamma^\nu(x) = -1 \qquad \mu,\nu = 0,1,2,3,$$

completely determines S.

In some non-generic fluid we can define the coordinate system adapted to the fluid as that in which the components of $\gamma(x)$ are

$$\begin{cases} \gamma^0 = 1/(-g_{00})^{1/2} & \\ \gamma^i = 0 & i = 1,2,3, \\ \gamma_\mu = g_{\mu 0}/(-g_{00})^{1/2} & \end{cases}$$

and the totally adapted or syncronic coordinates are those in which:

$$\begin{cases} \gamma^0 = 1 \\ \gamma^i = 0 \\ \gamma_\mu = g_{\mu 0} \end{cases}$$

Let θ_x be the one-dimensional subspace of the γ-colineal vectors in the tangent vector space at the point x, T_x and Σ_x the orthogonal 3-plane: θ_x and Σ_x are called time and space associated to the point x (Cattaneo 1961).

Any vector $V\varepsilon\, T_x$ can be decomposed univocally in the sum of two vectors $A\varepsilon\theta_x$ and $N\varepsilon\Sigma_x$

$$V = A + N \qquad (A\varepsilon\theta_x,\ N\varepsilon\Sigma_x) \quad .$$

The temporal and spatial projections can now be defined as

$$\mathcal{P}_\theta(V_\mu) \equiv A_\mu = -\gamma_\mu\gamma_0 V^\nu$$
$$\mathcal{P}_\Sigma(V_\mu) \equiv N_\mu = (g_{\mu\nu} + \gamma_\mu\gamma_\nu)V^\nu \equiv \gamma_{\mu\nu}V^\nu \quad .$$

These expressions represent the natural decomposition of a vector V in the physical reference frame, S.

We can now define the temporal norm

$$||V||_\theta = g_{\mu\nu}A^\mu A^\nu = A_\mu A^\mu = -\gamma_\mu\gamma_\nu V^\mu V^\nu \quad ,$$

and the spatial norm

$$||V||_\Sigma = g_{\mu\nu}N^\mu N^\nu = -\gamma_{\mu\nu}\, V^\mu V^\nu \quad ,$$

and it is now clearly seen that the tensors $-\gamma_\mu\gamma_\nu$ and $\gamma_{\mu\nu}$ play, respectively, the role of temporal and spatial metric tensor, a property which will be useful for deriving the field equation.

3. Projections of second rank tensors.

The following obvious notation indicates the natural projections of the tensor $t_{\mu\nu}$

$$\mathcal{P}_{\Sigma\Sigma}(t_{\mu\nu}) = \gamma_{\mu\alpha}\gamma_{\mu\beta}\ t^{\alpha\beta} \qquad \mathcal{P}_{\theta\theta}(t_{\mu\nu}) = -\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta t^{\alpha\beta} \quad ,$$
$$\mathcal{P}_{\theta\Sigma}(t_{\mu\nu}) = -\gamma_\mu\gamma_\alpha\gamma_{\nu\beta}t^{\alpha\beta} \qquad \mathcal{P}_{\Sigma\theta}(t_{\mu\nu}) = -\gamma_{\mu\alpha}\gamma_\nu\gamma_\beta t^{\alpha\beta} \quad .$$

If these definitions are applied to the metric tensor, the following interesting properties are obtained:

$$\mathcal{P}_{\Sigma\Sigma}(g_{\mu\nu}) = \gamma_{\mu\alpha}\gamma_{\mu\beta}\ g^{\alpha\beta} = \gamma_{\mu\beta}(g\nu^\beta + \gamma_\nu\gamma^\beta) = \gamma_{\mu\nu} \qquad (1\text{-a})$$

$$\mathcal{P}_{\Sigma\theta}(g_{\mu\nu}) = \mathcal{P}_{\theta\Sigma}(g_{\mu\nu}) = 0 \qquad (1\text{-b})$$

$$\mathcal{P}_{\theta\theta}(g_{\mu\nu}) = \gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta g^{\alpha\beta} = -\gamma_\alpha\gamma_\beta \qquad (1\text{-c})$$

The spatial and temporal metric tensors turn out to be the $\Sigma\Sigma$ and $\theta\theta$ projections of the space-time metric tensor $g_{\mu\nu}$.

The tensor index will be

i) completely spatial if $\gamma^\mu t \ldots \mu \ldots = 0$, or

ii) completely temporal if $\gamma^{\mu\nu} t \ldots \mu \ldots = 0$

4. Differentiation rules: transversal and longitudinal derivation.

We define the transversal or spatial derivative as:

$$\tilde{\partial}_\mu \phi = \gamma_\mu{}^\nu \, \partial_\nu \phi$$

and the longitudinal or temporal one as

$$\bar{\partial}_\mu \phi = -\gamma_\mu \gamma^0 \partial_0 \phi$$

They are obviously the spatial and temporal projections of the complete derivative

$$\partial_\mu \phi = \tilde{\partial}_\mu \phi + \bar{\partial}_\mu \phi$$

Analogously the covariant derivation can be defined as

$$\begin{aligned} \tilde{\nabla}_\mu S_\nu &= \mathcal{P}_{\Sigma\Sigma}(\nabla_\mu S_\nu) \\ &= \gamma_\mu{}^\alpha \gamma_\nu{}^\beta \, \nabla_\alpha S_\beta \\ &= \tilde{\partial}_\mu S_\nu - \tilde{\Gamma}^\alpha_{\mu\nu} \, S_\alpha \end{aligned}$$

where

$$\tilde{\Gamma}^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \, (\tilde{\partial}_\mu \gamma_{\nu\beta} + \tilde{\partial}_\nu \gamma_{\beta\mu} - \tilde{\partial}_\beta \gamma_{\mu\nu})$$

are the spatial Christoffel symbols.

The above definition is easily generalized for tensors:

$$\begin{aligned} \tilde{\nabla}_\mu S_{\theta\rho} &= \mathcal{P}_{\Sigma\Sigma\Sigma}(\nabla_\mu S_{\theta\rho}) \\ &= \tilde{\partial}_\mu \, S_{\theta\rho} - \tilde{\Gamma}^\nu_{\mu\theta} \, S_{\nu\rho} - \tilde{\Gamma}^\nu_{\mu\rho} \, S_{\theta\nu} \end{aligned}$$

If the lines x^0 defining the physical reference system S constitute a congruence with a family of orthogonal hypersurfaces V_3, this covariant transversal derivation at the point x can be identified with the ordinary covariant derivation on the hypersurface V_3 with induced metric tensor $\gamma_{\mu\nu}$ through x. However, even in the case in which the orthogonal hypersurfaces do not exist the notion of transversal derivation has a meaning.

5. Intrinsic properties of a physical reference system

As we have already mentioned, a reference fluid S can be individualized by the unitary temporal vector field $\gamma(x)$. All first order characteristics of this fluid are contained in $\nabla_\mu\gamma_\nu$. Let us decompose $\nabla_\mu\gamma_\nu$ into its symmetric and antisymmetric parts

$$\nabla_\mu \gamma_\nu = \frac{1}{2} (K_{\mu\nu} + \Omega_{\mu\nu}) \qquad ,$$

where

$$K_{\mu\nu} = \nabla_\mu \gamma_\nu + \nabla_\nu \gamma_\mu$$

is called the Killing tensor and

$$\Omega_{\mu\nu} = \nabla_\mu \gamma_\nu - \nabla_\nu \gamma_\mu$$

is the vortex tensor. Another natural vector is the curvature of the lines x^0 = var., which is defined as:

$$C_\mu = \gamma^\nu \nabla_\nu \gamma_\mu \qquad (C \in \Sigma)$$

The corresponding spatial or temporal projections of these tensors are

$$\Omega_{\mu\nu} = \tilde{\Omega}_{\mu\nu} + C_\mu \gamma_\nu - \gamma_\mu C_\nu \qquad K_{\mu\nu} = \tilde{K}_{\mu\nu} - C_\mu \gamma_\nu - \gamma_\mu C_\nu$$

$$\tilde{\Omega}_{\mu\nu} = \gamma_0 \left(\tilde{\partial}_\mu \frac{\gamma_\nu}{\gamma_0} - \tilde{\partial}_\nu \frac{\gamma_\mu}{\gamma_0}\right) \qquad \tilde{K}_{\mu\nu} = \gamma^0 \partial_0 \gamma_{\mu\nu}$$

We will say that

i) if $C_\mu = 0$, the fluid is geodesic (the lines x^0 = var. are geodesics in V_4),

ii) if $\tilde{\Omega}_{\mu\nu} = 0$, the fluid is curl-free,

iii) if $\tilde{K}_{\mu\nu} = 0$, the fluid is static or rigid.

The condition of curl-free or non-rotational fluid ii), is equivalent to

$$\gamma_\mu \Omega_{\nu\rho} + \gamma_\nu \Omega_{\rho\mu} + \gamma_\rho \Omega_{\mu\nu} = 0 \qquad (2) ,$$

which can easily be proven with a little algebra. This property guarantees the existence of ortoghonal 3-planes.

We will also use the important relation

$$\nabla_\mu \gamma_\nu = \frac{1}{2} (\tilde{K}_{\mu\nu} + \tilde{\Omega}_{\mu\nu}) - \gamma_\mu C_\nu$$

The Christoffel symbols can now be written in terms of the tensors we have introduced, as

$$\Gamma^\rho_{\mu\nu} = \tilde{\Gamma}^\rho_{\mu\nu} - \frac{1}{2} g^{\rho\theta} \{\gamma_\mu (\tilde{K}_{\nu\theta} + \Omega_{\nu\theta}) + \gamma_\nu (\tilde{K}_{\mu\theta} + \Omega_{\mu\theta}) + \gamma_\theta (Q_{\mu\nu} - \tilde{K}_{\mu\nu})\} ,$$

where the non-tensorial quantity $Q_{\mu\nu}$ is defined as

$$Q_{\mu\nu} = \partial_\mu \gamma_\nu + \partial_\nu \gamma_\mu$$

6. The scalar field equation for a given reference fluid

We will now use the above definitions to find an expression for the scalar field equation in which the role of the reference fluid is explicitly indicated. For simplicity we will work with a massless though the generalization to the $m \neq 0$ case is inmediate.

A real massless scalar field is described by the classical action

$$S[\phi] = -\frac{1}{2} \int d^4x \, (\phi_{;\mu} \phi^{;\mu} + \xi R \phi^2) .$$

A variational principle leads to the Klein-Gordon equation

$$(\Box - \xi R)\phi = 0 \qquad (3) ,$$

where $\Box = g^{\mu\nu} \nabla_\mu \partial_\nu$ and ξ is the coupling constant.

We will consider an irrotational fluid, i.e., $\tilde{\Omega}_{\mu\nu} = 0$. It can be proven that an adapted coordinate system exists and we have

$$\gamma_\mu = (g_{00}^{1/2} , 0, 0, 0)$$

Applying the definitions introduced above, expression (3) can be written as

$$(\gamma^{\mu}\gamma^{\nu} + \gamma^{\mu\nu}) \{\partial_{\mu\nu}\phi - \left[\tilde{\Gamma}^{\rho}_{\mu\nu} - \frac{1}{2} g^{\rho\theta} (\gamma_{\mu}\{\tilde{K}_{\nu\theta} + \Omega_{\nu\theta}\} + \gamma_{\nu}\{\tilde{K}_{\mu\theta} + \Omega_{\mu\theta}\} +$$

$$+ \gamma_{\theta}\{Q_{\mu\nu} - \tilde{\tilde{K}}_{\mu\nu}\})\right]\partial_{\rho}\phi\} - \xi R\phi =$$

$$= - (\gamma^{0})^{2}\partial_{00}\phi - \Delta_{S}\phi - C^{\rho}\partial_{\rho}\phi + \frac{1}{2}\left\{g^{\mu\nu}Q_{\mu\nu} - \gamma^{\mu\nu}\tilde{K}_{\mu\nu}\right\}\gamma^{0}\partial_{0}\phi - \xi R\phi = 0 \qquad (4)$$

III. CHARACTERIZATION OF NON-INERTIAL VACUUM STATES IN FLAT SPACE-TIME

We will make a systematic analysis of the natural vacuum-states for observers whose world lines are the trajectories of the conformal Killing vector fields in flat space-time and will point out some of their properties.

The notation that will be used follows.

A conformal Killing vector satisfies the equation

$$\mathcal{L}_{K^{\mu}} g_{\mu\nu} = \lambda(x)\, g_{\mu\nu} \qquad (5)$$

Condition (2) for the existence of hypersurfaces orthogonal to the vector field can be written in terms of the conformal Killing vector as

$$K_{[\mu}K_{\nu;\rho]} = 0 \quad ,$$

where now, the vector $\gamma(x)$ characterizing the fluid is obviously

$$\gamma^{\mu} = (-K^{2})^{-1/2}\, K^{\mu} \qquad (6)$$

and coincides with the trajectories of the conformal Killing vectors, all of which have constan acceleration.

The following important theorems were stated by Brown *et al.* (1982):

1. If K^{μ} is a Killing vector field of the metric $\Omega^{2}g_{\mu\nu}$, it is also a conformal Killing vector field for the metric $g_{\mu\nu}$.

2. If K^{μ} is a conformal Killing vector field for $g_{\mu\nu}$, it is also a globally tempora Killing vector field of the space-time with metric tensor $(-g_{\theta\rho}K^{\theta}K^{\rho})^{-1} g_{\mu\nu}$. Moreover, if K^{μ} is curl-free, this space-time is ultrastatic (i.e., it admits a constant Killing vector field : the norm of the temporal Killing vector g_{00} = cons.).

3. All ultrastatic conformally flat space-time is locally Minkowski, the Einstein static or the open Einstein universe.

There are 9 conformal Killing vector fields in flat space satisfying equation (5) which have already been described by Brown *et al.* (1982, 1983). Table 1 summarizes the properties of Minkowski space regions where these vector fields are temporal $\tau(K)$ is the set $K^{2} = g_{\mu\nu}K^{\mu}K^{\nu} < 0$.

Let us now think each one of these conformal Killing vector fields as a physical reference fluid such as it was introduced in the previous section. The world lines of the parti cles which constitute the fluid can be identified with the trajectories of the conformal Killin vectors.

With a little algebra the following useful relation is found (see Appendix I)

$$\tilde{K}_{\mu\nu} = (-K^{2})^{-1/2}\, \lambda(x)\gamma_{\mu\nu}$$

when K^{μ} is a conformal Killing vector.

TABLE 1

PROPERTIES OF NON-INERTIAL OBSERVERS AND THEIR CORRESPONDING VACUUM STATES IN MINKOWSKI SPACE-TIME

OBSERVER	$[-K(x)^2]$	$\lambda(x)$	TRAJECTORY	$\bar{\tau}(K)$	arc element	Σ_v
1. Minkowski	1	0	x^1=cons., x^2=cons. x^3=cons.	the whole space	$-(dx^0)^2+(dx^1)^2+(dx^2)^2+(dx^3)^2$	x =cons.
2. Rindler	ξ^{-2}	0	$(x^1)^2-(x^0)^2$=cons>0 x^2=cons., x^3=cons.	$x^2 > t^2$	$-\xi^2 dt'^2+d\xi^2+(dx^2)^2+(dx^3)^2$	t'=cons.
3. Milne	$e^{2t'}$	1	t=Ar, θ=cons. ϕ=cons.	$\|t\| > r$	$e^{2t'}\{-dt'^2+\xi^{-2}[d\xi^2+(dx^2)^2+(dx^3)^2]$	$t'=\pm\infty$
4.	$(t'^2-r'^2)^{-2}$	t'	$(r-c)^2-t^2=c^2$ θ=cons., ϕ=cons.	the whole space except the light cone of the origin	$(t'^2-r'^2)^{-2}(-dt'^2+dr'^2+r'^2d\Omega^2)$	t'=0
5.	$(t'+x')^{-2}$	2(t'+x')	$t^2-x^2-\rho^2=c(t+x)-1$ $\rho=A(t+x)$, ψ=cons. $\rho^2=y^2+z^2$	the whole space except the plane t+x = 0	$(t'+x')^{-2}(-dt'^2+dx'^2+dy'^2+dz'^2)$	none
6.	$(\cosh t'-\cosh r')^{-2}$	-2t'	$(r-c)^2-t^2=c^2-1$ θ=cons., ϕ=cons.	$\{\|t+r\|<1;\ \|t-r\|<1\}$ $\{\|t+r\|>1;\ \|t-r\|>1\}$	$\frac{1}{4}\,\mathrm{csch}^2\,\frac{1}{2}(t'+r')\,\mathrm{csch}^2\,\frac{1}{2}(t'-r')ds'^2$ $\frac{1}{4}\,\mathrm{sch}^2\,\frac{1}{2}(t'+r')\,\mathrm{sch}^2\,\frac{1}{2}(t'-r')ds'^2$ $ds'^2= -dt'^2+dr'^2+\sinh^2 r'\ d\Omega^2$	t'=0
7.	$(\sinh x'-\sinh t')^{-2}$	x'	$[x^2-(1-A^2)^{1/2}t+c]^2=$ $=1-c^2$ ψ=cons., ρ =At $\|A\|<1$, $\|c\|<1$	the union of the interiors of the cones $t^2=\rho^2+(x\pm1)^2$ minus their inters.	$\frac{1}{4}\,\mathrm{sch}^2(x'+t')\,\mathrm{csch}^2\,\frac{1}{2}(x'-t')ds'^2$ $ds'^2= -dt'^2+\xi^{-2}(d\xi^2+d\rho^2+\rho^2 d\psi^2)$	none
8.	(cost'+cosx')	2t'	$(r-c)^2-t^2=1+c^2$ $c\leq\infty$ θ =cons., ϕ =cons.	the whole space	$\frac{1}{4}\sec^2\frac{1}{2}(t'+r')\sec^2(t'-r')ds'^2$ $ds'^2= -dt'^2+dr'^2+\sin^2 r'\ d\Omega^2$	t'=0,¶,2¶
9.	$\xi^{-2}(\xi e^{-\tau}-1)^2$	4(1+t+x)	$\rho=A(t+x)$; ψ=cons. (t-x)(t+x+1)= =(t+x)$[B+A^2(t+x+$ $+1)]$	the intersection of the half sp. t+x+1>0 with the past light cone of the origin	$\xi^2(\xi e^{-2}-1)$. $\cdot[-d\tau^2+\xi^2(d\xi^2+d\rho'^2+\rho'^2 d\psi^2)$	$\tau = \infty$

Introducing now this expression into equation (4) it turns out that

$$\partial_{00}\phi + (-K^2)\Delta_S\phi - (\frac{3}{2}\lambda - \gamma^0\partial_0\gamma_0)\partial_0\phi - C^\rho\partial_\rho\phi = 0 \quad (7)$$

where $\gamma^{ij}\gamma_{ij} = 3$, R = 0 and theorem 3, written in the form

$$\eta_{\mu\nu} = (-K^2)g'_{\mu\nu}$$

have been used to find an easier expression ($g'_{\mu\nu}$ is one of the three ultrastatic Minkowski, static or open Einstein universes).

There is not a general solution to this and being $\lambda(x) = \lambda(x^0, x^i)$ it cannot be solved by separation of variables.

It can be shown (Brown *et al.* 1982) using theorem 2 that these three spaces $g'_{\mu\nu}$ possess globally temporal Killing vector fields:

$$K'_\mu = g'_{\mu\nu}K^\nu \quad (8)$$

As it has already been said (Unruh 1976) in this case there exists a natural definition of positive frequency modes with respect to these Killing vectors having the temporal dependence exp ($\pm i\omega t$). The corresponding vacuum state is the ground state of an Hamiltonian defined as

$$H = \int_\Sigma d\Sigma\ T_{00} \quad ,$$

where

$$T_{\mu\nu} = (1-2\xi)\phi_{;\mu}\phi_{;\nu} + (2\xi - \frac{1}{2})\, g_{\mu\nu}g^{\rho\sigma}\phi_{;\rho}\phi_{;\sigma} - 2\xi\phi_{;\mu\nu}\phi + \frac{1}{2}\,\xi g_{\mu\nu}\phi\,\Box\,\phi$$

$$- \xi\left[R_{\mu\nu} - \frac{1}{2}\,Rg_{\mu\nu} + \frac{3}{2}\,\xi Rg_{\mu\nu}\right]\phi^2 + \frac{1}{2}\,(1-3\xi)m^2g_{\mu\nu}\phi^2$$

The solutions to equation (7) that we can assume to have more physical content are are the *conformal modes*. If ϕ' is the ultrastatic solution, then

$$\phi = (-K^2)^{-1/2}\phi' \quad (9)$$

Brown *et al.* (1982, 1983) have presented a systematic analysis of the conformal Feynman propagators corresponding to the 9 conformal Killing vector fields mentioned above and they obtained an interesting compact expression to evaluate the energy-momentum tensor in terms of K^μ. However, we are interested in the physical properties of the conformal vacuum states as they can help to clarify the role played by the different observers more than the conformal structure of the regions $\tau(K)$. We do not want to go out of Minkowki's space. In this sense we apply a highly physical method to test the *conformal modes*.

2. Hamiltonian diagonalization

This criterium has been used in various opportunities to select vacuum states (Mamaev *et al.* 1976, Castagnino *et al.* 1975, Fulling 1979). It has also been severely questioned as it presents ambiguities when there is an annisotropic background geometry (Fulling 1979). Moreover, it predicts a massive particle creation whith finite energy density when the scalar field is conformally coupled to the gravitational field, although the energy density is infinite when other couplings are used (Unruh 1976, Fulling 1979).

However, recent works favour the conformal coupling even for massive fields (Marinov 1980, Nelson and Panangaden 1982). For example, a scalar field with quartic autointeraction can only be renormalized if it is conformally coupled to the curvature at least in the small autointeraction limit (Nelson and Panangaden 1982). Therefore, it seems a reasonable task to regain confidence in the method of Hamiltonian diagonalization. Moreover, as we are now working with a massless field in flat space this criterium can hardly be objected.

We call it instantaneous diagonalization as the Cauchy data which diagonalize the Hamiltonian on a given hypersurface could not coincide with those sharing the same property at any other instant. We will use a general expression for the Hamiltonian and the mathematical method used to diagonalize it will contain all the 9 observers in an equivalent way. However, it must be noticed that the introduction of the reference fluid is a natural way to understand that the hypersurfaces on which the Hamiltonian is defined are those orthonormal to the observer's trajectories and, therefore, they are different in each case. The conformal modes (9) should also be normalized on these particular surfaces.

Using (7) we can write

$$H = \frac{1}{2}\int d\Sigma \, \{\phi^2_{,0} - g_{00}\gamma^{ij}\phi_{,i}\phi_{,j}\}$$

$$= \frac{1}{2}\int d\Sigma \, \{\phi^2_{,0} + \gamma^{ij\,\prime}\phi_{,i}\phi_{,j}\} \tag{10}$$

The expression

$$\nabla_i(\phi\gamma^{ij}\phi_{,j}) = -\nabla_S\phi + \gamma^{ij}\phi_{,i}\phi_{,j} \tag{11}$$

holds in every space-time. The following integral performed on a compact surface is always null

$$\int_\Sigma d\Sigma \, (\nabla_i Y^i) = 0 \tag{12}$$

From (11) and (12) we can, therefore, write

$$H = \frac{1}{2}\int d\Sigma \, \{\phi^2_{,0} + \phi\Delta'_S\phi\} \tag{13}$$

Second-quantizing the field into its positive and negative frequency parts

$$\Phi = \int d^3\vec{k} \, (a_{\vec{k}}\,\phi_{\vec{k}} + a^+_{\vec{k}}\,\phi^*_{\vec{k}})$$

and taking into account the eigenvalue equation

$$\Delta'_S f_{\vec{k}} = \varepsilon^2_{\vec{k}} f_{\vec{k}}$$

where $\varepsilon^2_{\vec{k}}$ is the eignevalue that corresponds to the autofunction $f_{\vec{k}}$, it can easily be proven that the Cauchy data diagonalizing the Hamiltonian (13), are

$$\phi_{\vec{k}} = \frac{e^{i\alpha}}{\sqrt{2\varepsilon_{\vec{k}}}} f_{\vec{k}} \quad , \quad \phi^*_{\vec{k}} = \frac{e^{-i\alpha}}{\sqrt{2\varepsilon_{\vec{k}}}} f^*_{\vec{k}}$$

$$\phi_{\vec{k}} = i\sqrt{\frac{\varepsilon_{\vec{k}}}{2}}\, e^{i\alpha} f_{\vec{k}} \quad , \quad \phi^*_{\vec{k}} = -i\sqrt{\frac{\varepsilon_{\vec{k}}}{2}}\, e^{-i\alpha} f^*_{\vec{k}} \tag{14}$$

If we take the *conformal modes* (9) and their derivatives and evaluate them at a fixed time, it is immediately seen that they verify the Cauchy data (14) only on the hypersurfaces satisfying equation

$$\partial_0\left[(-K^2)^{-1/2}\right] = 0 \tag{15}$$

where ∂_0 denotes derivation with respect to the time coordinate x^0.

Table 1 includes in its last column the surfaces, called Σ_V, where condition (15) i satisfied. Figures 1, 2, 3 and 4 show the observer's trajectories in the most simple cases and the orthonormal hypersurfaces are also indicated.

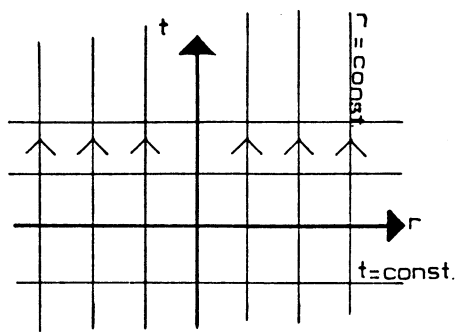

Fig. 1. Temporal future-directed trajectories of Minkowski Killing vector field. Horizontal lines indicate the {t = cons.} hypersurfaces.

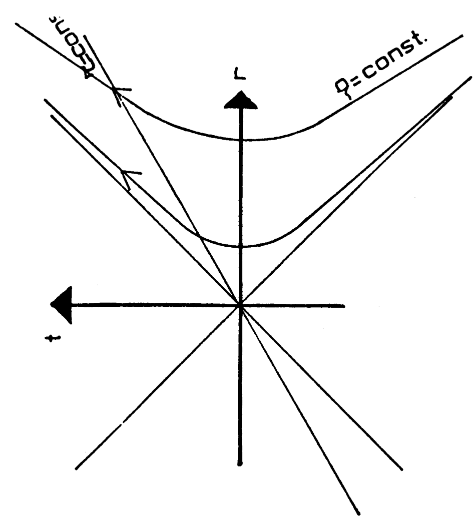

Fig. 2. Temporal future-directed trajectories of the Rindler observer.{ τ = cons.} lines indicate the orthonormal hypersurfaces.

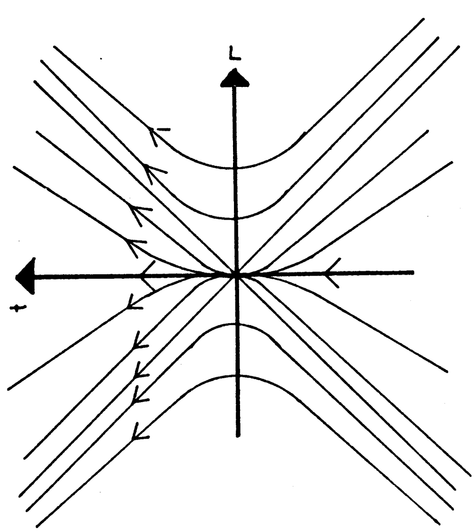

Fig. 4. Temporal future-directed trajectories of the observer following the orbits of K_3.

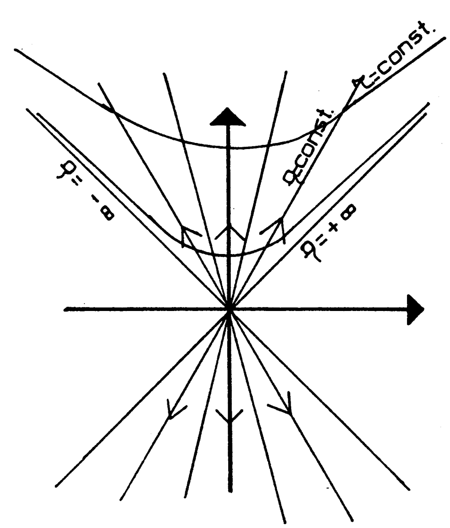

Fig. 3. Temporal future-directed trajectories of the Milne observer. {τ = cons.} hypersurfaces are orthonormal to the trajectories.

V. CONCLUSIONS

The results we have obtained display interesting features of the conformal vacuum states.

In the first place, the fact that *good* vacuum states exist only in some particular hypersurfaces does not mean that there will be particle creation between the surfaces verifying equation (15) for each one of the 9 cases. Indeed the *good* particle model is the same on every *privileged* surface in each case, i.e., the *conformal modes*. Therefore, there will not be particle creation as could be expected from the conformal triviality of the situation. The Bogoliubov transformation will yield $\beta(\tau, \tau') = 0$.

The analysis we have made suggests that a *good* vacuum definition could be reasonably expected on some surfaces though not on the whole space-time. Notice that we do not refer to the *real* vacuum in the sense mentioned at the Introduction. This fact is not disturbing as it is usually supposed in the literature that a *good* vacuum definition is only possible in some particular or trivial cases, such as adiabatic *in* or *out* regions (Parker 1969). The surfaces verifying equation (15) are those in which K^{μ} is an *instantaneously* temporal Killing vector and then the situation fits into the trivial case referred to in the preceding section (equation (8).

APPENDIX I

Using equation (6) it can be written that

$$K_{\mu\nu} = -\frac{1}{(-K^2)^{1/2}}\left(\frac{\partial K}{\partial x^{\mu}}\gamma_{\nu} + \gamma_{\mu}\frac{\partial K}{\partial x^{\nu}}\right) + (-K^2)^{-1/2}\lambda g_{\mu\nu} \quad .$$

Now being

$$\tilde{K}_{\mu\nu} = \wp_{\Sigma\Sigma}(K_{\mu\nu})$$

and

$$\wp_{\Sigma}(\gamma_{\mu}) = 0 \quad , \quad \wp_{\Sigma\Sigma}(g_{\mu\nu}) = \gamma_{\mu\nu} \quad ,$$

it obviously turns out that

$$\tilde{K}_{\mu\nu} = (-K^2)^{-1/2}\lambda\,\gamma_{\mu\nu} \quad .$$

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