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A FAST AND RELIABLE METHOD FOR COMPUTING FREE-BOUND EMISSION COEFFICIENTS FOR HYDROGENIC IONS

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RESUMEN

Se presenta una fórmula aproximada para el cálculo del coeficiente de emisión libre-ligado en el caso de iones hidrogenoides. La aproximación se obtiene mediante una manipulación del factor de Gaunt (libre-ligado) que intencionalmente distingue la dependencia en la temperatura y en la composición iónica. Las pruebas numéricas indican que la fórmula así derivada es muy precisa, rápida y fácil de usar, características que convierten al cálculo de la contribución libre-ligado de una región ionizada con temperatura y composición iónica variables en una tarea simple y breve.

ABSTRACT

An approximate formula for the computation of the free-bound emission coefficient for hydrogenic ions is presented. The approximation is obtained through a manipulation of the (free-bound) Gaunt factor which intentionally distinguish the dependence on frequency from the dependence on temperature and ionic composition. Numerical tests indicate that the derived formula is very precise, fast and easy to use, making the calculation of the free-bound contribution from an ionized region of varying temperature and ionic composition a very simple and time saving task.

Key words: **EMISSION COEFFICIENTS – PHYSICAL PROCESSES**

I. INTRODUCTION

The calculation of the continuous emission by an ionized region requires the integration, along the line of sight, of the transfer equation. One therefore needs the corresponding emission and absorption coefficients as functions of the local electron density, temperature and frequency. For thermal processes the emission and absorption coefficients are related through Kirckhoff's law. Free-free and free-bound transitions are among the most relevant processes for the continuous emission in ionized nebulae. The actual computation of the emission coefficients for these processes is usually performed using the semiclassical Kramers formula corrected by the quantum-mechanical, or Gaunt, factor. Proper calculations of the Gaunt factors using hypergeometric functions are those by Karzas and Latter (1961), Brussaard and van de Hulst (1962) and Gayet (1970). Menzel and Pekeris (1935) and Burgess (1958) have also given approximate formulae for the Gaunt factors which are easy to use and precise.

Several authors (see for instance Sibille, Lunel and Bergeat 1974; Waters and Lamers 1984) have calculated the continuous (free-bound and free-free) emission coefficients or Gaunt factors over a wide range of frequencies, temperatures and ionic abundances. These results, however, cannot be easily used in the calculation of the emission by an ionized region of varying temperature. Also, the emission in frequencies other than those tabulated by these authors have to be estimated by interpolation over the available computations.

It is the purpose of this paper to present an approximate but reliable formula for the free-bound emission coefficient of an ionized plasma (in the hydrogenic approximation), which explicitly shows its dependence on frequency, temperature and ionic composition. This formula is mainly intended to be used in those calculations where temperature, density and ionic composition vary along the line of sight and thus require a large amount of computations. In §II the formula is derived and compared to exact computations; while in §III the conclusions are given.

II. THE APPROXIMATE FORMULA

The free-free and free-bound continuum energy emitted per unit solid angle, unit time and unit frequency interval by free electrons colliding with positive ions of net charge Z in the hydrogenic approximation can be written as (see for example Brussaard and van de Hulst 1962)

$$j_\nu = \frac{1}{4\pi} N_+ N_e \frac{32Z^2 e^4 h}{3m_e^2 c^3} \left[\frac{\pi h\nu_0}{3KT}\right]^{1/2} \exp(-h\nu/KT) \left[\bar{g}(\nu, T) + f(\nu, T)\right] , \quad (1)$$

where $\bar{g}(\nu,T)$ is the Gaunt factor averaged over a Maxwellian distribution at temperature T for free-free emission; and $f(\nu,T)$ stands for the free-bound emission and is defined by

$$f(\nu, T) \equiv 2\frac{h\nu_0 Z^2}{KT} \sum_{n=m}^{\infty} n^{-3} \exp(h\nu_0 Z^2/n^2 KT)\, g_n(\nu) , \quad (2)$$

where m (the principal quantum number of the lower bound level for which emission at the frequency ν can occur) is given by $m-1 < (\nu_0 Z^2/\nu)^{1/2} < m$; and $g_n(\nu)$ is the Gaunt factor for the transition of a free electron with energy

$$-h\nu_0 Z^2/n^2 + h\nu$$

to the n^{th} bound level. The other symbols have their usual meaning, with $h\nu_0$ being the hydrogen ionization potential from its ground state.

For hydrogenic atoms the $g_n(\nu)$ free-bound Gaunt factors can be approximated by (Menzel and Pekeris 1935; Burgess 1958; Seaton 1960; Aller and Liller 1968; Sibille, Lunel and Bergeat 1974)

$$g_n(\nu) = 1 + 0.1728\left[\left(\frac{\nu}{\nu_0 Z^2}\right)^{1/3} - 2n^{-2}\left(\frac{\nu}{\nu_0 Z^2}\right)^{-2/3}\right] - 0.0496\left[\left(\frac{\nu}{\nu_0 Z^2}\right)^{2/3} - \frac{2}{3}n^{-2}\left(\frac{\nu}{\nu_0 Z^2}\right)^{-1/3} + \frac{2}{3}n^{-4}\left(\frac{\nu}{\nu_0 Z^2}\right)^{-4/3}\right] \quad (3)$$

Substitution of equation (3) in equation (2) gives,

$$f(\nu, T) = 2\frac{h\nu_0 Z^2}{KT}\left\{\left[1 + 0.1728\left(\frac{\nu}{\nu_0 Z^2}\right)^{1/3} - 0.0496\left(\frac{\nu}{\nu_0 Z^2}\right)^{2/3}\right] A_m + \left[0.0331\left(\frac{\nu}{\nu_0 Z^2}\right)^{-1/3} - 0.3456\left(\frac{\nu}{\nu_0 Z^2}\right)^{-2/3}\right] B_m - \left[0.0331\left(\frac{\nu}{\nu_0 Z^2}\right)^{-4/3}\right] C_m\right\} ; \quad (4)$$

where,

$$A_m(T/Z^2) \equiv \sum_{n=m}^{\infty} n^{-3} \exp(h\nu_0 Z^2/n^2 KT)$$

$$B_m(T/Z^2) \equiv \sum_{n=m}^{\infty} n^{-5} \exp(h\nu_0 Z^2/n^2 KT) \quad (5)$$

$$C_m(T/Z^2) \equiv \sum_{n=m}^{\infty} n^{-7} \exp(h\nu_0 Z^2/n^2 KT)$$

are functions of T/Z^2 only (for a given m).

Thus, substitution of equation (4) in the second part of equation (1) gives the exact emission coefficient (under the hydrogenic approximation) for free-bound transitions in a way which explicitly distinguishes the temperature dependence from the frequency one.

Tables 1 (Z = 1) and 2(Z = 2) give the values of A_m, B_m and C_m for m = 1,2, ..., 50 as a function of temperature, obtained by summing up to n = 5000. The wavelengths in the second column correspond to the upper limit of the interval in which the given value is to be used.

As we see the calculations of A_m, B_m and C_m involve infinite summations. Thus, their evaluation in problems where temperature and ionic composition vary along the line of sight becomes a rather time consuming task. A reliable approximation which does not require so many calculations is therefore desired. Such an approximation can be obtained in the customary way through

$$
\left.
\begin{aligned}
A_m(T/Z^2) &\approx \sum_{n=m}^{\ell-1} n^{-3} \exp(h\nu_0 Z^2/n^2 KT) + \int_{\ell}^{\infty} x^{-3} \exp(h\nu_0 Z^2/x^2 KT)\, dx \approx \\
&\approx \sum_{n=m}^{\ell-1} n^{-3} \exp(h\nu_0 Z^2/n^2 KT) + \frac{1}{2}\left(\frac{KT}{h\nu_0 Z^2}\right)\left[\exp(h\nu_0 Z^2/\ell^2 KT) - 1\right] , \\
B_m(T/Z^2) &\approx \sum_{n=m}^{\ell-1} n^{-5} \exp(h\nu_0 Z^2/n^2 KT) + \int_{\ell}^{\infty} x^{-5} \exp(h\nu_0 Z^2/x^2 KT)\, dx \approx \\
&\approx \sum_{n=m}^{\ell=1} n^{-5} \exp(h\nu_0 Z^2/n^2 KT) + \frac{1}{2}\left(\frac{KT}{h\nu_0 Z^2}\right)^2 \left\{ \frac{h\nu_0 Z^2}{\ell^2 KT} - 1 \right] \exp(h\nu_0 Z^2/\ell^2 KT) + 1 \right\} , \\
C_m(T/Z^2) &\approx \sum_{n=m}^{\ell-1} n^{-7} \exp(h\nu_0 Z^2/n^2 KT) + \int_{\ell}^{\infty} x^{-7} \exp(h\nu_0 Z^2/x^2 KT)\, dx \approx \\
&\approx \sum_{n=m}^{\ell-1} n^{-7} \exp(h\nu_0 Z^2/n^2 KT) + \frac{1}{2}\left(\frac{KT}{h\nu_0 Z^2}\right)^3 \left\{ \left[\left(\frac{h\nu_0 Z^2}{\ell^2 KT}\right)^2 - 2\left(\frac{h\nu_0 Z^2}{\ell^2 KT}\right) + \right.\right. \\
&\qquad \left.\left. + 2\right] \exp(h\nu_0 Z^2/\ell^2 KT) - 2 \right\} ;
\end{aligned}
\right\} \qquad (6)
$$

where ℓ is an integer greater than m + 1; the greater ℓ, the better the approximation. Numerical tests indicate that for ℓ = m + 3, equation (4) with A_m, B_m and C_m calculated using equation (6) gives an approximation for $f(\nu,T)$ better than 5% for Z = 1 and better than 6% for Z = 2 in the temperature range $T \geqslant 1.25 \times 10^3$ °K.

If the terms involving B_m and C_m are left out when computing $f(\nu,T)$, the errors are less than 23% if $\lambda > 912$ A and less than 10% if $\lambda > 1.0$ μm, for Z = 1 and $T \geqslant 1.25 \times 10^3$ °K. For Z = 2 the errors are 18% and 10% respectively. This approximation is similar to that used by Waters and Lamers (1984).

Tha mass absorption coefficient (Mihalas 1970) may also be calculated from

$$
a_\nu(b-f) = \frac{2f_H}{\mu' m_H U_H(T)} \frac{64\pi^2 \mu_H^2 e^{10}}{3\sqrt{3} m c h^6 \nu^3} \frac{KT}{2h\nu_0 Z^2} \exp(-h\nu/KT)\, f(\nu, T) ; \qquad (7)
$$

where $f(\nu,T)$ is given by equation (4).

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TABLE 1*a*

A_m VALUES FOR Z = 1 CALCULATED BY SUMMING FROM n = m UP TO n = 5000 IN EQUATION (5)

m	λ (μm)	T (°K) 5.0E03	7.5E03	1.0E04	1.25E04	1.5E04	1.75E04	2.0E04	∞
1	9.113E-02	5.178E+13	1.389E+09	7.196E+06	3.060E+05	3.727E+04	8.287E+03	2.684E+03	1.233E+00
2	3.645E-01	3.367E+02	2.462E+01	6.767E+00	3.158E+00	1.918E+00	1.352E+00	1.044E+00	2.221E-01
3	8.201E-01	1.406E+00	4.845E-01	2.924E-01	2.186E-01	1.812E-01	1.589E-01	1.443E-01	9.095E-02
4	1.458E+00	1.693E-01	1.003E-01	7.836E-02	6.793E-02	6.189E-02	5.798E-02	5.525E-02	5.022E-02
5	2.278E+00	5.681E-02	4.207E-02	3.645E-02	3.352E-02	3.173E-02	3.052E-02	2.965E-02	3.221E-02
6	3.281E+00	2.851E-02	2.350E-02	2.140E-02	2.026E-02	1.954E-02	1.904E-02	1.868E-02	2.257E-02
7	4.465E+00	1.738E-02	1.519E-02	1.422E-02	1.368E-02	1.334E-02	1.310E-02	1.292E-02	1.677E-02
8	5.832E+00	1.183E-02	1.071E-02	1.020E-02	9.910E-03	9.722E-03	9.591E-03	9.494E-03	1.298E-02
9	7.381E+00	8.632E-03	7.996E-03	7.701E-03	7.531E-03	7.420E-03	7.342E-03	7.284E-03	1.037E-02
10	9.113E+00	6.606E-03	6.217E-03	6.034E-03	5.927E-03	5.858E-03	5.809E-03	5.772E-03	8.483E-03
11	1.103E+01	5.235E-03	4.983E-03	4.863E-03	4.793E-03	4.747E-03	4.714E-03	4.690E-03	7.076E-03
12	1.312E+01	4.260E-03	4.089E-03	4.007E-03	3.959E-03	3.927E-03	3.905E-03	3.888E-03	5.996E-03
13	1.540E+01	3.539E-03	3.419E-03	3.361E-03	3.327E-03	3.305E-03	3.289E-03	3.277E-03	5.148E-03
14	1.786E+01	2.991E-03	2.904E-03	2.861E-03	2.837E-03	2.820E-03	2.808E-03	2.800E-03	4.470E-03
15	2.050E+01	2.562E-03	2.498E-03	2.466E-03	2.448E-03	2.436E-03	2.427E-03	2.420E-03	3.918E-03
16	2.333E+01	2.221E-03	2.173E-03	2.149E-03	2.134E-03	2.125E-03	2.118E-03	2.113E-03	3.464E-03
17	2.634E+01	1.945E-03	1.907E-03	1.889E-03	1.878E-03	1.871E-03	1.866E-03	1.862E-03	3.085E-03
18	2.953E+01	1.718E-03	1.689E-03	1.674E-03	1.665E-03	1.660E-03	1.656E-03	1.652E-03	2.765E-03
19	3.290E+01	1.529E-03	1.506E-03	1.494E-03	1.487E-03	1.482E-03	1.479E-03	1.477E-03	2.493E-03
20	3.645E+01	1.370E-03	1.351E-03	1.342E-03	1.336E-03	1.332E-03	1.330E-03	1.328E-03	2.259E-03
21	4.019E+01	1.235E-03	1.219E-03	1.212E-03	1.207E-03	1.204E-03	1.202E-03	1.200E-03	2.057E-03
22	4.411E+01	1.119E-03	1.106E-03	1.100E-03	1.096E-03	1.093E-03	1.092E-03	1.090E-03	1.881E-03
23	4.821E+01	1.019E-03	1.008E-03	1.003E-03	9.996E-04	9.975E-04	9.960E-04	9.949E-04	1.727E-03
24	5.249E+01	9.313E-04	9.224E-04	9.180E-04	9.154E-04	9.136E-04	9.124E-04	9.115E-04	1.591E-03
25	5.695E+01	8.549E-04	8.474E-04	8.437E-04	8.414E-04	8.400E-04	8.389E-04	8.381E-04	1.470E-03
26	6.160E+01	7.876E-04	7.812E-04	7.780E-04	7.761E-04	7.749E-04	7.740E-04	7.733E-04	1.363E-03
27	6.643E+01	7.280E-04	7.225E-04	7.198E-04	7.182E-04	7.171E-04	7.163E-04	7.157E-04	1.267E-03
28	7.144E+01	6.749E-04	6.702E-04	6.679E-04	6.665E-04	6.655E-04	6.649E-04	6.644E-04	1.181E-03
29	7.664E+01	6.275E-04	6.234E-04	6.214E-04	6.202E-04	6.194E-04	6.188E-04	6.184E-04	1.104E-03
30	8.201E+01	5.849E-04	5.814E-04	5.796E-04	5.785E-04	5.778E-04	5.773E-04	5.770E-04	1.033E-03
31	8.757E+01	5.465E-04	5.434E-04	5.419E-04	5.410E-04	5.404E-04	5.399E-04	5.396E-04	9.699E-04
32	9.331E+01	5.119E-04	5.091E-04	5.078E-04	5.070E-04	5.064E-04	5.061E-04	5.058E-04	9.119E-04
33	9.924E+01	4.804E-04	4.780E-04	4.768E-04	4.761E-04	4.756E-04	4.753E-04	4.750E-04	8.590E-04
34	1.053E+02	4.517E-04	4.496E-04	4.486E-04	4.479E-04	4.475E-04	4.472E-04	4.470E-04	8.106E-04
35	1.116E+02	4.256E-04	4.237E-04	4.228E-04	4.222E-04	4.218E-04	4.216E-04	4.214E-04	7.662E-04
36	1.181E+02	4.017E-04	4.000E-04	3.991E-04	3.986E-04	3.983E-04	3.981E-04	3.979E-04	7.254E-04
37	1.248E+02	3.797E-04	3.782E-04	3.774E-04	3.770E-04	3.767E-04	3.765E-04	3.763E-04	6.877E-04
38	1.316E+02	3.595E-04	3.581E-04	3.575E-04	3.571E-04	3.568E-04	3.566E-04	3.565E-04	6.529E-04
39	1.386E+02	3.409E-04	3.397E-04	3.391E-04	3.387E-04	3.384E-04	3.383E-04	3.381E-04	6.207E-04
40	1.458E+02	3.237E-04	3.226E-04	3.220E-04	3.217E-04	3.215E-04	3.213E-04	3.212E-04	5.908E-04
41	1.532E+02	3.077E-04	3.067E-04	3.062E-04	3.059E-04	3.057E-04	3.056E-04	3.055E-04	5.630E-04
42	1.607E+02	2.929E-04	2.920E-04	2.916E-04	2.913E-04	2.911E-04	2.910E-04	2.909E-04	5.372E-04
43	1.685E+02	2.792E-04	2.784E-04	2.780E-04	2.777E-04	2.776E-04	2.775E-04	2.774E-04	5.131E-04
44	1.764E+02	2.664E-04	2.657E-04	2.653E-04	2.651E-04	2.649E-04	2.648E-04	2.647E-04	4.905E-04
45	1.845E+02	2.545E-04	2.538E-04	2.535E-04	2.532E-04	2.531E-04	2.530E-04	2.529E-04	4.695E-04
46	1.928E+02	2.433E-04	2.427E-04	2.424E-04	2.422E-04	2.421E-04	2.420E-04	2.419E-04	4.497E-04
47	2.013E+02	2.329E-04	2.323E-04	2.320E-04	2.319E-04	2.318E-04	2.317E-04	2.316E-04	4.312E-04
48	2.100E+02	2.231E-04	2.226E-04	2.223E-04	2.222E-04	2.221E-04	2.220E-04	2.220E-04	4.138E-04
49	2.188E+02	2.140E-04	2.135E-04	2.132E-04	2.131E-04	2.130E-04	2.129E-04	2.129E-04	3.974E-04
50	2.278E+02	2.053E-04	2.049E-04	2.047E-04	2.045E-04	2.045E-04	2.044E-04	2.043E-04	3.820E-04

TABLE 1*b*

B_m VALUES FOR Z = 1 CALCULATED BY SUMMING FROM n = m UP TO n = 5000 IN EQUATION (5)

m	λ (μm)	T (°K) 5.0E03	7.5E03	1.0E04	1.25E04	1.5E04	1.75E04	2.0E04	∞
1	9.113E-02	5.178E+13	1.389E+09	7.196E+06	3.060E+05	3.727E+04	8.286E+03	2.683E+03	1.038E+00
2	3.645E-01	8.397E+01	6.081E+00	1.646E+00	7.547E-01	4.502E-01	3.119E-01	2.371E-01	3.733E-02
3	8.201E-01	1.462E-01	4.749E-02	2.738E-02	1.978E-02	1.596E-02	1.371E-02	1.225E-02	5.871E-03
4	1.458E+00	8.696E-03	4.805E-03	3.597E-03	3.031E-03	2.707E-03	2.499E-03	2.354E-03	1.667E-03
5	2.278E+00	1.668E-03	1.165E-03	9.771E-04	8.804E-04	8.217E-04	7.824E-04	7.543E-04	6.470E-04
6	3.281E+00	5.359E-04	4.221E-04	3.753E-04	3.500E-04	3.342E-04	3.233E-04	3.155E-04	3.041E-04
7	4.465E+00	2.268E-04	1.913E-04	1.759E-04	1.673E-04	1.619E-04	1.581E-04	1.553E-04	1.624E-04
8	5.832E+00	1.134E-04	9.989E-05	9.380E-05	9.034E-05	8.812E-05	8.657E-05	8.542E-05	9.494E-05
9	7.381E+00	6.345E-05	5.749E-05	5.474E-05	5.317E-05	5.214E-05	5.143E-05	5.090E-05	5.941E-05
10	9.113E+00	3.844E-05	3.553E-05	3.416E-05	3.337E-05	3.286E-05	3.250E-05	3.223E-05	3.917E-05
11	1.103E+01	2.472E-05	2.318E-05	2.245E-05	2.203E-05	2.175E-05	2.155E-05	2.141E-05	2.692E-05
12	1.312E+01	1.666E-05	1.579E-05	1.538E-05	1.514E-05	1.498E-05	1.486E-05	1.478E-05	1.914E-05
13	1.540E+01	1.166E-05	1.114E-05	1.089E-05	1.075E-05	1.065E-05	1.058E-05	1.053E-05	1.400E-05
14	1.786E+01	8.412E-06	8.092E-06	7.937E-06	7.846E-06	7.786E-06	7.743E-06	7.711E-06	1.049E-05
15	2.050E+01	6.228E-06	6.022E-06	5.922E-06	5.863E-06	5.824E-06	5.796E-06	5.776E-06	8.015E-06
16	2.333E+01	4.712E-06	4.576E-06	4.509E-06	4.470E-06	4.444E-06	4.425E-06	4.412E-06	6.235E-06
17	2.634E+01	3.633E-06	3.540E-06	3.495E-06	3.468E-06	3.450E-06	3.438E-06	3.428E-06	4.925E-06
18	2.953E+01	2.848E-06	2.783E-06	2.751E-06	2.732E-06	2.720E-06	2.711E-06	2.704E-06	3.944E-06
19	3.290E+01	2.264E-06	2.218E-06	2.196E-06	2.182E-06	2.173E-06	2.167E-06	2.162E-06	3.196E-06
20	3.645E+01	1.824E-06	1.790E-06	1.774E-06	1.764E-06	1.757E-06	1.753E-06	1.749E-06	2.619E-06
21	4.019E+01	1.485E-06	1.461E-06	1.449E-06	1.441E-06	1.437E-06	1.433E-06	1.431E-06	2.166E-06
22	4.411E+01	1.222E-06	1.204E-06	1.195E-06	1.189E-06	1.186E-06	1.183E-06	1.181E-06	1.808E-06
23	4.821E+01	1.015E-06	1.001E-06	9.943E-07	9.902E-07	9.874E-07	9.855E-07	9.840E-07	1.521E-06
24	5.249E+01	8.503E-07	8.396E-07	8.342E-07	8.311E-07	8.289E-07	8.274E-07	8.263E-07	1.289E-06
25	5.695E+01	7.177E-07	7.093E-07	7.052E-07	7.027E-07	7.010E-07	6.999E-07	6.990E-07	1.100E-06
26	6.160E+01	6.099E-07	6.034E-07	6.001E-07	5.982E-07	5.969E-07	5.960E-07	5.953E-07	9.440E-07
27	6.643E+01	5.218E-07	5.166E-07	5.140E-07	5.124E-07	5.114E-07	5.107E-07	5.101E-07	8.150E-07
28	7.144E+01	4.490E-07	4.448E-07	4.428E-07	4.415E-07	4.407E-07	4.401E-07	4.397E-07	7.074E-07
29	7.664E+01	3.885E-07	3.851E-07	3.835E-07	3.825E-07	3.818E-07	3.814E-07	3.810E-07	6.170E-07
30	8.201E+01	3.379E-07	3.352E-07	3.338E-07	3.330E-07	3.325E-07	3.321E-07	3.318E-07	5.406E-07
31	8.757E+01	2.952E-07	2.930E-07	2.919E-07	2.913E-07	2.908E-07	2.905E-07	2.903E-07	4.757E-07
32	9.331E+01	2.591E-07	2.573E-07	2.564E-07	2.559E-07	2.555E-07	2.552E-07	2.551E-07	4.203E-07
33	9.924E+01	2.284E-07	2.269E-07	2.261E-07	2.257E-07	2.254E-07	2.252E-07	2.250E-07	3.727E-07
34	1.053E+02	2.021E-07	2.008E-07	2.002E-07	1.998E-07	1.996E-07	1.994E-07	1.993E-07	3.317E-07
35	1.116E+02	1.795E-07	1.784E-07	1.779E-07	1.776E-07	1.774E-07	1.772E-07	1.771E-07	2.962E-07
36	1.181E+02	1.600E-07	1.591E-07	1.586E-07	1.584E-07	1.582E-07	1.581E-07	1.580E-07	2.654E-07
37	1.248E+02	1.430E-07	1.423E-07	1.419E-07	1.417E-07	1.415E-07	1.414E-07	1.413E-07	2.384E-07
38	1.316E+02	1.283E-07	1.276E-07	1.273E-07	1.271E-07	1.270E-07	1.269E-07	1.268E-07	2.148E-07
39	1.386E+02	1.154E-07	1.148E-07	1.145E-07	1.144E-07	1.143E-07	1.142E-07	1.141E-07	1.941E-07
40	1.458E+02	1.040E-07	1.036E-07	1.033E-07	1.032E-07	1.031E-07	1.030E-07	1.030E-07	1.758E-07
41	1.532E+02	9.408E-08	9.367E-08	9.347E-08	9.335E-08	9.327E-08	9.322E-08	9.317E-08	1.596E-07
42	1.607E+02	8.528E-08	8.493E-08	8.476E-08	8.466E-08	8.459E-08	8.454E-08	8.450E-08	1.452E-07
43	1.685E+02	7.749E-08	7.719E-08	7.704E-08	7.695E-08	7.689E-08	7.685E-08	7.682E-08	1.324E-07
44	1.764E+02	7.057E-08	7.031E-08	7.018E-08	7.010E-08	7.005E-08	7.001E-08	6.998E-08	1.210E-07
45	1.845E+02	6.441E-08	6.418E-08	6.407E-08	6.400E-08	6.395E-08	6.392E-08	6.390E-08	1.108E-07
46	1.928E+02	5.890E-08	5.870E-08	5.860E-08	5.854E-08	5.850E-08	5.848E-08	5.846E-08	1.017E-07
47	2.013E+02	5.397E-08	5.380E-08	5.371E-08	5.366E-08	5.363E-08	5.360E-08	5.358E-08	9.345E-08
48	2.100E+02	4.955E-08	4.940E-08	4.932E-08	4.927E-08	4.924E-08	4.922E-08	4.921E-08	8.605E-08
49	2.188E+02	4.557E-08	4.544E-08	4.537E-08	4.533E-08	4.530E-08	4.528E-08	4.527E-08	7.936E-08
50	2.278E+02	4.199E-08	4.187E-08	4.181E-08	4.177E-08	4.175E-08	4.173E-08	4.172E-08	7.331E-08

TABLE 1*c*

C_m VALUES FOR Z = 1 CALCULATED BY SUMMING FROM n = m UP TO n = 5000 IN EQUATION (5)

m	λ (μm)	T (°K) 5.0E03	7.5E03	1.0E04	1.25E04	1.5E04	1.75E04	2.0E04	∞
1	9.113E-02	5.178E+13	1.389E+09	7.196E+06	3.060E+05	3.727E+04	8.286E+03	2.683E+03	1.008E+00
2	3.645E-01	2.097E+01	1.513E+00	4.075E-01	1.858E-01	1.102E-01	7.592E-02	5.745E-02	8.349E-03
3	8.201E-01	1.577E-02	5.009E-03	2.839E-03	2.024E-03	1.617E-03	1.379E-03	1.224E-03	5.368E-04
4	1.458E+00	4.968E-04	2.667E-04	1.961E-04	1.633E-04	1.447E-04	1.327E-04	1.244E-04	7.953E-05
5	2.278E+00	5.755E-05	3.919E-05	3.240E-05	2.893E-05	2.683E-05	2.543E-05	2.443E-05	1.849E-05
6	3.281E+00	1.228E-05	9.475E-06	8.332E-06	7.717E-06	7.333E-06	7.072E-06	6.882E-06	5.695E-06
7	4.465E+00	3.692E-06	3.064E-06	2.794E-06	2.643E-06	2.548E-06	2.482E-06	2.434E-06	2.123E-06
8	5.832E+00	1.379E-06	1.198E-06	1.118E-06	1.072E-06	1.043E-06	1.022E-06	1.007E-06	9.083E-07
9	7.381E+00	5.975E-07	5.357E-07	5.074E-07	4.911E-07	4.806E-07	4.733E-07	4.678E-07	4.315E-07
10	9.113E+00	2.888E-07	2.646E-07	2.533E-07	2.468E-07	2.425E-07	2.396E-07	2.373E-07	2.224E-07
11	1.103E+01	1.516E-07	1.411E-07	1.362E-07	1.333E-07	1.314E-07	1.301E-07	1.291E-07	1.224E-07
12	1.312E+01	8.501E-08	8.008E-08	7.773E-08	7.636E-08	7.546E-08	7.482E-08	7.435E-08	7.112E-08
13	1.540E+01	5.026E-08	4.778E-08	4.659E-08	4.589E-08	4.543E-08	4.511E-08	4.487E-08	4.321E-08
14	1.786E+01	3.104E-08	2.973E-08	2.909E-08	2.872E-08	2.847E-08	2.830E-08	2.817E-08	2.727E-08
15	2.050E+01	1.990E-08	1.917E-08	1.881E-08	1.860E-08	1.846E-08	1.836E-08	1.829E-08	1.778E-08
16	2.333E+01	1.317E-08	1.274E-08	1.253E-08	1.241E-08	1.233E-08	1.227E-08	1.223E-08	1.193E-08
17	2.634E+01	8.951E-09	8.695E-09	8.570E-09	8.496E-09	8.447E-09	8.412E-09	8.386E-09	8.207E-09
18	2.953E+01	6.233E-09	6.074E-09	5.996E-09	5.950E-09	5.920E-09	5.898E-09	5.882E-09	5.770E-09
19	3.290E+01	4.432E-09	4.331E-09	4.281E-09	4.252E-09	4.232E-09	4.219E-09	4.208E-09	4.136E-09
20	3.645E+01	3.211E-09	3.145E-09	3.113E-09	3.093E-09	3.081E-09	3.072E-09	3.065E-09	3.018E-09
21	4.019E+01	2.366E-09	2.322E-09	2.300E-09	2.287E-09	2.279E-09	2.272E-09	2.268E-09	2.236E-09
22	4.411E+01	1.769E-09	1.739E-09	1.725E-09	1.716E-09	1.710E-09	1.706E-09	1.703E-09	1.681E-09
23	4.821E+01	1.341E-09	1.321E-09	1.310E-09	1.304E-09	1.300E-09	1.297E-09	1.295E-09	1.280E-09
24	5.249E+01	1.030E-09	1.015E-09	1.008E-09	1.003E-09	1.001E-09	9.986E-10	9.970E-10	9.864E-10
25	5.695E+01	7.993E-10	7.889E-10	7.837E-10	7.806E-10	7.786E-10	7.771E-10	7.760E-10	7.684E-10
26	6.160E+01	6.270E-10	6.194E-10	6.157E-10	6.134E-10	6.119E-10	6.109E-10	6.101E-10	6.046E-10
27	6.643E+01	4.965E-10	4.910E-10	4.882E-10	4.866E-10	4.855E-10	4.847E-10	4.841E-10	4.801E-10
28	7.144E+01	3.967E-10	3.926E-10	3.905E-10	3.893E-10	3.885E-10	3.879E-10	3.875E-10	3.845E-10
29	7.664E+01	3.195E-10	3.164E-10	3.149E-10	3.140E-10	3.134E-10	3.129E-10	3.126E-10	3.103E-10
30	8.201E+01	2.593E-10	2.570E-10	2.558E-10	2.551E-10	2.547E-10	2.543E-10	2.541E-10	2.524E-10
31	8.757E+01	2.120E-10	2.102E-10	2.093E-10	2.088E-10	2.084E-10	2.082E-10	2.080E-10	2.066E-10
32	9.331E+01	1.744E-10	1.730E-10	1.723E-10	1.719E-10	1.717E-10	1.715E-10	1.713E-10	1.703E-10
33	9.924E+01	1.444E-10	1.433E-10	1.428E-10	1.425E-10	1.423E-10	1.421E-10	1.420E-10	1.412E-10
34	1.053E+02	1.202E-10	1.194E-10	1.190E-10	1.187E-10	1.186E-10	1.184E-10	1.184E-10	1.177E-10
35	1.116E+02	1.007E-10	1.000E-10	9.968E-11	9.948E-11	9.935E-11	9.926E-11	9.919E-11	9.870E-11
36	1.181E+02	8.473E-11	8.420E-11	8.394E-11	8.378E-11	8.367E-11	8.360E-11	8.354E-11	8.315E-11
37	1.248E+02	7.165E-11	7.123E-11	7.102E-11	7.089E-11	7.081E-11	7.075E-11	7.071E-11	7.039E-11
38	1.316E+02	6.087E-11	6.053E-11	6.036E-11	6.026E-11	6.019E-11	6.015E-11	6.011E-11	5.986E-11
39	1.386E+02	5.194E-11	5.166E-11	5.153E-11	5.145E-11	5.139E-11	5.135E-11	5.132E-11	5.112E-11
40	1.458E+02	4.450E-11	4.428E-11	4.416E-11	4.410E-11	4.405E-11	4.402E-11	4.400E-11	4.383E-11
41	1.532E+02	3.828E-11	3.809E-11	3.800E-11	3.795E-11	3.791E-11	3.788E-11	3.786E-11	3.773E-11
42	1.607E+02	3.304E-11	3.289E-11	3.282E-11	3.277E-11	3.274E-11	3.272E-11	3.270E-11	3.259E-11
43	1.685E+02	2.863E-11	2.850E-11	2.844E-11	2.840E-11	2.838E-11	2.836E-11	2.835E-11	2.825E-11
44	1.764E+02	2.489E-11	2.478E-11	2.473E-11	2.470E-11	2.468E-11	2.466E-11	2.465E-11	2.458E-11
45	1.845E+02	2.170E-11	2.162E-11	2.157E-11	2.155E-11	2.153E-11	2.152E-11	2.151E-11	2.144E-11
46	1.928E+02	1.898E-11	1.891E-11	1.888E-11	1.885E-11	1.884E-11	1.883E-11	1.882E-11	1.877E-11
47	2.013E+02	1.665E-11	1.659E-11	1.656E-11	1.655E-11	1.653E-11	1.652E-11	1.652E-11	1.647E-11
48	2.100E+02	1.465E-11	1.460E-11	1.458E-11	1.456E-11	1.455E-11	1.454E-11	1.454E-11	1.450E-11
49	2.188E+02	1.293E-11	1.288E-11	1.286E-11	1.285E-11	1.284E-11	1.283E-11	1.283E-11	1.280E-11
50	2.278E+02	1.143E-11	1.139E-11	1.138E-11	1.137E-11	1.136E-11	1.135E-11	1.135E-11	1.132E-11

TABLE 2*a*

A_m VALUES FOR Z = 2 CALCULATED BY SUMMING FROM n= m UP TO n = 5000 IN EQUATION (5)

m	λ (μm)	T (°K) 5.0E03	7.5E03	1.0E04	1.25E04	1.5E04	1.75E04	2.0E04	∞
1	2.278E-02	7.188E+54	3.725E+36	2.681E+27	8.763E+21	1.930E+18	4.713E+15	5.178E+13	1.233E+00
2	9.113E-02	6.472E+12	1.737E+08	8.995E+05	3.826E+04	4.663E+03	1.038E+03	3.367E+02	2.221E-01
3	2.050E-01	4.616E+04	4.320E+02	4.229E+01	1.062E+01	4.280E+00	2.256E+00	1.406E+00	9.095E-02
4	3.645E-01	4.339E+01	3.333E+00	9.629E-01	4.691E-01	2.951E-01	2.140E-01	1.693E-01	5.022E-02
5	5.695E-01	1.477E+00	3.168E-01	1.536E-01	1.016E-01	7.800E-02	6.495E-02	5.681E-02	3.221E-02
6	8.201E-01	2.256E-01	8.453E-02	5.355E-02	4.126E-02	3.490E-02	3.106E-02	2.851E-02	2.257E-02
7	1.116E+00	7.094E-02	3.651E-02	2.679E-02	2.243E-02	1.999E-02	1.845E-02	1.738E-02	1.677E-02
8	1.458E+00	3.255E-02	2.026E-02	1.621E-02	1.425E-02	1.311E-02	1.236E-02	1.183E-02	1.298E-02
9	1.845E+00	1.849E-02	1.297E-02	1.097E-02	9.949E-03	9.334E-03	8.925E-03	8.632E-03	1.037E-02
10	2.278E+00	1.197E-02	9.095E-03	7.978E-03	7.389E-03	7.028E-03	6.783E-03	6.606E-03	8.483E-03
11	2.757E+00	8.433E-03	6.774E-03	6.097E-03	5.732E-03	5.504E-03	5.348E-03	5.235E-03	7.076E-03
12	3.281E+00	6.299E-03	5.267E-03	4.831E-03	4.591E-03	4.440E-03	4.336E-03	4.260E-03	5.996E-03
13	3.850E+00	4.908E-03	4.229E-03	3.934E-03	3.769E-03	3.665E-03	3.592E-03	3.539E-03	5.148E-03
14	4.465E+00	3.947E-03	3.480E-03	3.272E-03	3.156E-03	3.081E-03	3.029E-03	2.991E-03	4.470E-03
15	5.126E+00	3.253E-03	2.920E-03	2.769E-03	2.684E-03	2.629E-03	2.591E-03	2.562E-03	3.918E-03
16	5.832E+00	2.733E-03	2.489E-03	2.377E-03	2.313E-03	2.272E-03	2.243E-03	2.221E-03	3.464E-03
17	6.584E+00	2.333E-03	2.149E-03	2.065E-03	2.016E-03	1.984E-03	1.962E-03	1.945E-03	3.085E-03
18	7.381E+00	2.018E-03	1.877E-03	1.811E-03	1.773E-03	1.749E-03	1.731E-03	1.718E-03	2.765E-03
19	8.224E+00	1.765E-03	1.655E-03	1.603E-03	1.573E-03	1.553E-03	1.539E-03	1.529E-03	2.493E-03
20	9.113E+00	1.558E-03	1.471E-03	1.429E-03	1.405E-03	1.389E-03	1.378E-03	1.370E-03	2.259E-03
21	1.005E+01	1.387E-03	1.316E-03	1.283E-03	1.263E-03	1.251E-03	1.242E-03	1.235E-03	2.057E-03
22	1.103E+01	1.243E-03	1.186E-03	1.158E-03	1.142E-03	1.132E-03	1.124E-03	1.119E-03	1.881E-03
23	1.205E+01	1.121E-03	1.074E-03	1.051E-03	1.038E-03	1.029E-03	1.023E-03	1.019E-03	1.727E-03
24	1.312E+01	1.017E-03	9.775E-04	9.587E-04	9.476E-04	9.403E-04	9.351E-04	9.313E-04	1.591E-03
25	1.424E+01	9.266E-04	8.938E-04	8.780E-04	8.686E-04	8.625E-04	8.581E-04	8.549E-04	1.470E-03
26	1.540E+01	8.483E-04	8.206E-04	8.071E-04	7.992E-04	7.940E-04	7.903E-04	7.876E-04	1.363E-03
27	1.661E+01	7.797E-04	7.561E-04	7.447E-04	7.379E-04	7.335E-04	7.303E-04	7.280E-04	1.267E-03
28	1.786E+01	7.193E-04	6.991E-04	6.893E-04	6.835E-04	6.796E-04	6.769E-04	6.749E-04	1.181E-03
29	1.916E+01	6.658E-04	6.484E-04	6.399E-04	6.349E-04	6.316E-04	6.292E-04	6.275E-04	1.104E-03
30	2.050E+01	6.181E-04	6.031E-04	5.957E-04	5.914E-04	5.885E-04	5.864E-04	5.849E-04	1.033E-03
31	2.189E+01	5.755E-04	5.624E-04	5.560E-04	5.522E-04	5.497E-04	5.479E-04	5.465E-04	9.699E-04
32	2.333E+01	5.372E-04	5.257E-04	5.201E-04	5.168E-04	5.146E-04	5.130E-04	5.119E-04	9.119E-04
33	2.481E+01	5.027E-04	4.926E-04	4.877E-04	4.847E-04	4.828E-04	4.814E-04	4.804E-04	8.590E-04
34	2.634E+01	4.714E-04	4.625E-04	4.582E-04	4.556E-04	4.539E-04	4.527E-04	4.517E-04	8.106E-04
35	2.791E+01	4.431E-04	4.352E-04	4.313E-04	4.290E-04	4.275E-04	4.264E-04	4.256E-04	7.662E-04
36	2.953E+01	4.172E-04	4.102E-04	4.068E-04	4.047E-04	4.033E-04	4.024E-04	4.017E-04	7.254E-04
37	3.119E+01	3.936E-04	3.873E-04	3.842E-04	3.824E-04	3.812E-04	3.803E-04	3.797E-04	6.877E-04
38	3.290E+01	3.719E-04	3.663E-04	3.636E-04	3.619E-04	3.608E-04	3.601E-04	3.595E-04	6.529E-04
39	3.465E+01	3.520E-04	3.470E-04	3.445E-04	3.431E-04	3.421E-04	3.414E-04	3.409E-04	6.207E-04
40	3.645E+01	3.337E-04	3.292E-04	3.270E-04	3.256E-04	3.248E-04	3.241E-04	3.237E-04	5.908E-04
41	3.830E+01	3.168E-04	3.127E-04	3.107E-04	3.095E-04	3.087E-04	3.081E-04	3.077E-04	5.630E-04
42	4.019E+01	3.012E-04	2.975E-04	2.956E-04	2.946E-04	2.938E-04	2.933E-04	2.929E-04	5.372E-04
43	4.212E+01	2.867E-04	2.833E-04	2.817E-04	2.807E-04	2.800E-04	2.795E-04	2.792E-04	5.131E-04
44	4.411E+01	2.732E-04	2.701E-04	2.686E-04	2.677E-04	2.671E-04	2.667E-04	2.664E-04	4.905E-04
45	4.613E+01	2.607E-04	2.579E-04	2.565E-04	2.557E-04	2.551E-04	2.548E-04	2.545E-04	4.695E-04
46	4.821E+01	2.490E-04	2.464E-04	2.452E-04	2.444E-04	2.439E-04	2.436E-04	2.433E-04	4.497E-04
47	5.033E+01	2.381E-04	2.358E-04	2.346E-04	2.339E-04	2.335E-04	2.331E-04	2.329E-04	4.312E-04
48	5.249E+01	2.279E-04	2.257E-04	2.247E-04	2.241E-04	2.236E-04	2.233E-04	2.231E-04	4.138E-04
49	5.470E+01	2.183E-04	2.164E-04	2.154E-04	2.148E-04	2.144E-04	2.142E-04	2.140E-04	3.974E-04
50	5.695E+01	2.094E-04	2.076E-04	2.067E-04	2.061E-04	2.058E-04	2.055E-04	2.053E-04	3.820E-04

TABLE 2*b*

B_m VALUES FOR Z = 2 CALCULATED BY SUMMING FROM n= m UP TO n = 5000 IN EQUATION (5)

m	λ (μm)	T (°K) 5.0E03	7.5E03	1.0E04	1.25E04	1.5E04	1.75E04	2.0E04	∞
1	2.278E-02	7.188E+54	3.725E+36	2.681E+27	8.763E+21	1.930E+18	4.713E+15	5.178E+13	1.038E+00
2	9.113E-02	1.618E+12	4.341E+07	2.249E+05	9.562E+03	1.165E+03	2.592E+02	8.397E+01	3.733E-02
3	2.050E-01	5.126E+03	4.783E+01	4.648E+00	1.155E+00	4.587E-01	2.382E-01	1.462E-01	5.871E-03
4	3.645E-01	2.675E+00	1.997E-01	5.571E-02	2.622E-02	1.598E-02	1.127E-02	8.696E-03	1.667E-03
5	5.695E-01	5.551E-02	1.117E-02	5.129E-03	3.248E-03	2.408E-03	1.950E-03	1.668E-03	6.470E-04
6	8.201E-01	5.460E-03	1.885E-03	1.127E-03	8.333E-04	6.837E-04	5.946E-04	5.359E-04	3.041E-04
7	1.116E+00	1.164E-03	5.510E-04	3.836E-04	3.100E-04	2.695E-04	2.441E-04	2.268E-04	1.624E-04
8	1.458E+00	3.805E-04	2.192E-04	1.677E-04	1.432E-04	1.290E-04	1.198E-04	1.134E-04	9.494E-05
9	1.845E+00	1.609E-04	1.054E-04	8.581E-05	7.596E-05	7.009E-05	6.621E-05	6.345E-05	5.941E-05
10	2.278E+00	8.036E-05	5.755E-05	4.887E-05	4.436E-05	4.161E-05	3.976E-05	3.844E-05	3.917E-05
11	2.757E+00	4.500E-05	3.434E-05	3.007E-05	2.779E-05	2.638E-05	2.542E-05	2.472E-05	2.692E-05
12	3.281E+00	2.736E-05	2.188E-05	1.960E-05	1.836E-05	1.758E-05	1.705E-05	1.666E-05	1.914E-05
13	3.850E+00	1.770E-05	1.467E-05	1.337E-05	1.266E-05	1.220E-05	1.189E-05	1.166E-05	1.400E-05
14	4.465E+00	1.202E-05	1.024E-05	9.459E-06	9.024E-06	8.746E-06	8.553E-06	8.412E-06	1.049E-05
15	5.126E+00	8.474E-06	7.380E-06	6.893E-06	6.618E-06	6.441E-06	6.318E-06	6.228E-06	8.015E-06
16	5.832E+00	6.165E-06	5.466E-06	5.149E-06	4.969E-06	4.853E-06	4.772E-06	4.712E-06	6.235E-06
17	6.584E+00	4.603E-06	4.140E-06	3.929E-06	3.807E-06	3.729E-06	3.674E-06	3.633E-06	4.925E-06
18	7.381E+00	3.513E-06	3.198E-06	3.052E-06	2.969E-06	2.914E-06	2.876E-06	2.848E-06	3.944E-06
19	8.224E+00	2.731E-06	2.512E-06	2.409E-06	2.350E-06	2.312E-06	2.284E-06	2.264E-06	3.196E-06
20	9.113E+00	2.158E-06	2.002E-06	1.928E-06	1.886E-06	1.858E-06	1.838E-06	1.824E-06	2.619E-06
21	1.005E+01	1.730E-06	1.616E-06	1.562E-06	1.531E-06	1.511E-06	1.496E-06	1.485E-06	2.166E-06
22	1.103E+01	1.404E-06	1.320E-06	1.280E-06	1.256E-06	1.241E-06	1.230E-06	1.222E-06	1.808E-06
23	1.205E+01	1.152E-06	1.089E-06	1.059E-06	1.041E-06	1.029E-06	1.021E-06	1.015E-06	1.521E-06
24	1.312E+01	9.544E-07	9.065E-07	8.835E-07	8.701E-07	8.612E-07	8.550E-07	8.503E-07	1.289E-06
25	1.424E+01	7.981E-07	7.611E-07	7.434E-07	7.330E-07	7.261E-07	7.213E-07	7.177E-07	1.100E-06
26	1.540E+01	6.727E-07	6.440E-07	6.301E-07	6.220E-07	6.166E-07	6.128E-07	6.099E-07	9.440E-07
27	1.661E+01	5.713E-07	5.486E-07	5.377E-07	5.313E-07	5.270E-07	5.240E-07	5.218E-07	8.150E-07
28	1.786E+01	4.884E-07	4.704E-07	4.617E-07	4.566E-07	4.532E-07	4.508E-07	4.490E-07	7.074E-07
29	1.916E+01	4.201E-07	4.057E-07	3.987E-07	3.946E-07	3.919E-07	3.899E-07	3.885E-07	6.170E-07
30	2.050E+01	3.635E-07	3.518E-07	3.462E-07	3.428E-07	3.406E-07	3.390E-07	3.379E-07	5.406E-07
31	2.189E+01	3.161E-07	3.066E-07	3.020E-07	2.993E-07	2.975E-07	2.962E-07	2.952E-07	4.757E-07
32	2.333E+01	2.763E-07	2.685E-07	2.647E-07	2.625E-07	2.610E-07	2.599E-07	2.591E-07	4.203E-07
33	2.481E+01	2.426E-07	2.362E-07	2.330E-07	2.312E-07	2.299E-07	2.291E-07	2.284E-07	3.727E-07
34	2.634E+01	2.139E-07	2.086E-07	2.059E-07	2.044E-07	2.034E-07	2.027E-07	2.021E-07	3.317E-07
35	2.791E+01	1.893E-07	1.849E-07	1.827E-07	1.814E-07	1.806E-07	1.799E-07	1.795E-07	2.962E-07
36	2.953E+01	1.682E-07	1.645E-07	1.627E-07	1.616E-07	1.608E-07	1.603E-07	1.600E-07	2.654E-07
37	3.119E+01	1.500E-07	1.468E-07	1.453E-07	1.444E-07	1.438E-07	1.433E-07	1.430E-07	2.384E-07
38	3.290E+01	1.342E-07	1.315E-07	1.302E-07	1.294E-07	1.289E-07	1.285E-07	1.283E-07	2.148E-07
39	3.465E+01	1.204E-07	1.181E-07	1.170E-07	1.163E-07	1.159E-07	1.156E-07	1.154E-07	1.941E-07
40	3.645E+01	1.083E-07	1.064E-07	1.055E-07	1.049E-07	1.045E-07	1.042E-07	1.040E-07	1.758E-07
41	3.830E+01	9.778E-08	9.611E-08	9.529E-08	9.480E-08	9.448E-08	9.425E-08	9.408E-08	1.596E-07
42	4.019E+01	8.848E-08	8.704E-08	8.633E-08	8.591E-08	8.563E-08	8.543E-08	8.528E-08	1.452E-07
43	4.212E+01	8.026E-08	7.901E-08	7.840E-08	7.803E-08	7.779E-08	7.762E-08	7.749E-08	1.324E-07
44	4.411E+01	7.297E-08	7.189E-08	7.136E-08	7.104E-08	7.083E-08	7.068E-08	7.057E-08	1.210E-07
45	4.613E+01	6.650E-08	6.556E-08	6.510E-08	6.482E-08	6.464E-08	6.451E-08	6.441E-08	1.108E-07
46	4.821E+01	6.073E-08	5.991E-08	5.951E-08	5.926E-08	5.910E-08	5.899E-08	5.890E-08	1.017E-07
47	5.033E+01	5.558E-08	5.486E-08	5.450E-08	5.429E-08	5.415E-08	5.405E-08	5.397E-08	9.345E-08
48	5.249E+01	5.096E-08	5.033E-08	5.002E-08	4.983E-08	4.971E-08	4.962E-08	4.955E-08	8.605E-08
49	5.470E+01	4.682E-08	4.626E-08	4.598E-08	4.582E-08	4.571E-08	4.563E-08	4.557E-08	7.936E-08
50	5.695E+01	4.308E-08	4.259E-08	4.235E-08	4.220E-08	4.211E-08	4.204E-08	4.199E-08	7.331E-08

TABLE 2c

C_m VALUES FOR Z = 2 CALCULATED BY SUMMING FROM n = m UP TO n = 5000 IN EQUATION (5)

m	λ (μm)	T (°K) 5.0E03	7.5E03	1.0E04	1.25E04	1.5E04	1.75E04	2.0E04	∞
1	2.278E-02	7.188E+54	3.725E+36	2.481E+27	8.763E+21	1.930E+18	4.713E+15	5.178E+13	1.008E+00
2	9.113E-02	4.045E+11	1.085E+07	5.622E+04	2.350E+03	2.912E+02	6.476E+01	2.097E+01	8.349E-03
3	2.050E-01	5.695E+02	5.304E+00	5.135E-01	1.269E-01	5.012E-02	2.586E-02	1.577E-02	5.368E-04
4	3.645E-01	1.659E-01	1.220E-02	3.348E-03	1.552E-03	9.330E-04	6.503E-04	4.968E-04	7.953E-05
5	5.695E-01	2.142E-03	4.182E-04	1.872E-04	1.163E-04	8.492E-05	6.795E-05	5.755E-05	1.849E-05
6	8.201E-01	1.404E-04	4.664E-05	2.716E-05	1.972E-05	1.596E-05	1.373E-05	1.228E-05	5.695E-06
7	1.116E+00	2.106E-05	9.590E-06	6.518E-06	5.184E-06	4.455E-06	4.000E-06	3.692E-06	2.123E-06
8	1.458E+00	5.072E-06	2.819E-06	2.111E-06	1.779E-06	1.588E-06	1.464E-06	1.379E-06	9.083E-07
9	1.845E+00	1.641E-06	1.041E-06	8.323E-07	7.285E-07	6.669E-07	6.263E-07	5.975E-07	4.315E-07
10	2.278E+00	6.462E-07	4.500E-07	3.763E-07	3.384E-07	3.153E-07	2.998E-07	2.888E-07	2.224E-07
11	2.757E+00	2.926E-07	2.179E-07	1.883E-07	1.726E-07	1.629E-07	1.564E-07	1.516E-07	1.224E-07
12	3.281E+00	1.468E-07	1.149E-07	1.018E-07	9.470E-08	9.026E-08	8.722E-08	8.501E-08	7.112E-08
13	3.850E+00	7.973E-08	6.485E-08	5.854E-08	5.506E-08	5.287E-08	5.136E-08	5.026E-08	4.321E-08
14	4.465E+00	4.608E-08	3.862E-08	3.538E-08	3.357E-08	3.242E-08	3.163E-08	3.104E-08	2.727E-08
15	5.126E+00	2.801E-08	2.404E-08	2.228E-08	2.130E-08	2.066E-08	2.022E-08	1.990E-08	1.778E-08
16	5.832E+00	1.775E-08	1.553E-08	1.454E-08	1.397E-08	1.361E-08	1.335E-08	1.317E-08	1.193E-08
17	6.584E+00	1.165E-08	1.036E-08	9.768E-09	9.432E-09	9.215E-09	9.063E-09	8.951E-09	8.207E-09
18	7.381E+00	7.875E-09	7.095E-09	6.736E-09	6.530E-09	6.396E-09	6.302E-09	6.233E-09	5.770E-09
19	8.224E+00	5.463E-09	4.977E-09	4.751E-09	4.620E-09	4.536E-09	4.476E-09	4.432E-09	4.136E-09
20	9.113E+00	3.876E-09	3.564E-09	3.418E-09	3.334E-09	3.279E-09	3.240E-09	3.211E-09	3.018E-09
21	1.005E+01	2.804E-09	2.600E-09	2.503E-09	2.447E-09	2.411E-09	2.385E-09	2.366E-09	2.236E-09
22	1.103E+01	2.065E-09	1.928E-09	1.863E-09	1.825E-09	1.800E-09	1.782E-09	1.769E-09	1.681E-09
23	1.205E+01	1.545E-09	1.451E-09	1.406E-09	1.380E-09	1.362E-09	1.350E-09	1.341E-09	1.280E-09
24	1.312E+01	1.172E-09	1.106E-09	1.075E-09	1.056E-09	1.044E-09	1.036E-09	1.030E-09	9.864E-10
25	1.424E+01	9.002E-10	8.538E-10	8.315E-10	8.185E-10	8.099E-10	8.038E-10	7.993E-10	7.684E-10
26	1.540E+01	6.997E-10	6.663E-10	6.503E-10	6.408E-10	6.346E-10	6.302E-10	6.270E-10	6.046E-10
27	1.661E+01	5.496E-10	5.253E-10	5.136E-10	5.067E-10	5.021E-10	4.989E-10	4.965E-10	4.801E-10
28	1.786E+01	4.359E-10	4.180E-10	4.093E-10	4.042E-10	4.008E-10	3.985E-10	3.967E-10	3.845E-10
29	1.916E+01	3.488E-10	3.355E-10	3.290E-10	3.252E-10	3.226E-10	3.209E-10	3.195E-10	3.103E-10
30	2.050E+01	2.815E-10	2.714E-10	2.665E-10	2.636E-10	2.617E-10	2.603E-10	2.593E-10	2.524E-10
31	2.189E+01	2.288E-10	2.212E-10	2.174E-10	2.152E-10	2.138E-10	2.127E-10	2.120E-10	2.066E-10
32	2.333E+01	1.874E-10	1.815E-10	1.786E-10	1.769E-10	1.758E-10	1.750E-10	1.744E-10	1.703E-10
33	2.481E+01	1.545E-10	1.499E-10	1.477E-10	1.464E-10	1.455E-10	1.449E-10	1.444E-10	1.412E-10
34	2.634E+01	1.281E-10	1.246E-10	1.228E-10	1.218E-10	1.211E-10	1.206E-10	1.202E-10	1.177E-10
35	2.791E+01	1.069E-10	1.041E-10	1.027E-10	1.019E-10	1.013E-10	1.010E-10	1.007E-10	9.870E-11
36	2.953E+01	8.965E-11	8.743E-11	8.634E-11	8.569E-11	8.526E-11	8.496E-11	8.473E-11	8.315E-11
37	3.119E+01	7.559E-11	7.381E-11	7.294E-11	7.242E-11	7.208E-11	7.184E-11	7.165E-11	7.039E-11
38	3.290E+01	6.403E-11	6.261E-11	6.191E-11	6.149E-11	6.122E-11	6.102E-11	6.087E-11	5.986E-11
39	3.465E+01	5.450E-11	5.334E-11	5.278E-11	5.244E-11	5.222E-11	5.206E-11	5.194E-11	5.112E-11
40	3.645E+01	4.658E-11	4.564E-11	4.518E-11	4.491E-11	4.473E-11	4.460E-11	4.450E-11	4.383E-11
41	3.830E+01	3.997E-11	3.921E-11	3.883E-11	3.861E-11	3.846E-11	3.835E-11	3.828E-11	3.773E-11
42	4.019E+01	3.444E-11	3.381E-11	3.350E-11	3.332E-11	3.320E-11	3.311E-11	3.304E-11	3.259E-11
43	4.212E+01	2.978E-11	2.926E-11	2.901E-11	2.885E-11	2.875E-11	2.868E-11	2.863E-11	2.825E-11
44	4.411E+01	2.584E-11	2.541E-11	2.520E-11	2.507E-11	2.499E-11	2.493E-11	2.489E-11	2.458E-11
45	4.613E+01	2.250E-11	2.214E-11	2.196E-11	2.186E-11	2.179E-11	2.174E-11	2.170E-11	2.144E-11
46	4.821E+01	1.965E-11	1.935E-11	1.920E-11	1.911E-11	1.906E-11	1.901E-11	1.898E-11	1.877E-11
47	5.033E+01	1.721E-11	1.696E-11	1.684E-11	1.676E-11	1.672E-11	1.668E-11	1.665E-11	1.647E-11
48	5.249E+01	1.512E-11	1.491E-11	1.481E-11	1.474E-11	1.470E-11	1.467E-11	1.465E-11	1.450E-11
49	5.470E+01	1.332E-11	1.314E-11	1.306E-11	1.300E-11	1.297E-11	1.294E-11	1.293E-11	1.280E-11
50	5.695E+01	1.177E-11	1.162E-11	1.154E-11	1.150E-11	1.147E-11	1.145E-11	1.143E-11	1.132E-11

III. CONCLUSION

The approximate formula for the calculation of the free-bound contribution to the continuum energy derived in the preceding lines has been found to be very precise, fast, and easy to evaluate in any micro-computer for most temperatures of interest. Its main feature is, however, an enormous simplification in the calculation of the free-bound emission by an ionized region of varying temperature and ionic composition.

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