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GROWTH OF ISOTHERMAL PERTURBATIONS DURING AND AFTER THE RECOMBINATION ERA

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ABSTRACT: A study of the evolution of isothermal perturbations is given during and after the recombination era. Effects due to recombination, nonlinear growth and the decoupling of matter and radiation are taken into account. A result of particular interest is that isothermal fluctuations of mass very much smaller than the Jeans mass at the beginning of the recombination era (i.e. $M \ll M_J \simeq 10^5 M_\odot$) are not damped to extinction. In particular, isothermal fluctuations of mass 25-200 $M_\odot$ survive. The results thus indicate that massive Population III stars in the mass range 25-200 $M_\odot$ need not have formed by fragmentation from massive clouds, but could have formed from perturbations which survived directly from the recombination era.

Key words: isothermal perturbations, recombination era, Population III stars

I. INTRODUCTION

It is well known that two possible primordial spectra of perturbations exist, adiabatic and isothermal (e.g. Efstathiou and Silk 1983, Rees 1982a,b, Silk 1982, Vettolani et al 1984). Hierarchical build-up of matter is generally associated with an isothermal model (Efstathiou and Silk 1983). The hierarchical model simulates well the observed distribution of galaxies and its correlation function (Vettolani et al 1984).

Hogan (1978), Kashlinsky and Rees (1983), and others emphasize the importance of nonlinear effects of isothermal perturbations.

A power law spectra for isothermal fluctuations $\delta\rho/\rho \propto M^{-\alpha}$ (e.g. Hogan 1978, Kashlinsky and Rees 1983, Gott 1977, Efstathiou and Silk 1983, White and Rees 1978, Carr and Rees 1984) is generally adopted. Assuming this power law distribution for the isothermal perturbations, we will have at recombination $\delta\rho/\rho > 1$ (nonlinear perturbations) below certain masses. The masses in the range 10^6-$10^8 M_\odot$ are suggested to have $\delta\rho/\rho \sim 1$ (e.g. Kashlinsky an Rees 1983).

The importance of a detailed study of the evolution of isothermal perturbations during the recombination era was emphasized by Carr and Rees (1984).

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Up to the present, a detailed calculation of the evolution of isothermal erturbations, for masses $M \lesssim 10^4 M_\odot$, taking into account recombination processes, has not been ade. Peebles (1969) has performed such calculations for $M \gtrsim 10^4 M_\odot$ in the linear regime.

Of particular interest is the mass spectrum of Population III stars. As indicated y Silk (1983), for example, Population III stars must have contained massive stars in order to ynthesize the heavy elements seen in extreme Population II stars. Silk (1983) analyzed the ossibility that massive Population III stars were formed by fragmentation from a massive cloud ue to a thermal instability.

An oxygen enhancement with respect to iron in extreme Population II stars is ndicative that the Population III stars had an excess of massive stars. An enhancement of [O/Fe] $\sim$0.7 dex has been found (Barbuy 1983). Barbuy (1983) suggests that the early stars had asses in the range $10 < M/M_\odot < 500$, the most plausible explanation of her data being that here was an early enhancement of O (and N as well) produced by massive stars, with C and Fe oming later, these elements being partly (probably) produced by intermediate mass stars. An nhancement [O/Fe] $\sim$0.7 dex is also seen in the Virgo Cluster gas (Canizares et al. 1982).

In section II we discuss the basic equations used and subsequent sections resent the results of the calculations.

I. BASIC EQUATIONS

1. Hydrodynamic Equations

Following Weinberg (1972) and others, we use the Newtonian approximation for the ydrodynamic equations

$$\rho \frac{d\vec{v}}{dt} + \vec{v}.\vec{\nabla}\rho = 0 \quad (1) \; ; \quad \rho \frac{d\vec{v}}{dt} + \vec{\nabla}P + \rho\vec{\nabla}\phi + \frac{\sigma b T^4}{m_p c} x_e \rho (\vec{v} - H\vec{r}) = 0 \quad (2) \; ;$$

$$\nabla^2\phi = 4\pi G\rho \quad (3) \; ; \quad P = N\rho \, k_B \, T \, (1 + x_e) \quad (4) \; ; \quad \frac{dU}{dt} = -\mathcal{L} + \frac{P}{\rho^2} \frac{d\rho}{dt} \quad (5)$$

Here ρ is the matter density, P is the matter pressure, ϕ is the gravitational otential, $\sigma b T^4 x_e \rho(\vec{v}-H\vec{r})/m_p c$ is the photon drag, x_e $(=n_e/n)$ is the degree of ionization, here n_e is the electron density, U is the internal energy of the cloud and $\mathcal{L}$ is the cooling unction which we describe later. The expansion of the universe is also taken into account in he calculation.

2. Degree of ionization

The calculation of the degree of ionization is not simple, since photon ecombination and primordial photon radiation can reionize hydrogen. Our calculation of the legree of ionization is based on the analysis of Peebles (1968). Another calculation which ave similar results was made by Zel'Dovich (1968).

3. The Energy Equation

For the internal energy we have:

$$U = \frac{3}{2} N k_B T_e (1 + x_e) + \phi_1 \quad (6)$$

where U is the internal energy per unit mass. The first term on the right side represents the kinetic energy per unit mass and ϕ_1 is the gravitational energy per unit mass of the cloud. Using (6) and (5) we have:

$$\frac{3}{2} N k_B \dot{T}_e (1 + x_e) + \frac{3}{2} N k_B T_e \dot{x}_e + \dot{\phi}_1 = -\mathcal{L} + \frac{P}{\rho^2} \dot{\rho} \quad (7)$$

where a dot represents the total derivative in time. As we have remarked earlier, $\mathcal{L}$ is the cooling function, that is, the energy loss minus the energy gain. We include Compton heating (Weymann 1965), thermal conductivity (Spitzer 1962), photoionization due to primordial and recombination photons, and recombination. Collisional ionization was found not to be important for the densities and temperatures considered. Each electron recombination involves an energy loss of $k_B T_e$ (Schwartz et al 1972). The Lyman-α photons do not contribute to the cooling, since their optical depth is very large (Hasegawa et al 1981 and Lepp and Shull 1984). Thus we have:

$$\mathcal{L} = - k_B T_e N \dot{x} + \frac{4\sigma a\, T_R^4\, x_e\, N k_B}{m_e c} (T_e - T_R) + \frac{1}{\rho} \vec{\nabla} (\kappa \vec{\nabla} T_e) \quad (8)$$

where σ is the Thompson cross section, T_R is the radiation temperature, κ is the coeficient of thermal conductivity and "a" is (4/c) the Stefan-Boltzmann constant.

4. The Perturbed Equations

For the perturbed equations we have:

$$\rho = \bar{\rho} + \rho_1 \quad (9) \quad ; \quad P_C = P_A + P_1 \quad (10) \; ; \; \vec{v} = \bar{\vec{v}} + \vec{v}_1 \quad (11) \; ; \; \phi = \phi_1 \quad (12) \; ; \; T_C = T_M^A + T_{1M} \; (13)$$

$$T_M^A = \bar{T} + T_1^{AR} \quad (14) \quad ; \quad T_C = \bar{T} + T_1^{CR} \quad (15) \quad ; \quad x_C = x_A + x_1 \quad (16)$$

where $\bar{\rho} = \rho_o (R_o/R)^3$, ρ_o is the present density, R the scale factor, P_C the cloud pressure, P_A the ambient pressure, $\bar{\vec{v}} = \vec{r}\ \dot{R}/R$ the comoving velocity, $\bar{T}$ or T_R the radiation temperature, T_M^A the ambient temperature, T_C the cloud temperature, x_A ou $\bar{x}$ the ambient degree of ionization, and x_C the cloud degree of ionization. Substituting (9)-(16) in (1)-(5) and assuming all perturbations of the form:

$$a(\vec{r},t) = a(t)\exp(i\, \frac{\vec{k}.\vec{r}}{R}) \quad (17)$$

we obtain:

$$\dot{\rho}_1 = - 3\frac{\dot{R}}{R} \rho_1 - \frac{\omega_1}{R} (\bar{\rho} + 2\rho_1) \quad (18)$$

$$\left(\frac{\dot{\omega}_1}{R}\right) + \left(\frac{\omega_1}{R}\right)\left(\frac{\dot{R}}{R} + \frac{\sigma b T^4}{m_p c} x_C\right) + \left(\frac{\omega_1}{R}\right)^2 + 4\pi G \rho_1 - N k_B \frac{T_M^A (1+\bar{x})}{(1 + \rho_1/\bar{\rho})} \frac{k^2}{R^2} \left[\frac{\rho 1}{\bar{\rho}} + \frac{T_{1M}}{T_M^A} + \frac{x_1}{1 + \bar{x}} + \frac{2\rho_1 T_{1M}}{\bar{\rho}\ T_M^A} + \frac{2\rho_1 x_1}{\bar{\rho}\ (1+\bar{x})} + \frac{2 x_1 T_{1M}}{(1+ \bar{x}) T_M^A} + \frac{3\rho_1 T_{1M}\, x_1}{(1+\bar{x}) T_M^A\, \bar{\rho}} \right] = 0 \quad (19)$$

where x_A and x_C are given by:

$$\dot{x}_A = -\frac{\alpha(\bar{T})}{1+\beta_c/\Lambda_{2s1s}}\left[N\bar{\rho}x_A^2(1+T^{AR}/\bar{T})^{-1/2} - (1-x_A)\,\bar{T}^{3/2}(2\pi m_e k_B)^{3/2}\exp\left(-\frac{\chi}{k_B T}\right)\right] \quad (20)$$

$$\dot{x}_C = -\frac{\alpha(\bar{T})}{1+\beta_c/\Lambda_{2s1s}}\left[N\,x_C^2\,\bar{\rho}\,(1+\rho_1/\bar{\rho})(1+T_1^{CR}/\bar{T})^{-1/2} - (1-x_C)\,\bar{T}^{3/2}\,(2\pi m_e k_B)^{3/2}\exp\left(-\frac{\chi}{k_B T}\right)\right] \quad (21)$$

where β_c is the rate of photoionization, and α is the coeficient for recombination.

$$\dot{T}_1^{CR} = \frac{\frac{-Nk_B}{2}(T+T_1^{CR})\dot{x}_C - \dot{\phi}_1 + \frac{4\sigma a T_R^4\, x_C B k_B}{m_e c}\, T_1^{CR} + \frac{P}{\rho^2}\dot{\rho} + \frac{1}{\rho}\vec{\nabla}(k\vec{\nabla}T_C)}{\frac{3}{2}\,Nk_B(1+x_C)} - \dot{\bar{T}} \quad (22)$$

$$\dot{T}_1^{AR} = -\dot{\bar{T}} + \frac{8\sigma a T_R^4 x_A}{3m_e c(1+x_A)}(T_R - T_m^A) - T_m^A\left(2\frac{\dot{R}}{R} + \frac{\dot{x}_A}{3(1+x_A)}\right) \quad (23)$$

This last equation represents the decoupling between matter and radiation. It is obtained from Eq. (22) assuming that no perturbations are present. Thus Eq. (23) is completely analogous to that obtained by Peebles (1968).

III. CALCULATIONS

We studied the perturbations $\delta\rho/\rho = 0.1$ and $\delta\rho/\rho = 10^{-3}$ for a flat universe $\Omega h^2 = 1.0$ and for an open universe $\Omega h^2 = 0.025$, for a range of λ values $0.01 \leq \lambda/\lambda_J \leq 5$. We begin the calculation at a temperature of 4000 K, when the degree of ionization begins to be different from unity.

The Jeans mass is about $\sim 10^5\ M_\odot$ for $\Omega h^2 = 1.0$ and $\sim 10^6\ M_\odot$ for $\Omega h^2 = 0.025$ at the beginning of our calculations. The range $0.01 \leq \lambda/\lambda_J \leq 5$ studied therefore involves masses in the range $0.2 \lesssim M/M_\odot \lesssim 10^7$ for $\Omega h^2 = 1.0$, for example.

Let us define:

$$R_1 = \frac{(\delta/\delta_i)_{\delta_i = 10^{-1}}}{(\delta/\delta_i)_{\delta_i = 10^{-3}}} \quad (24)$$

where $\delta_i \equiv \delta\rho/\rho$. R_1 is thus the ratio of the amplification of an initial perturbation $\delta_i = 0.1$ to the amplification of an initial perturbation $\delta_i = 10^{-3}$. As seen in Fig. 1, the amplification for a $\delta_i = 0.1$ perturbation is greater than for a $\delta_i = 10^{-3}$ and this difference depends, also, on mass. Thus the solution for $\delta_i = 0.1$ is already nonlinear. At 500 K for $\lambda = 5\lambda_J$ ($\sim 10^7\ M_\odot$), for example, the amplification for $\delta_i = 0.1$ is twice as large as for $\delta_i = 10^{-3}$.

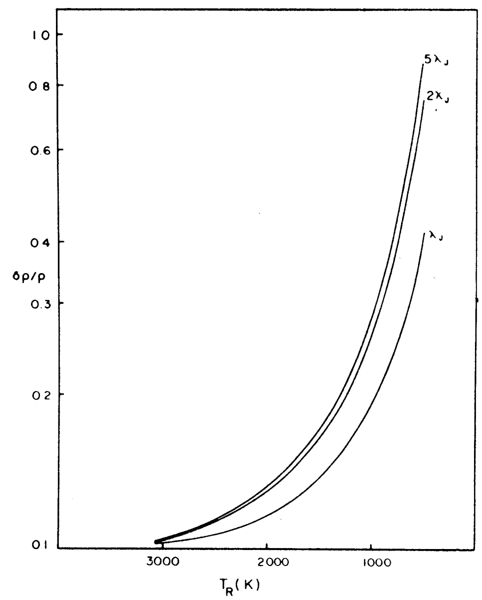

Fig. 1. The ratio R_1 as a function of temperature, as defined by Eq. (24).

The dependence of the growth of $\delta\rho/\rho$ on the type of universe is examined in Fig. 2, where we compare, for $\lambda = 2\lambda_J$, the evolution of $\delta\rho/\rho$ in a flat universe ($\Omega h^2 = 1.0$)

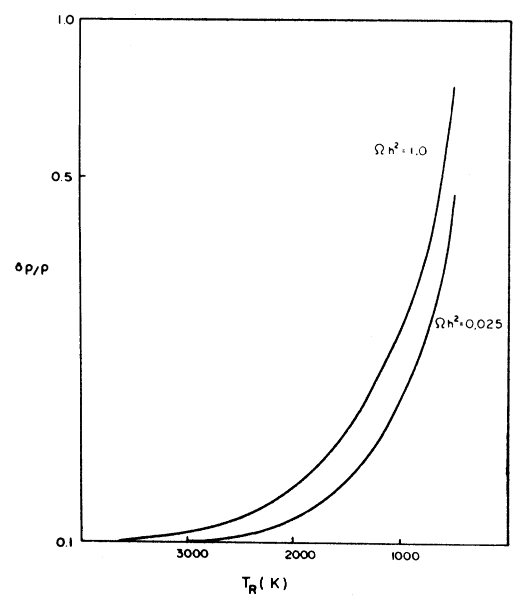

Fig. 2. Comparison of the $\lambda = 2\lambda_J$ for $\Omega h^2 = 1.0$ and $\Omega h^2 = 0.025$.

and in an open universe ($\Omega h^2 = 0.025$), as a function of the temperature. As can be noted, for $\Omega h^2 = 0.025$ the growth is less than that for $\Omega h^2 = 1.0$.

In Fig. 3 we study the evolution of masses greater than the Jeans mass for $\lambda/\lambda_J = 1$, 2, and 5 for $\Omega h^2 = 1.0$. We note that the curves of growth are similar, with the greater growth occuring for greater masses.

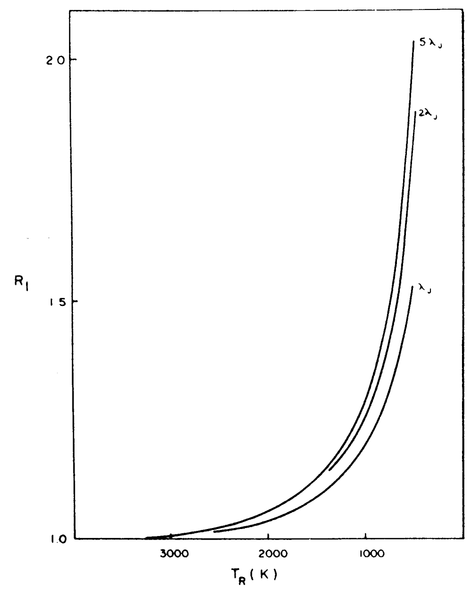

Fig. 3. The growth of isothermal perturbations $\delta\rho/\rho$ for $\lambda/\lambda_J = 1,2$, and 5.

In Fig.4 we study the evolution of masses smaller than M_J for $\lambda/\lambda_J = 1/2$, 1/5, 1/10 and 1/20. (We found for $\lambda/\lambda_J = 1/100$ that no significant residual perturbation remains below the recombination era.) For $\lambda = \lambda_J/5$ (1600 $M_\odot$) we have two oscillations, $\lambda = \lambda_J/10$ (200 $M_\odot$) four oscillations and $\lambda = \lambda_J/20$ (25 $M_\odot$) nine oscillations, between the recombination and the present epoch. It is noted that residual perturbations persist for these masses.

Larger initial density contrasts ($\delta\rho/\rho > 1$) and the formation of the hydrogen molecule will considerably increase the value of $\delta\rho/\rho$ at recent epochs. The possibility thus exists that Population III stars of mass $25 < M/M_\odot < 200$ could have formed from perturbations which survived directly from the recombination era and need not necessarily have been formed from the fragmentation of massive clouds.

The number of oscillations which we obtained for $\lambda/\lambda_J < 1$ as a result of our calculations is smaller than the rough estimates given by Carr and Rees (1984): whereas we

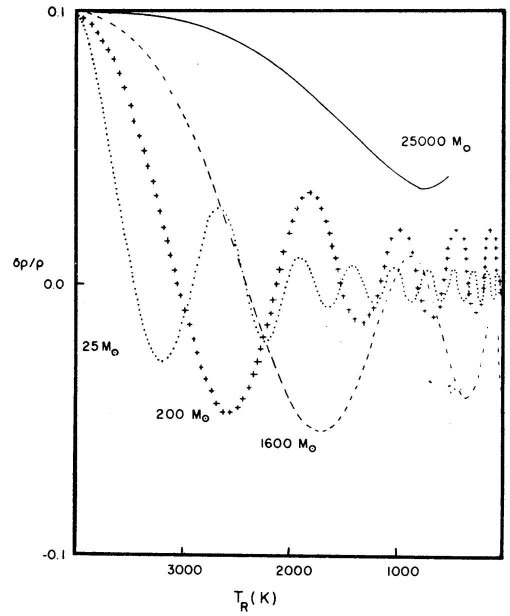

Fig. 4. The evolution of $\delta\rho/\rho$ for M = 25000 $M_\odot$ (λ/λ_J = 1/2), M = 1600 $M_\odot$ (λ/λ_J = 1/5), M = 200 $M_\odot$ (λ/λ_J = 1/10), and M = 25 $M_\odot$ (λ/λ_J = 1/20) for a flat universe (Ωh^2 = 1.0).

obtained for λ/λ_J = 1/5, 1/10 and 1/20, the number of oscillations 2, 4 and 9, respectively, Carr and Rees (1984) estimated 5, 10 and 20 oscillations, respectively.

An effect also mentioned by Carr and Rees (1984) and other authors is that residual oscillations can be damped due to nonlinear transfer of energy to higher harmonics. Using their criteria, we find that this process is not important in the calculations of Fig. 4.

Our results for M > 10^4 $M_\odot$ are in good agreement with the calculations of Peebles (1969) in the linear regime.

As mentioned above, initial density contrasts $\delta\rho/\rho > 1$ and the formation of the hydrogen molecule will increase $\delta\rho/\rho$ near the present epoch, aiding the formation of Population III stars. For example for $\delta\rho/\rho \propto M^{-\alpha}$ (e.g. Carr and Rees 1984), we have with α = 1/2 (e.g. Carr and Rees 1984) and $\delta\rho/\rho \sim 1$ for 10^7 $M_\odot$ (e.g. Kashlinsky and Rees 1983) $\delta\rho/\rho \sim 10^3$ for M $\sim$ 10 $M_\odot$. Such calculations are presently being made, the results of which will be reported elsewhere.

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