Rev. Mexicana Astron. Astrof. **12**, 46-48 (1986)

A COSMOLOGICAL MODEL WITH CYLINDRICAL SYMMETRY[1]

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I. INTRODUCTION.

In recent times Paul Birch (1982) has exhibited radioastronomical data by which it could be inferred the existence of cosmic rotation. There has been a certain polemics as to the interpretation of the data (Phinney, Webster; Birch 1983). Anyhow the possibility of such rotation, is worthwhile being investigated in the theoretical domain. In this summary we exhibit the assumptions and preliminary results as to the possibility of Mach's principle being valid in a universe with expansion and rotation. The differential equations of the present cosmological model are extremely complex and a numerical computation program is under way.

II. MODIFIED BIANCHI III METRIC.

The following time transformation,

$$d\tilde{t} = dt + F(t,\chi,\phi)d\phi, \quad (1)$$

is introduced into a Bianchi III metric (a closed cylindrical universe) and the metric of our cosmological model comes to be:

$$ds^2 = g_{11}(t,\chi,\phi)d\chi^2 + g_{22}(t,\chi,\phi)d\phi^2 + g_{33}(t,\chi,\phi)d\zeta^2 + $$
$$+ 2g_{02}(t,\chi,\phi)d\phi dt + dt^2, \quad (2)$$

$$g_{11} = g_{33}. \quad (3)$$

III. VELOCITY FIELD, STRESS TENSOR AND EINSTEIN'S EQUATIONS.

The velocity field is given by:

$$u^1 \neq 0,\ u^2 \neq 0,\ u^3 = 0,\ u^0 = 1, \quad (4)$$

1. Work partially supported by FINEP-ITA.

The last condition is adopted with the aim of seeking the compatibility of Mach's principle with the present universe model.

The energy-momentum tensor is of the perfect fluid type and it can be shown that,

$$R_{03} = R_{13} = R_{33} \equiv 0, \tag{5}$$

and that the Einstein equations are reduced to four non-linear differential equations.

IV. BOUNDARY CONDITIONS AND MACH'S PRINCIPLE.

The given metric field leads to the result:

$$0 = g_{11}(u^1)^2 + g_{22}(u^2)^2 + 2g_{02}(u^2). \tag{6}$$

Since a local Minkovsky metric is assumed, which means,

$$\lim_{\chi\to 0} g_{02}/\tilde{g}_{22} = 0, \quad (\tilde{g}_{22} \text{ : Bianchi III}) \tag{7}$$

it comes for the local observer:

$$\lim_{\chi\to 0} u^1 = 0, \qquad \lim_{\chi\to 0} u^2 = 0. \tag{8}$$

By the fact that conditions (8) are always valid for the local observer, the following conclusions come forth:

$$\lim_{\chi\to 0} \frac{du^1}{dt} = 0, \qquad \lim_{\chi\to 0} \frac{du^2}{dt} = 0, \tag{9}$$

bearing in mind that ds=dt.

Thus every local observer is inertial and every distant observer is non inertial. Therefore any inertial reference system in this space-time is essentially relative.

Hence, Newton's problem of an absolute inertial frame is non-existent. Observe that in the present context the cosmic fluid does not necessarily move in geodesics as in the case of the standard cosmological models, and so, there is not a cosmic inertial system for all comoving observers; in the present case the observers are not comoving. The standard models imitate Newton's hypothesis of an absolute inertial system; not the present.

It seems possible (depending in part on the solution of the differential equations) that Mach's principle (at least in its original formulation) can be verified in this model. Why ?

Looking at Newton's bucket experiment through a relativistic viewpoint, proper time comes to be the privileged coordinate, not necessarily the space frame variables. For,

$$ds' = g_{00}^{\frac{1}{2}}(r)ds, \qquad (10)$$

ds' : proper time of rotating observer at r,
ds : proper time of the inertial frame.

The component $g_{00}(r)$ states the gravitational interaction between the local rotating system with the rest of the universe. For the universal time dt=ds (last condition 4) is determined by the Einstein equations (time is not independent of matter !).

Mach's principle:

Standard Models

a) Every comoving observer is inertial. Existence of a cosmic inertial system.
b) Local accelerations referred to this cosmic system.
c) Local accelerations resulting from gravitational interaction through cosmic time:

$$ds' = g_{00}^{\frac{1}{2}}(r)ds,$$

$$ds = dt$$

Present Model

a) Every local observer is inertial. Non-existence of a cosmic inertial system. Possibility of inversion of the role played by two separate observers. That is not possible in Newton's problem. Inertia is essentially relative.
b) Local accelerations referred to the local inertial system.
c) Same.

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