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CORRECTIONS AND IMPROVEMENTS IN THE EFFECTIVENESS AND EXTENSION OF THE APPLICABILITY RANGE OF THE MODEL WINK OF ECLIPSING BINARIES.

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SUMMARY: Errors and inconsistencies still present in the program WINK (even after the corrections and improvements of all the WINK status Reports - WSR - now at the no. 10) have been detected and corrected. Modifications of the program have been implemented in order to improve the effectiveness of the model during the solution of the inverse problem of finding the parameters of a system from the light curves . Other modifications are introduced in order to extend the validity range of the model to closer and more deformed systems. They are: a) a control (and eventual limitation) of the sizes of components; b) introduction of the 4th and 5th order terms of Chandrasekhar's equations in the definition of the semi-major axes of the ellipsoids which represent the stars; and c) a more exact calculation of the local gravity in the treatment of the gravity darkening. Such an extension does *not* enable WINK to analyse semi-detached or closer systems due to the geometrical assumptions of the model (ellipsoidal stars). However, after the introduction of the cited modifications, the WINK solutions for such systems become much closer to the solutions produced by models which are more realistic geometrically (eg. the Wilson-Devinney model) than the solutions produced by the original WINK model.

Key words: Eclipsing Binaries, numerical models

I - INTRODUCTION

The WINK model (Wood, 1971a, 1971b, 1972) has been extensively in use since its publication and has been changed and corrected though the WSR, now at the no. 10 (Etzel and Wood, 1982). However there are some errors and inconsistencies still uncorrected in the versions updated and corrected with the WSR. The changes now reported have been detected by Vaz (1984) and are being simultaneously communicated to Dr. P.B. Etzel, who, together with Dr. D.B. Wood, is taking care of the WSR.

II - CORRECTIONS

II-1 - Annular and atmospheric eclipses

The geometrical treatment of annular (sub-routine ANNECL) and atmospheric (ATMECL) eclipses is in error. The correction of these errors has small effects on the published solutions due to the facts that: a) the annular eclipses seldom occur and when present are normally of short duration, having a relatively small influence on the solution, and b) almost nobody dares to assume atmospheric eclipses in the solutions.

II-2 - The analytical partial derivatives

The partial derivatives of the light intensity with respect to the normalization parameters - quadrature magnitude (Q) and third light (L3) - are the only ones which can be calculated analytically. The partial derivatives with respect to Q - $\partial L/\partial Q$ - are calculated using the "normalized calculated intensity" and not the "normalized observed intensity" as should be the case. The correct equation to be used in the program (SOLVE1) should be:

$$\partial L/\partial Q \ (=D(I,NVAR)) = -0.921034 \ L_{norm,obs} \ (=0.921034*LUM(I)) \qquad (1)$$

where $L_{norm,obs}$ is the normalized observed intensity.

When one is close to the solution there is, of course, very little difference between the two definitions, but this is not the case when one starts solving a system. The correct normalization is known to have influence on the solutions, and the use of eq. (1) improves significantly the effectiveness of the solution in these initial stages (Vaz, 1984).

II-3 - The reflection effect

Wood (1973, 1974) introduced in the treatment of the reflection effect some approximations for the calculation of the bolometric incident flux provenient from the companion star, F^*:

$$F^* = A \ f(T_{eff,s}) \ g(u_s) \ h(\cos z) \qquad (2)$$

where A is the apparent area of the source star for the illuminated point on the surface of the reflecting star, $f(T_{eff,s})$ is a function of the effective temperature of the source star, u_s is the *bolometric* limb-darkening coefficient of the source star, and cos z is the cosine of the zenith distance of the centre of the source star for the illuminated point. When the spherical irradiating star is entirely above the horizon of the illuminated point,

$$g(u_s) = 1 - u_s/3. \qquad (3)$$

In the case of a distorted star, $g(u_s)$ is only slightly different from eq. (3). The form of $g(u_s)$ becomes different when the source star is partially below the horizon, also. In this case, however, the influence on the system light is small and we can use eq. (3) in all cases with good approximation.

It is clear that the total energy emmited by the source star is independent of limb-darkening, but stellar disks are *not* of uniform brightness bolometrically. Even simple grey atmosphere models, for which the exact theoretical limb-darkening law can be deduced (see Mihalas, 1978 p. 72), present a bolometric limb-darkening coefficient of 0.639. Apparently there are very few works (e.g. Grygar, 1965) on these bolometric coefficients for normal stars, but the influence of these coefficients must be taken into account in the calculation of eq. (2).

The function h(cos z) can be approximated by (Wood, 1973):

$$h(\cos z) = A + B \cos z + C \qquad (4a)$$

where

$$C = \begin{cases} 0 & \text{(source star above the horizon)} \\ A_1 + B_1 \cos z & \text{(" \quad " partially below the horizon)} \end{cases} \qquad (4b)$$

(C is a correction for the penumbral zones). The coefficients A, B, A_1, and B_1 can be approximated by polynomials of the relative radii of the reflecting and source stars. However, the polynomial coefficients published by Wood (1975) are wrong, inducing errors in the calculations of h(cos z) of up to 100% (Vaz, 1984). A satisfactory approximation is:

$$\text{coefficient } (A,B,A_1, \text{ or } B_1) = \sum_{i=1}^{3} \sum_{j=1}^{4} a_{ij} \, r_R^{(i-1)} \, r_S^{(j-1)} \qquad (5)$$

where r_R, r_S are the relative radii (relative to the orbital separation) of the reflecting and source stars, respectively, and the coefficients a_{ij} are given in Table 1.

The correction factor introduced by Wood (1976) in order to take correct account of the geometrical sizes of the zones used in the calculation of the bolometric incident flux, F^*, (since two extra zones should have been considered in the original derivation, and the zones would therefore become smaller) is spurious. Wood did not take into account the re-distribution in position of the smaller zones, and this re-arrangement compensates the smaller areas. Therefore this correction factor should not be considered.

TABLE 1. The coefficients a_{ij}

	A	B	A_1	B_1
a_{11}	-.0018167041	.99643308	.001365895	-.52081436
a_{12}	.041187171	-.01938372	-.03261647	-.0013132542
a_{13}	-.059758633	.094374851	.042438164	-.0089261314
a_{21}	.031235491	-.006012087	.42151396	-.064085118
a_{22}	-.68578226	.62239663	.42224965	-.41049702
a_{23}	1.6054241	-2.4368603	-.62050305	3.4409550
a_{31}	-.10616891	-.24242064	.059194053	.25842428
a_{32}	2.3082648	-2.3967460	-1.2103113	3.9927836
a_{33}	-6.3639234	6.8966633	2.9400545	-14.447191
a_{41}	.10674626	-.18033679	-.11772629	-.025408053
a_{42}	-1.5578767	.62633707	.29841723	-2.8981216
a_{43}	10.520706	-12.991757	-6.6742502	23.154907

Another fact is that Wood's approximation, eq (2), calculated with the use of eqs. (3), (4), (5), and Table 1, was generated for spherical stars, but is valid when the reflecting star being considered is an ellipsoid (Vaz,1984). The only necessary modification to extend the approximation to ellipsoidal reflecting stars (the source star is still considered spherical in the calculation of A) is to correct the calculation of cos z, now taking account for the ellipsoidal form of the reflecting star.

II-4 - The calculation of the bolometric flux

The WINK model defines the temperature of a illuminated point on one of an the components by

$$T_{heated} = T_{unheated} \; (1 + wF^*/F)^{1/4} \tag{6}$$

where F^* is the bolometric incident flux -eq. (2) -, F is the bolometric flux emitted by the reflecting star at that point in the absence of irradiation, and w is the bolometric reflection albedo. Recently, Vaz and Nordlund (1985) have analysed the reflection effect in grey atmospheres and were able to express their results in a practical way for use in models of eclipsing binaries. The calculation of F is in error when the reflecting star happens to be the secondary star (the reflection effect is considered for both stars). In section V-3, below, it is suggested a complete substitution of Wood's approximating equations by Chandraskhar's (1933b) equation for ellipsoidal stars.

The effects of corrections II-3 and II-4 (using Chandrasekhar's equation) is to eliminate systematic trends in the determination of the elements of the system. The amount of the effect is dependent on the system, but it can reach up to 10% in the radii and 5% in the orbital inclination.

III - IMPROVEMENTS IN THE EFFECTIVENESS

III-1 - Correlation between the parameters

It is well known that the numerical methods can simulate a high correlation between certain parameters. In the case of eclipsing binaries, we have for example, the correlation between k, the ratio of radii, and all the other parameters, which is very high when k ~ 1 and $T_A \sim T_B$ - Andersen *et al.*, 1980; Vaz and Andersen, 1984 - or between w, the bolometric reflection albedo, and β, the gravity darkening exponent - Vaz, 1984, 1985. Therefore it is very convenient to control the numerical correlation between the parameters adjusted by the least squares differential correction method while performing a solution, in order not to permit two highly (numerically) correlated parameters to be simultaneously free during the iterations. The correlation coefficients are very dynamic, changing significantly in the course of the solution, and have to be controled continuously. In general, when the program is approaching the solution, the correlation coefficients become smaller but this is a very local effect around the solution point. Then, the correlation coefficients can be used as a secure guide in the solution process and the introduction of their calculation in the program requires very small changes in the computer code.

III-2 - The character of the numerical partial derivatives

The numerical partial derivatives of the light intensity with respect to the orbital inclination, i, *must* be symmetrical. The asymmetrical definition used in the program, with a backwards step, is a poor approximation, since the effect of i on the light curves is strongly variable, and dependent on the other geometrical parameters (though mainly on the stellar relative radii). Furthermore it can be shown by numerical tests that the program works less effectively when $\partial L/\partial i$ is calculated asymmetrically (Vaz, 1984). The original steps and definitions for the other numerical partial derivatives were controled and found to be satisfactory.

III-3 - Economic version

The most expensive part of the WINK program is the calculation of the numerical partial derivatives for the least squares solution. The partial derivatives with respect to Q and L3 can be calculated analytically and take practically no computer time. For that reason we introduced in the program the option of using the numerical partial derivatives calculated for a certain iteration in subsequent iterations, recalculating only the partial derivatives which have an analytical definition. The number of "cheap" iterations between two normal is determined by the user. The cheap iterations are used to find a better starting point for the following normal iteration. After each cheap iteration the new fit is tested. If the fit becomes worse than the previous fit, then the program automatically breaks the chain of cheap iterations, and the next iteration will be a normal one. Otherwise the number of cheap iterations determined by the user is performed prior to doing the next normal iteration.

Experience shows that an economic iteration will always improve the fit to the observations, even if the starting point is far from the wanted solution and the partial derivatives change significantly after each iteration, due to the necessarily large corrections. When the starting point is relatively close to the solution, then the cheap iterations have practically the same effect as the normal ones because the partial derivatives change only little as the solution is approached.

After some test runs it proved convenient to permit 2-3 cheap iterations between any two normal iterations. Test runs starting at the same point and having the same number of normal iterations show that the runs with cheap iterations between the normal ones obtain better fits. Sometimes they converge to the desired solution with fewer normal iterations.

IV - IMPROVEMENTS IN THE TREATMENT OF ECCENTRIC SYSTEMS

The effects of orbital eccentricity on the light curves of eclipsing binary systems manifest themselves in the position of the central phase of the secondary minimum, relative to that of the primary minimum $(T_2-T_1)/P$, and on the relative durations of the minima, D_{sec}/D_{pri}. Both quantities $(T_2-T_1)/P$ and D_{sec}/D_{pri}, in particular $(T_2-T_1)/P$, can be determined with high accuracy from the observed light curves, and can be used for fixing the correct values of e (orbital eccentricity) and ω (position angle of periastron), consistent with i, a_o and k, by an iterative process, instead of adjusting e and ω by the least squares method. The errors involved in the process are usually smaller than when least squares adjustments are used, and the effectiveness of the whole process is very high, saving computer time. Application of the program with the improved treatment of e and ω on the eccentric systems KM Hya (Andersen and Vaz, 1984) and PV Pup (Vaz and Andersen, 1984), for which the values of e and ω could be controlled by the high quality spectroscopic data, turned out to be extremely successful.

V - EXTENSION OF THE APPLICABILITY RANGE OF THE MODEL

V-1 - Roche Lobes

A limit is imposed on the sizes of the components, using, when necessary, the volume of the corresponding Roche lobe as the criterion to limit the volume of the tri-axial ellipsoids which represent the stars. This prevents the known tendency of WINK to produce solutions in which the stars are larger than the corresponding Roche lobe. (Provoost, 1980; Andersen *et al.* 1983).

V-2 - The order of approximation of the semi-major axes

The 4th and 5th order of Chandrasekhar's (1933a) equations for the semi-major axes are introduced in order to take into account the asymmetric tidal deformations of the figure for the stars. The 4th order term is considered to be equivalent to a displacement of the geometrical centres of the ellipsoids relative to centre of mass, so that the ellipsoids get a little closer to each other. In distorted systems, the form of the ellipsoids gets closer to the corresponding equipotential surface after introduction of the 5th order term.

After introduction of modifications V-1 and V-2 it is possible to use the modified WINK model to derive the starting parameter set to be used by, for instance, the Wilson-Devinney (WD) model (Wilson and Devinney, 1971, 1973; Wilson *et al.*, 1972; Wilson and Biermann, 1976; Leung and Wilson, 1977; Wilson, 1979), which is much more expensive, and therefore save computer time. In this sense, and in this sense only, the applicability of the WINK model has been extended: for very close systems the intrinsic geometric approximations of WINK are inadequate for a definitive solution, but when used with care the WINK model can provide reasonable solutions to be improved by other models.

V-3 - Modifications in the treatment of the gravity darkening

V-3-a - The bolometric flux

The ratio of the local bolometric flux to that at e reference point is

$$F_{local}/F_{ref} = (g_{local}/g_{ref})^{\beta} \qquad (7)$$

where β=1 for stars with atmospheres in radiative equilibrium (von Zeipel, 1924) or β=0.32 for stars with deep convective layers (Lucy, 1967). Wood (1978) introduced the approximation

$$g_{local}/g_{ref} = 1 + k(r_{local}/r_{ref} - 1) \qquad (8)$$

where the radii are the radius vectors for the ellipsoids defined by Chandrasekhar (1933a). The approximation (8) does imply problems as the coefficient k is *not* constant and in fact becomes very large at certain points (Wood, 1978). However eq. (8) is not necessary, since Chandrasekhar (1933b) derived expressions for the surface gravity within the same philosophy and to the same degree of approximation as for the surface geometric form for the stars. Chandresekhar's equations are then introduced in order to improve the correctness and consistency of the model.

V-3-b - The monochromatic gravity darkening

Eq. (7) is valid only bolometrically. Following Kopal (1959), we can expand the monochromatic flux in a Taylor series in the neighbourhood of T_{ref} and write:

$$F_{\lambda}/F_{\lambda,ref} = 1 + b\ (T/T_{ref} - 1) \qquad (9)$$

where

$$b = (\partial \ln F_{\lambda}/\partial \ln T)_{T_{ref}} \qquad (10)$$

Using Stefan's law and eq. (7) we can write

$$F_{\lambda}/F_{\lambda,ref} \simeq 1 + b\beta\ (g/g_{ref} - 1) \qquad (11)$$

In the WINK model, b in eq. (10) is calculated using the Planck function as an approximation for F_{λ}, even in the version with stellar atmospheres (after WSR no. 7). Ao soon as we have a table of the flux as a function of temperature and log g it is easy, and more consistent, to calculate numerically the partial derivative of eq. (10). This is of course a little more time consuming, but the gain in internal consistency and exactness compensates for the slightly longer computing time.

VI - CONCLUSIONS

Figure 1 shows the residuals Δ Mag (O-C) for the normal points of V Pup, which is a semi-detached, almost contacting system(as generated by Andersen *et al.* (1983) with observations by Clausen *et al.* (1983)) from various theoretical light curves. The signs (+) represent

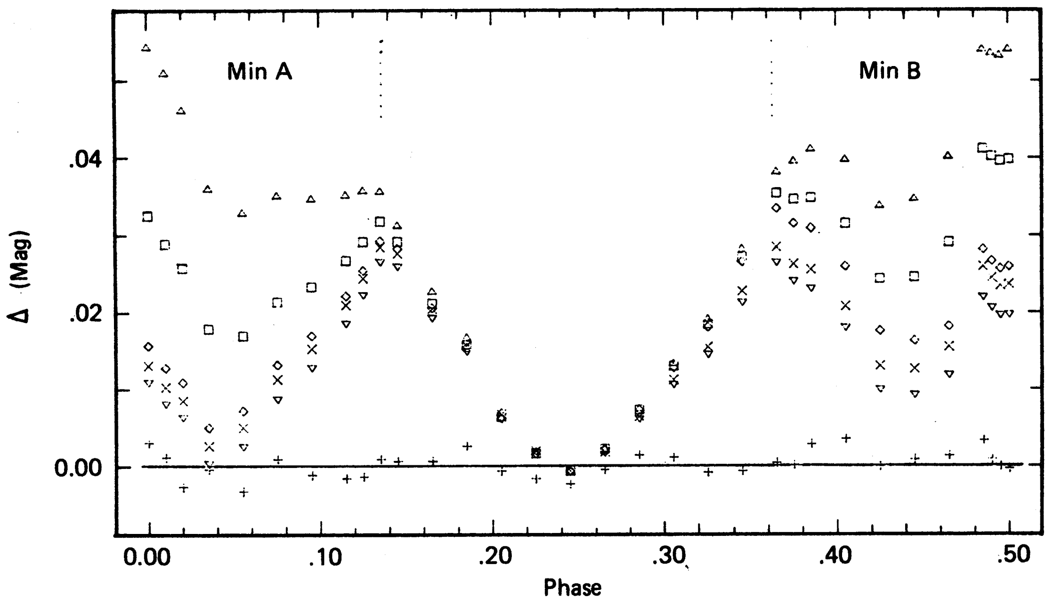

Fig. 1 Theoretical light curves for V Pup

the residuals for the theoretical solution obtained with the WD model by Andersen *et al.* (1983). The other symbols correspond to theoretical light curves generated by WINK with the correct parameter set, obtained by Andersen *et al.* (1983) with the WD model. The only difference is the number of modifications introduced in the WINK model, as follows. The symbols (Δ) show the residuals corresponding to the theoretical light curve generated by WINK modified so that the size of the secondary component is limited to the corresponding Roche lobe. The criterion used for the limiting radius is the mean radius of the equipotential surface. The (□) correspond to WINK now using the "volume radius" as the criterion for the limiting surface of the secondary. The models corresponding to the symbols (Δ) and (□) preserve all the other geometrical approximations of the original WINK. The (◇) correspond to the (□) model implemented with the 4th and 5th order terms of Chandrasekhar's equations. The (×) represent the (◇) model implemented with the correct calculation of the surface gravity of Section V-3-a, with the coefficient b, eq. (10), being calculated using the Planck approximation. The symbols (∇) correspond to the (×) model but with b calculated by applying the definition (eq. (10)) and numerical derivatives in the tables defining the atmospheres of the stars (from Kurucz (1979) tables).

It is again stressed that it was the *same* set of parameters for all the models of Fig. 1, the only difference being the corresponding modification of WINK. It is clear that after each modification the WINK model became more realistic, reproducing the observations better in the mean. It should be remembered that the WINK model is definitely not valid for this very distorted system and that the fit to the observations by the WINK theoretical light curves is expected to show large systematic deviations. However, the improvement shown in Fig. 1 is very significant because the natural consequence is that, when the program is applied in the solution mode, the model (∇), which best represents the observations with the "correct" set of parameters, will change this input set less than the model (Δ) will. In other words, the "final" set of parameters (i.e. obtained after a normal solution procedure) will then be closer to the correct set when the model (∇) is used.

As already pointed out the proposed modifications are *not* intended to replace the WD model, but to save time in its application in the analysis of deformed systems by using the improved WINK to provide very good starting elements to be used with the expensive WD program. It is worth mentioning that the WINK model after introduction of all these modifications still runs 5-10 times faster (and, therefore, cheaper) than the WD model.

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