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# VALIDATION OF SAGNIER'S THEORY BY ITS APPLICATION TO THE MAJOR PLANETS

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ABSTRACT. In this paper, the theory of Sagnier was applied to the major planets, in order to have a first evaluation of this theory which was built, primarily, to study the galilean satellites of Jupiter. In this application we included the first order of the intermediary solution and the solution of the linear variational equations. Results from other theories have been used as basis for comparisons. The results and their comparisons are shown by means of tables of coefficients and proper frequencies.

Key words: planetary theory, major planets.

## I. INTRODUCTION

In 1981, J. L. Sagnier proposed a new method to construct the analytical series representing the parameters of the dynamical system formed by the galilean satellites of Jupiter. In this paper we present the results of the application of that method to the system formed by the four major planets. This application aims at a knowledge of the method and at an independent check of some of its parts. The choice of the major planets arised from the fact that several good theories exist for this system, thus allowing a sure frame of reference.

## II. RESULTS AND COMPARISONS

We give here some results with respect to the intermediary solution and the proper frequencies. For all calculations we used the integration constants given in Simon and Bretagnon (1975). All constants are shown in Table I.

Table I
Integration constants and masses

| | Júpiter | Saturn | Uranus | Neptune |
|---|---|---|---|---|
| n(''day$^{-1}$) | 299.1283 | 120.4547 | 42.2309 | 21.5349 |
| $\varepsilon$ | 316°11'38'' | 158 19 01 | 99 07 42 | 194 24 02 |
| $e_o$ | 0.048417 | 0.055719 | 0.046331 | 0.009000 |
| $i_o$ | 1°18'21".2 | 2 29 26.0 | 0 46 22.0 | 1 46 28.1 |
| $\bar{\omega}_o$ | 13°31'34'' | 92 04 39 | 172 17 27 | 47 26 23 |
| $h_o$ | 99 56 55 | 113 13 37 | 73 43 36 | 131 13 51 |
| reciprocal masses | 1047.357 | 3498.1 | 22759 | 19332 |

Table II shows the intermediary solution, represented by the coefficients of complex exponentials. It is compared with first order intermediary solution by Duriez (1979). In both cases it is the solution at the first order with respect to the masses, not depending on eccentricities and inclinations.

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TABLE II

Coeficients compared for the intermediary solution.Jupiter disturbed by Saturn.

| $^1\eta$ coeficient('')of exp is($1_S-1_J$) | | | $^2\eta$ coeficient ('') of exp is ($1_S-1_J$) | | | $^3\eta$ coeficient('')of exp i($s1_S-(s-1)1_J$) | | |
|---|---|---|---|---|---|---|---|---|
| S | Gomes | Duriez | S | Gomes | Duriez | S | Gomes | Duriez |
| | | | | | | -2 | 3.13 | 3.13 |
| 1 | -24.37 | -24.36 | 1 | -6.08 | -6.08 | -1 | -1.14 | -1.13 |
| 2 | -33.63 | -33.63 | 2 | -20.80 | -20.80 | 0 | -7.09 | -7.09 |
| 3 | -12.43 | -12.42 | 3 | -9.53 | -9.52 | 1 | -16.53 | -16.53 |
| | | | | | | 2 | 134.88 | 134.91 |
| | | | | | | 3 | 22.41 | 22.41 |
| 4 | -5.24 | -5.23 | 4 | -4.57 | -4.56 | 4 | 8.10 | 8.09 |
| 5 | -2.37 | -2.36 | 5 | -2.25 | -2.24 | 5 | 3.47 | 3.46 |

Proper frequencies are shown in Table III for plane and spatial cases. They are very little different from proper frequencies given by other theories, like classical Laplace-Lagrange and Duriez (1979) at first order. That can be confirmed by examining the analytical form of the matrix that originates the eigenvalues, which has the same form, for instance, in Sagnier's theory and in Laplace-Lagrange theory. The only differences are due to different meaning of the constants used in each theory.

TABLE III

Proper frequencies in"/day

| $e \exp i\varpi$ | $\mathrm{Sin}(I)\exp i\Omega$ |
|---|---|
| 0.01018427 | 0.0 |
| 0.06154837 | -0.0710480 |
| 0.00742530 | -0.0079874 |
| 0.00174425 | -0.0018668 |

## III. CONCLUSIONS.

Our aim has been to verify the theory of Sagnier, by its application to the major planets. In order to reach that purpose we have made use of results from other theories, which served as basis for comparisons. Concordance has been found to a sufficiently good approximation. A good deal of Sagnier's theory was revised, mainly in its algebraic developments.

No effort was made to find superior order terms because we did not aim at an ephemeris construction. Nevertheless some peculiar features in superior orders of Sagnier's theory might possibly give interesting results when applied to planets or satellites.

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