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ON THE VERTEX DEVIATION OF YOUNG STARS

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RESUMEN

Se muestra que la desviación del vértex de estrellas jóvenes es una consecuencia natural del carácter epicíclico de los movimientos estelares en la galaxia. Si las estrellas son suficientemente jóvenes (edades del orden de 1.8×10^7 años), la desviación del vértex es de alrededor de -30° y no resulta muy afectada por la formación de estrellas en un brazo o por las velocidades sistemáticas en el momento de su nacimiento; las estrellas de edades mayores (edades del orden de 6.5×10^7 años) sufren los efectos de modelado de sus condiciones de nacimiento.

ABSTRACT

We show that the vertex deviation of young stars is a natural consequence of the epicyclic character of the stellar motions in the galaxy. If the stars are young enough (ages of about 1.8×10^7 years), the vertex deviation is about -30° and it is not much affected by the formation of stars in an arm or their systematic velocities at birth; older stars (ages of about 6.5×10^7 years) suffer the modelling effects of their birth conditions.

Key words: **GALAXY-STRUCTURE**

I. INTRODUCTION

The elementary theory of stellar systems with rotational symmetry shows that if the distribution of peculiar stellar velocities is ellipsoidal, then the longest axis of the velocity ellipsoid (i.e. the vertex direction) should point toward the galactic center (see, e.g. Ogorodnikov, 1965). This prediction agrees with the observational results for old stars, but young stars exhibit an appreciable vertex deviation in the plane of the Galaxy. See, e.g. Delhaye (1965), for a general review of the subject, Nordström (1936) and Filin (1957) for the particular case of the young stars.

Several authors attempted to explain the vertex deviation within the framework of theories that predict regions of star formation, like spiral arms, and the velocities the stars have when they are born. Among others, we can mention the work of Yuan (1971) and of Hilton and Bash (1982); the latter used the observational results of Filin (1957) for comparison. While these attempts are interesting, in the present paper we want to sound a word of caution regarding the use of the vertex deviation as a check for theories. Using the epicyclic approximation, we show that the vertex deviation is just a natural consequence of the stellar motions in the galaxy (section II) and then, in section III, we present simple numerical models to explore the limits of our analytical derivations; in section IV we discuss the different causes that affect the vertex deviation for young stars, and in section V we summarize our results.

II. A SIMPLE ANALYTICAL THEORY

As we are interested in the motions of young stars, whose orbits do not depart too much from circles, it is reasonable to use the epicyclic approximation for the present investigation; Woolley (1970) also used that approximation to investigate the vertex deviation, but he was interested in stars older than those we will deal with.

Adopting the usual convention for the coordinates in the rotating frame of reference, ξ and η, and the residual velocities, u and v, we have (see, e.g., Lindblad 1959):

$$\xi = C_1 + C \cos \kappa (t - t_0) \tag{1}$$

$$\eta = -2 A C_1 (t - t_1) - C (\omega/-B)^{1/2} \sin \kappa (t - t_0) \ , \tag{2}$$

$$u = -C \kappa \sin \kappa (t - t_0) \ , \tag{3}$$

$$v = 2 B C \cos \kappa (t - t_0) \ ; \tag{4}$$

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where t is time, ω is angular speed, κ is epicyclic frequency, A and B are Oort constants near the Sun, and C, C_1, t_0 and t_1 are parameters of the orbit.

Let us set $t = 0$ at present and $t = -\tau$ at the time the stars were born. As the stars we observe lie now near the Sun, we have:

$$0 = C_1 + C \cos \kappa t_0 \ , \qquad (5)$$

$$0 = 2 A C_1 t_1 + C (\omega/-B)^{1/2} \sin \kappa t_0 \ ; \qquad (6)$$

and if the components of the velocity of the star at birth are $(-V \cos l, V \sin l)$, then:

$$-V \cos l = C \kappa \sin \kappa (\tau + t_0) \ , \qquad (7)$$

$$V \sin l = 2 B C \cos \kappa (\tau + t_0) \ . \qquad (8)$$

Solving the system of the equations (5) through (8) we obtain the orbital parameters which, substituted in equations (1) through (4) allow us to obtain the present stellar velocity:

$$u_0 = -V \cos l \cos \kappa\tau - (\kappa V \sin l \sin \kappa\tau)/2B \ , \qquad (9)$$

$$v_0 = V \sin l \cos \kappa\tau - (2 B V \cos l \sin \kappa\tau)/\kappa \ ; \qquad (10)$$

and the stellar position at birth:

$$\xi_{-\tau} = [V (1 - \cos \kappa\tau) \sin l]/2B + (V \sin \kappa\tau \cos l)/\kappa \ , \qquad (11)$$

$$\begin{aligned} \eta_{-\tau} = & -(\omega/-B)^{1/2} V (1 - \cos \kappa\tau) \cos l/\kappa \ + \\ & + 2 A \tau V \cos l \sin \kappa\tau/\kappa \ - \\ & - A \tau V \sin l \cos \kappa\tau/B \ + \\ & + (\omega/-B)^{1/2} V \sin l \sin \kappa\tau/2 B \ . \end{aligned} \qquad (12)$$

It is easy to see that formulae (9) and (10) define an ellipse, whose axes point in the directions $l_{1,2}$ given by:

$$\operatorname{cotg} 2l_{1,2} = [(\kappa^2 + 4B^2) \operatorname{tg} \kappa\tau]/4B\kappa \ . \qquad (13)$$

In other words, if we assume that the stellar velocities have the *circular* velocity distribution, equations (7,8) when the stars are born, later on they acquire the *elliptical* velocity distribution, equations (9) and (10), whose vertex deviation is given by formula (13).

The vertex deviation of stars just born ($\tau = 0$), defined as a limit because the velocity distribution is circular, is $-45°$ ($B < 0$); if we consider older stars, the vertex deviation diminishes (in absolute value) reaching 0° for $\tau = \pi/2\kappa$, when the ratio of the semiaxes reaches a value of $4B^2/\kappa^2 \cong 0.4$, and then acquires positive values reaching a maximum of +45° for $\tau = \pi/\kappa$, when the distribution is circular again.

Figure 1 shows the velocity ellipse of equations (9) and (10) for $V = 10$ km s^{-1} and $\tau = 10^7$ y; the vertex deviation is $l = -35°$ and the minor and major semiaxes are 8.6 km s^{-1} and 11.6 km s^{-1}, respectively.

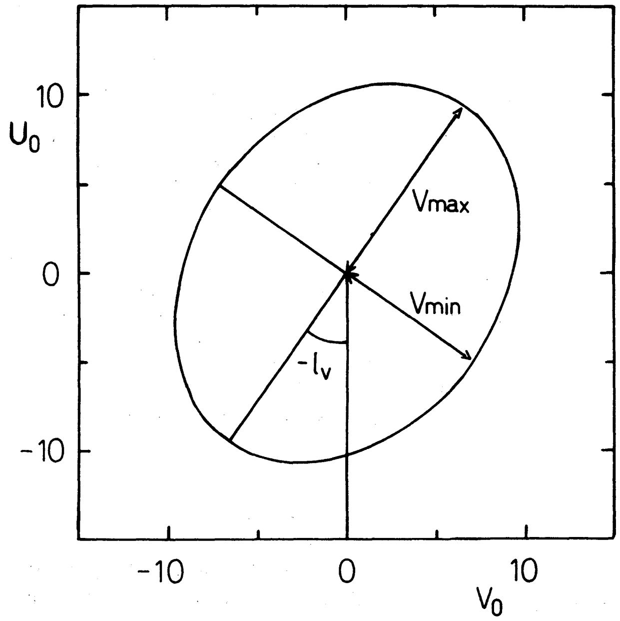

Fig. 1. Velocity ellipse given in equations (9) and (10) for $V = 10$ km s^{-1} and $\tau = 10^7$ y; the vertex deviation is $l_V = -35°$ and the minor and major semiaxes are 8.6 km s^{-1} and 11.6 km s^{-1}, respectively.

The model is too simple, and we will not push its consequences farther, but we note that it allows us to explain the large negative vertex deviation of very young stars, and the decreasing deviation and the increase in the ratio of the semiaxes of the velocity ellipse for older stars, as mere consequences of the epicyclic motion of the stars in the galaxy.

III. NUMERICAL MODELS

Several objections can be raised about the naive analysis of the previous section. To begin with, stars of a given spectral type and luminosity class do not have all the same age, but rather they have ages that span from zero to τ years. Besides, although it is reasonable to accept a spheroidal velocity distribution for the stars just born, different velocities have different probabilities: the larger the velocity the smaller the probability; the effect of systematic motions, in addition to the random ones, has not been considered either. Also, not all the stars used for kinematical studies lie in the immediate solar neighborhood; allowance for at least a few hundred parsecs should then be made in formulae (5) and (6). Finally, formulae (11) and (12) imply that the stars had to be born in certain places (again an ellipse) and, if stars were not born τ years ago in *all* those places, then the velocity ellipse given by equations (9) and (10) would be deformed, because parts of it would be lacking.

The effect of the systematic motions is the easiest to evaluate. If in formulae (7) and (8) we add, respectively, systematic terms u_i and v_i to the first members, it follows simply that in formulae (9) and (10) additional terms appear which are respectively:

$$\Delta u_0 = u_i \cos \kappa\tau - (\kappa/2B)\, v_i \sin \kappa\tau \qquad (14)$$

and

$$\Delta v_0 = (2B/\kappa)\ u_i \sin \kappa\tau + v_i \cos \kappa\tau. \qquad (15)$$

These terms change with stellar age, τ, but they are independent of longitude; therefore, the systematic motions of stars at birth have no direct incidence on the vertex deviation. They could, however, have some indirect effect because, for example, they also change the positions given in formulae (11) and (12) where the stars originated.

The most likely result of the other effects is probably a softening, or blurring, of the clear-cut results of the previous section and, as long as the stars are young enough, the main trends should not be much altered. For example, if we consider velocities of about 10 km s^{-1}, stars younger then 10^7 years could not have drifted more than 100 pc from their place of birth, and the modelling introduced by the presence of an arm about 1000 pc wide is probably irrelevant; alternatively, spectral types that include stars up to 10^8 years old would yield a mixture of negative and positive vertex deviations and, introducing thus, a considerable complication of our elementary scheme.

In order to obtain a quantitative check of the previous qualitative description we carried out some numerical simulations as follows. Every 10^6 years, and over a time span equal to the maximum stellar age, we generated a fixed number of stars, choosing at random their positions from a uniform distribution and their velocities from a circular Gaussian distribution. From those positions and velocities at birthtime we obtained, using the epicyclic approximation, the present positions and velocities; stars that lie at present within a given distance from the Sun were retained, and the others were rejected. We then used the final sample to derive the mean velocity and the parameters of the velocity ellipse (i.e. minimum and maximum velocity dispersions and vertex deviation).

Maximum stellar ages of 1.8×10^7 years or of 6.5×10^7 years were adopted for most models; the former corresponds to a spectral type of about B2.5, and the latter to one of about B5, according to the age scale adopted by Hilton and Bash (1982). A couple of models were run taking a maximum age of 13×10^7 years, a value large enough to yield a positive vertex deviation in

TABLE 1
PARAMETERS OF THE NUMERICAL MODELS

Model	Max. Age (10^7 y)	Radius Solar Neighb. (pc)	Region Star Formation	System u (km s^{-1})	System v (km s^{-1})
1	1.8	200	no limits	0	0
2	1.8	400	no limits	0	0
3	6.5	200	no limits	0	0
4	6.5	400	no limits	0	0
5	1.8	200	arm ($\xi > 0$)	0	0
6	1.8	400	arm ($\xi > 0$)	0	0
7	6.5	200	arm ($\xi > 0$)	0	0
8	6.5	400	arm ($\xi > 0$)	0	0
9	1.8	200	no limits	8	0
10	1.8	200	no limits	0	– 8
11	6.5	200	no limits	8	0
12	6.5	200	no limits	0	– 8
13	1.8	200	arm ($\xi > 0$)	8	0
14	1.8	200	arm ($\xi > 0$)	0	– 8
15	6.5	200	arm ($\xi > 0$)	8	0
16	6.5	200	arm ($\xi > 0$)	0	– 8
17	13.0	200	no limits	0	0
18	13.0	200	arm ($\xi > 0$)	0	0
19	1.8	200	no limits	8	– 8
20	6.5	200	no limits	8	– 8

TABLE 2

MEAN VELOCITIES AND PARAMETERS OF THE VELOCITY ELLIPSES

Model	$\langle u \rangle$ (km s^{-1})	$\langle v \rangle$ (km s^{-1})	σ_{max} (km s^{-1})	σ_{min} (km s^{-1})	$\sigma_{min}/\sigma_{max}$	l_v
1	0.6 ± 0.5	0.4 ± 0.4	9.3 ± 0.1	6.9 ± 0.2	0.74 ± 0.02	− 31.8 ± 6.4
2	0.5 ± 0.2	− 0.5 ± 0.9	9.5 ± 0.0	7.2 ± 0.1	0.76 ± 0.01	− 19.0 ± 6.9
3	0.6 ± 0.6	− 0.4 ± 0.1	11.1 ± 0.3	6.4 ± 0.2	0.58 ± 0.01	− 8.4 ± 1.2
4	− 0.6 ± 0.6	0.2 ± 0.3	11.7 ± 0.6	6.2 ± 0.1	0.54 ± 0.02	− 5.4 ± 0.5
5	− 3.0 ± 1.2	0.3 ± 0.5	8.8 ± 0.4	6.8 ± 0.2	0.78 ± 0.05	− 32.1 ± 1.0
6	− 1.7 ± 0.4	− 0.3 ± 0.3	9.5 ± 0.1	7.1 ± 0.2	0.75 ± 0.02	− 33.4 ± 8.6
7	− 6.9 ± 0.5	1.0 ± 0.2	9.4 ± 0.1	5.6 ± 0.1	0.59 ± 0.01	− 23.0 ± 2.7
8	− 5.9 ± 0.3	1.0 ± 0.0	9.7 ± 0.3	6.4 ± 0.0	0.67 ± 0.02	− 14.6 ± 5.0
9	− 3.9 ± 0.3	− 7.5 ± 0.7	9.8 ± 0.2	7.4 ± 0.1	0.75 ± 0.03	− 25.9 ± 10.4
10	6.7 ± 0.4	− 1.8 ± 0.6	9.3 ± 0.1	7.0 ± 0.2	0.76 ± 0.04	− 30.8 ± 1.5
11	− 9.2 ± 1.0	− 3.7 ± 0.1	11.3 ± 0.5	7.6 ± 0.0	0.68 ± 0.03	3.2 ± 3.7
12	3.5 ± 0.4	− 3.7 ± 0.2	11.8 ± 0.3	6.4 ± 0.4	0.54 ± 0.04	− 7.0 ± 2.0
13	− 6.4 ± 1.3	− 7.7 ± 0.7	8.6 ± 0.4	7.2 ± 0.2	0.83 ± 0.02	− 48.2 ± 8.7
14	2.8 ± 0.7	− 0.4 ± 0.2	8.7 ± 0.2	6.7 ± 0.4	0.77 ± 0.05	− 48.7 ± 11.5
15	− 12.7 ± 0.3	− 2.4 ± 0.2	10.2 ± 0.2	7.4 ± 0.3	0.73 ± 0.03	− 8.4 ± 3.7
16	− 6.6 ± 0.3	− 1.8 ± 0.5	10.5 ± 0.4	5.7 ± 0.1	0.54 ± 0.02	− 17.3 ± 3.8
17	− 0.4 ± 0.4	− 0.6 ± 0.2	10.6 ± 0.7	6.6 ± 0.4	0.62 ± 0.05	− 5.9 ± 2.6
18	− 3.6 ± 0.4	3.6 ± 0.2	10.0 ± 0.6	5.5 ± 0.2	0.56 ± 0.06	− 13.9 ± 1.8
19	4.4 ± 0.9	− 9.5 ± 0.8	9.6 ± 0.5	7.0 ± 0.4	0.73 ± 0.04	− 23.3 ± 7.3
20	− 4.7 ± 0.4	− 7.4 ± 0.4	12.2 ± 0.2	6.4 ± 0.2	0.53 ± 0.02	− 5.0 ± 2.2

our simple analytical model. The maximum radius for the solar neighborhood was also given two different values (200 pc and 400 pc), in different models, to check its effect on the kinematical properties.

Oort constants A and B were taken as 15 km s^{-1} kpc^{-1} and -10 km s^{-1} kpc^{-1}, respectively (Schmidt 1965); the velocity dispersion of the stars at birth was chosen as 8 km s^{-1}, a value similar to the one for gas clouds (see, e.g., Kerr 1968) and very young stars (see, e.g., Delhaye 1965).

The extension of the uniform spatial distribution of birthplaces was limited by the distance that a star would traverse, moving on a straight line with a speed equal to three times the velocity dispersion, in a time interval equal to the maximum stellar age. Thus, there is an extremely low probability that a star, whose velocity at birth follows the chosen law, reaches the solar neighborhood if it were born outside that region; in practice, this is equivalent to considering an unlimited region of star formation.

In order to obtain some insight into the influence of different conditions at birth, we also simulated stellar formation in a spiral arm limiting the birthplaces to the region with $\xi > 0$; the effect of systematic velocity components was investigated adding a constant value of 8 km s^{-1} in one or both coordinates to all the velocities at birth time.

Table 1 presents the different parameters and properties of the models. In each case, three different models were run keeping fixed those parameters and properties, and changing the seed number for the random number generator. Table 2 gives the mean values and root mean square error obtained in each case, from the three corresponding models, these are: the mean velocity components, $\langle u \rangle$ and $\langle v \rangle$; the maximum and minimum velocity dispersions, σ_{max} and σ_{min}; the ratio of the velocity dispersions; and the vertex deviation.

A sample of the results of our simulations is shown in Figure 2 which gives the distribution of the representative points on the velocity plane and the corresponding velocity ellipse. The sample is one of the models of case 7, i.e., for a stellar age of 6.5×10^7 years, with the stars ending up at less than 200 pc from the sun after being born in an arm. The corresponding parameters of the velocity ellipse are:

$$\langle u \rangle = -7.8 \text{ km s}^{-1} \qquad \langle v \rangle = +0.7 \text{ km s}^{-1}$$
$$\sigma_{max} = 9.6 \text{ km s}^{-1} \qquad \sigma_{min} = 5.4 \text{ km s}^{-1}$$
$$l_v = -17\overset{\circ}{.}6$$

The departure of the mean values from the origin and the vertex deviation are clearly noticeable in the figure.

IV. DISCUSSION

The smoothing or blurring caused by background stellar field comes out clearly from a comparison of the models that consider only stars nearer than 200 pc from the sun, with those that extend that limit to 400 pc (i.e., models 1 and 2, 3 and 4, 5 and 6, and 7 and 8): within the errors, the larger the region considered, the smaller (in absolute value) the vertex deviation. Other parameters, like mean velocity components, and velocity dispersions and their ratio, remain essentially unaltered when the radius of the solar neighborhood is changed.

Table 2 clearly shows the influence of the stellar age

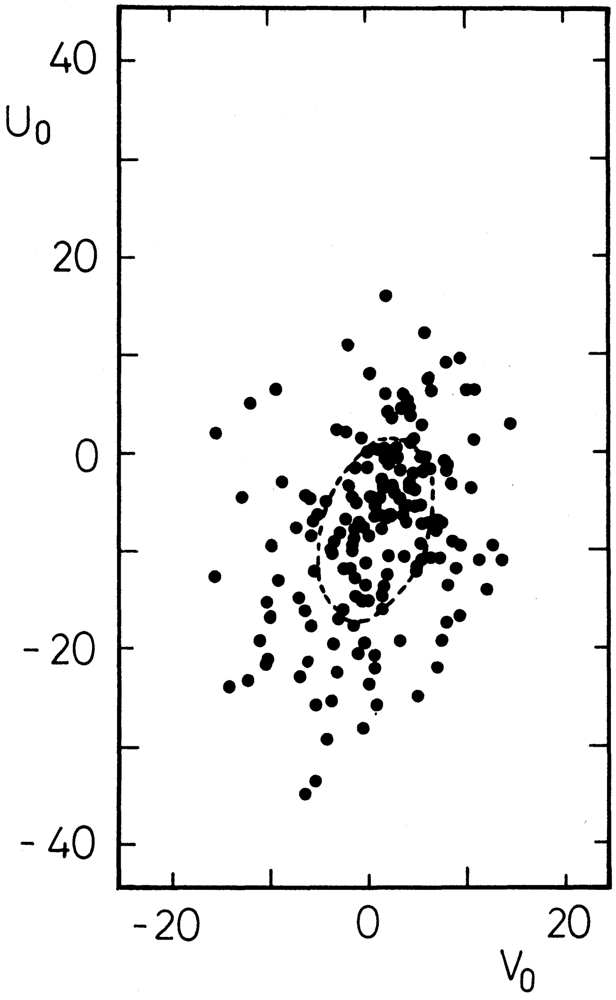

Fig. 2. Representative points on the velocity plane and velocity ellipse for one model of case 7.

on the ratio of velocity dispersions and on the vertex deviation: the older the stars, the smaller ratio of the dispersion and the absolute value of the vertex deviation. The simple picture described in Section II is however modified by the different modelling effects. First, except for the not significant value of model 11, all the vertex deviations are negative, although models 3, 4, 7, 8, 11, 12, 15, 16 and specially, 17 and 18 correspond to ages larger than $\tau = \pi/2\kappa$ (about 5×10^7 years). Also, dispersion ratios as low as $4\ B^2/\kappa^2$ (about 0.40) are never reached either; for $\tau = 6.5 \times 10^7$ years and 13×10^7 years, the dispersion ratios are rather similar to the theoretical steady state value of $[-B/(A-B)]^{1/2} \cong 0.63$.

The main changes take place for ages smaller than 6.5×10^7 years, as the values for 13×10^7 years do not differ appreciably from those for 6.5×10^7 years; notice, however, that the presence of an arm might alter the preceding conclusion about the vertex deviation (models 5, 7 and 18).

The effect of limiting the region of star formation to an "arm" (models 5 through 8 and 18) is very little for the youngest stars; the only difference between models 1 and 2 and models 5 and 6, seems to be that the mean u velocity is not negligible for the latter. For $\tau = 6.5 \times 10^7$ and 13×10^7 years, however, the effect of an arm seems to be an increase of the vertex deviation, in addition to the important mean u and v velocities.

The effect of the systematic initial velocity on the mean velocity is very clear, as had been anticipated in the previous section. For $\tau = 1.8 \times 10^7$ years there is no change in the vertex deviation (model 1 and models 9 and 10); for $\tau = 6.5 \times 10^7$ years, the vertex deviation of model 12 agrees very well with the one of model 3, but the departure of model 11 suggests a real difference.

The superposition of systematic velocity components seems to result in a linear superposition of the mean velocities for each component (compare models 9, 10 and 19, and models 11, 12 and 20) and, again, little or no effect on the velocity dispersions, their ratio and the vertex deviation.

The superposition of an arm with the systematic velocity (models 5, 9 and 13; 5, 10 and 14; 7, 11 and 15; 7, 12 and 16) shows again a linear superposition of the mean velocities for $\tau = 1.8 \times 10^7$ years, but a not so linear superposition for $\tau = 6.5 \times 10^7$ years. There is no apparent effect on the dispersions, but a curious one appears on the vertex deviation: for $\tau = 1.8 \times 10^7$ years there seems to be an addition of effects (models 5, 9 and 13, and models 5, 10 and 14), while for $\tau = 6.5 \times 10^7$ years there seems to be an averaging of effects (models 7, 11 and 15, and models 7, 12 and 16); the errors are very large, however, and it is difficult to establish the true result.

V. CONCLUSION

Assuming that: *a)* stars just born have a circular velocity distribution in the galactic plane, *b)* stellar formation proceeds at a constant rate over a time span similar to the maximum stellar age, and *c)* stellar orbits in the galactic plane are close enough to circles to be adequately represented by the epicyclic approximation, we have shown that both the vertex deviation and the increase with stellar age of the velocity dispersion ratio are direct consequences of the differential stellar motions, at least for stars whose maximum ages do not exceed a few times 10^7 years.

The presence of systematic velocity components when stars are born, or the fact that stars form in spiral arms, does not alter the previous conclusions. For stellar ages of about 10^8 years or larger, those phenomena may have an appreciable effect, however; as they also yield not negligible mean velocities, the observed values of the solar motion pose some limitations to the conditions that can be postulated for just born stars.

For stellar ages of about 10^8 years the ratio of velocity dispersions and the vertex deviation are close to the theoretical steady state values.

We would like to conclude by noting that our assumption *b)* may be questioned because star formation is not continuous, neither in space nor in time. The same problem is also present in previous attempts to explain the vertex deviation, however, and it emphasizes the difficulty, pointed out in the Introduction, to use the vertex deviation as a check for theories of star formation in spiral arms. Our results show that even under very simple hypotheses the main features of the deviation can be adequately explained.

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REFERENCES

Delhaye, J. 1965, in *Galactic Structure*, eds. A. Blaauw and M. Schmidt, 61.
Filin, A. Ia. 1957, *Astr. Zh.*, **34**, 525.
Hilton, J.L. and Bash, F. 1982, *Ap. J.*, **255**, 217.
Kerr, F.J. 1968, in *Nebulae and Interstellar Matter*, eds. B.M. Middlehurst and L.H. Aller, 575.
Lindblad, B. 1959, *Handbuch der Physik*, (Berlin: Springer-Verlag), LIII, 21.
Nordström, H. 1936, *Meddel. Lund. Ser. II*, No. 79.
Ogorodnikov, K.F. 1965, *Dynamics of Stellar Systems* (New York: Pergamon).
Schmidt, M. 1965, in *Galactic Structure*, eds. A. Blaauw and M. Schmidt, 513.
Woolley, R. 1970, in *IAU Symposium No. 38, The Spiral Structure of our Galaxy*, eds. W. Becker and G. Contopoulos, (Dordrecht: D Reidel), p. 423.
Yuan, C. 1971, *A.J.*, **76**, 664.

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