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AN APPLICATION OF INFORMATION THEORY TO ASTRONOMICAL PHOTOMETRY

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RESUMEN

En este trabajo, hemos considerado una aplicación de algunos conceptos básicos de la teoría de información a la fotometría estelar. Se analizan las aplicaciones en algunos parámetros relacionados con magnitudes y colores en el sistema de Johnson, en la zona espectral del infrarojo cercano.

ABSTRACT

In this paper, we consider the application of some basic knowledge from information theory to stellar photometry. We analyze their application to some parameters related with magnitudes and colors in the system of Johnson for the near infrared region.

Key words: INFORMATION THEORY – PHOTOMETRY

I. INTRODUCTION

The photometric system of the near infrared contains the bands J, H, K, L, M (1μ to 5μ). The color indices usually used in this system are combinations of the magnitudes provided by the filters of the mentioned bands ($J-H$, $H-K$, $K-L$, etc.). In the paper of Tignanelli and Feinstein (1983), the parameters $Q(\lambda)$ and $t(\lambda)$ were given in the following manner

$$Q(\lambda) = (K - |\lambda|) - \left[\frac{E(K - |\lambda|)}{E(V - K)} \cdot (V - K) \right], \quad (1)$$

where $|\lambda|$ is the infrared magnitude for any one of the bands included in the near infrared, V is the visual magnitude used for the color $(V-K)$ which has been taken as a measure of the reddening. Moreover, they also defined as

$$t(\lambda_2) = \{ |\mathrm{mag}(\lambda_1) + \mathrm{mag}(\lambda_3)| /2 \} - \mathrm{mag}(\lambda_2) \quad ,$$

which for the system of Johnson is related to the following combinations

$$t(H) = \{|J + K|/2\} - H$$

$$t(K) = \{|H + L|/2\} - K \quad (2)$$

$$t(L) = \{|K + M|/2\} - L \quad .$$

The color index is directly related to the temperature of the star and its spectral type. Then, there is some equivalence between classifications based on ranges of variation of the color index and those derived from spectral type. The color index in the near infrared is an indicator usually more sensitive to the stellar temperature than to luminosity class; any way, the mentioned authors show that some combination of the parameters $Q(\lambda)$ and $t(\lambda)$ provide a sensitive differentiation in temperature (spectral type) and luminosity class, mainly for late type stars (S.T. > M0).

This taxonomic quality of the parameters $Q(\lambda)$ and $t(\lambda)$ can be analysed using the information theory in the way of Brillouin (1964). This author has established a measure of information in the following manner

$$I_1 = K(\ell n P_0 - \ell n P_1) \quad (3)$$

In general, the last formula can be used to summarize an experiment in which two variables X and Y of ranges a and b are measured. Moreover, for this experiment we define a Cartesian system in which P_0 is represented by the area a, b, the whole range of variation of both variables, and being P_1 a partition of P_0 including the values of our observations.

In our study, we have some measures using the parameters $Q(\lambda)$ and $t(\lambda)$ for a set of late stars. The ranges of variations which are designated by $\Delta Q(\lambda)$ and $\Delta t(\lambda)$ are taken from literature (these quantities are identified with P_0 in formula (3)) and $\sigma_{Q(\lambda)}$, $\sigma_{t(\lambda)}$, the standard deviations of our observations (identified with P_1 in formula (3)).

Then, we define as measure of information

$$I_i = \log \Delta\delta_i \ \log \sigma_{\delta i} \quad , \qquad (4)$$

where δ_i denotes the variables Q(λ) and t(λ).

II. PHOTOMETRIC MEASURES AND INFORMATION THEORY

The calculated values for Q(λ) and t(λ) given by Tignanelli and Feinstein (1983) are included in Table 1 to 3 and their corresponding deviations $\Delta\delta_i$ and $\sigma_{\delta i}$ in Table 5.

The election of some base for the logarithm in expression (4) is equivalent to selecting some unit for the measure of information, since

$$\log_a X = \frac{1}{\log_b a} \log_b X \quad .$$

If the number 2 has been chosen for the base the unit is called a bit; in the same way, for the number e the unit is the nat (natural unit). For the number 10 the unit is called Hartley, because Hartley (1928) was the first author who defined the logarithmic measure for information. There are some combinations of photometric parameters which permit a classification of a sample according to the spectral type (S.T.) and luminosity class (L.C.). For instance, the combination Q(λ) vs. t(λ) is the best one, because it has a greater measure of information according to the relation given in (4).

To calculate the total information corresponding to every combination of parameters, we define

$$I_{i,j}^{L.C.} = I_i^{L.C.} + I_j^{L.C.} \quad , \qquad (5)$$

with i = Q(λ) , where λ = *J, H, L*

and j = t(λ′) , where λ′ = *H, K* .

The super-index L.C. of formula (5) indicates the luminosity classes which are considered in the calculation of the quantity of information using the expression (4). For example

$$I_{t(H)}^{I,III,V} = \log \left. \frac{\Delta\, t(H)}{\sigma_{t(H)}} \right]_{I,III,V} \quad . \qquad (6)$$

In the last formula Δt(H) is the absolute value of the difference between the highest and lowest values of the parameter t(*H*) corresponding to the luminosity classes I, III and V., and $\sigma_{t(H)}$ should be calculated using all values of the parameter t(*H*) (last columns of Tables 1, 2 and 3).

TABLE 1[a]

MEASURE OF PARAMETERS FOR MAIN SEQUENCE STARS

	Q(*J*)	Q(*H*)	Q(*L*)	Q(*M*)	t(*H*)	t(*K*)
M0	−.75849	.0735	.03406	...	1.445	.025
M1	−.78299	.0830	.04668	...	1.560	.020
M2	−.80749	.0825	.03930	...	1.665	.030
M3	−.83199	.0520	.04192	...	1.740	.050
M4	−.80649	.0515	.03454	...	1.845	.060
M5	−.80099	.0610	.04716	...	1.960	.055
M6	−.77549	.0705	...	...	2.075	.150
M7	−.77999	.0800	...	...	2.190	.155
M8	−.78449	.0795	...	...	2.295	.165

a. All values have five decimal digits. Then corresponding zeros at right places should be considered.

TABLE 2[a]

MEASURE OF PARAMETERS FOR GIANT STARS

	Q(*J*)	Q(*H*)	Q(*L*)	Q(*M*)	t(*H*)	t(*K*)
M0	−.62004	−.05787	−.01586	−.01668	1.660	0.45
M1	−.64506	−.05136	−.02167	−.01354	1.740	.045
M2	−.65073	−.06078	−.02111	−.00844	1.850	.050
M3	−.66675	−.06166	−.02819	−.00077	2.065	.055
M4	−.66032	−.05278	−.02398	−.02469	2.400	.050
M5	−.65984	−.05780	−.03522	...	2.790	.060
M6	−.62627	−.05712	−.01154	...	3.250	.050
M7	...	...	...	...	...	...
M8	...	...	...	...	...	...

a. All values have five decimal digits. Then, corresponding zeros at right places should be considered.

TABLE 3

MEASURE OF PARAMETERS FOR SUPER GIANT STARS[a]

	Q(*J*)	Q(*H*)	Q(*L*)	Q(*M*)	t(*H*)	t(*K*)
M0	−.50788	−.05131	−.01651	−.02715	1.700	.010
M1	−.49769	−.05467	−.02516	−.02452	1.770	.015
M2	−.51604	−.07096	−.03300	−.01909	1.905	.025
M3	−.54286	−.08125	...	−.01000	2.175	.030
M4	−.50332	−.05069	−.06841	...	2.480	.025
M5	−.50488	−.05829	...	...	2.830	.160
M6	...	...	...	...	...	...
M7	...	...	...	...	...	...
M8	...	...	...	...	...	...

a. All values have five decimal digits. Then, corresponding zeros at right places should be considered.

In Table 4 we report the values for $I^{L.C.}_{i,j}$ in decreasing order. These values correspond to some combinations Q(λ) vs. t(λ').

These combinations are related with the graphics shown in the paper of Tignanelli *et al.* already mentioned. In Table 4 decreasing values of $I^{L.C.}_{i,j}$ are related with a decreasing possibility of classification for the combination of parameters in the following sense: the combination Q(*J*) vs. t(*H*) present clustering for the luminosity classes and also a right sequence of the spectral types in all classes of luminosity. In a similar analysis, the combination Q(*L*) vs. t(*K*) also gives clustering for the luminosity classes, but the spectral types are only given in right order for the stars of type I.

The combination Q(*M*) vs. t(*H*) presents a clear differentiation of the spectral types for the classes I and III. Only these classes have been considered because there are only measurements of Q(*M*) available for them.

For decreasing values in the variables $I^{L.C.}_{i,j}$ presented in Table 4, we have the combination Q(*H*) vs. t(*H*) which gives a right ordering for the spectral types of all luminosity classes (I, III and V) although it only differentiates between the class V and from the I and III together, because the last two classes appear mixed. (Table 5).

Proceeding in the same way, we have the combinations Q(*J*) vs. t(*K*) and Q(*H*) vs. t(*K*). Here, the first combination separately cluster the classes of luminosity I, III and V, although it lacks ordering in the spectral types of any one of these classes. The second combination only differentiates between the class V and the two classes I and III, in this way, it resembles the combination Q(*H*) vs. t(*H*). Moreover, it is also unable to order the spectral types, that is, Q(*H*) vs. t(*K*) is the worst among those combinations analysed in this paper for taxonomic purposes, using the criterion according to formula (5).

TABLE 4

MEASURE OF INFORMATION FOR DIFFERENT COMBINATIONS OF PHOTOMETRIC PARAMETERS

Q(λ) vs. t(λ')	$I^{L.C.}_{i,j}$
Q(*J*) vs. t(*H*)	1.067 Hartley
Q(*L*) vs. t(*K*)	1.032 "
Q(*M*) vs. t(*H*)	1.014 "
Q(*H*) vs. t(*H*)	1.013 "
Q(*J*) vs. t(*K*)	0.970 "
Q(*H*) vs. t(*K*)	0.916 "

TABLE 5

RANGE OF VARIATION OF THE PARAMETERS FOR DIFFERENT LUMINOSITY CLASS

L.C. I, III and V	
ΔQ(*J*)	0.3343
$\sigma_{Q(J)}$	0.11567
Δt(*H*)	1.805
$\sigma_{t(H)}$	0.44754
Δt(*K*)	0.155
$\sigma_{t(K)}$	0.04798
ΔQ(*H*)	0.16425
$\sigma_{Q(H)}$	0.06437
ΔQ(*L*)	0.11557
$\sigma_{Q(L)}$	0.03468
L.C. I and III	
ΔQ(*M*) | I,III	0.02638
$\sigma_{Q(M)}$ | I,III	0.00826
Δt(*H*) | I,III	1.59
$\sigma_{t(H)}$ | I,III	0.49087

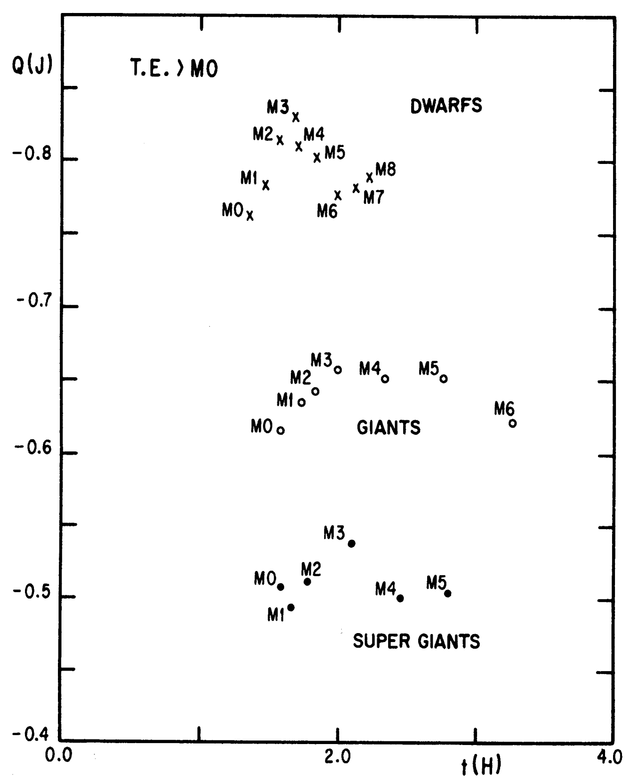

Fig. 1. Q(*J*) vs. t(*H*) for different spectral types.

Finally, the graph for the combinations Q(*J*) vs. t(*H*) and Q(*H*) vs. t(*K*) are shown in the Figures 1 and 2.

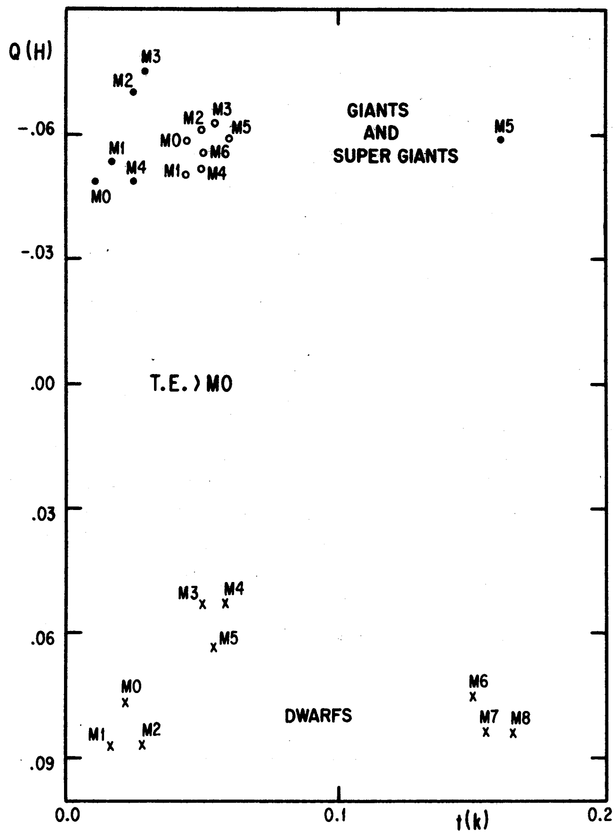

Fig. 2. Q(*H*) vs. t(*K*) for different spectral types.

III. CONCLUSIONS

From the preceding arguments it follows that some combinations of photometric parameters are suitable for grouping the stars according to their luminosity classes and spectral types. These combinations correspond to higher values of some magnitude which can be defined as a quantity of information in the manner given by relation (4). In other words, the possibilities of ordering the observations in separate groups increase with the values of the different combinations (Figures 1, 2).

This method is easily extended to any combination of photometric parameters through the whole electromagnetic spectrum. In this way, it should be possible to get the best selection of parameters for taxonomical purposes.

Finally, the values taken from the literature have been chosen from Golay (1974) and Koornneef (1982, 1983). Another reference of information theory related with this paper is Abramson (1981).

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