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# A HYDRODYNAMICAL THREE-COMPONENT MODEL OF THE UNIVERSE DURING THE RECOMBINATION ERA

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RESUMEN. El universo, durante la era de la recombinación, es considerado como un fluido constituído por hidrógeno neutro, plasma y radiación, interactuando a través de los procesos de fotorecombinación, fotoionización, y dispersión Compton. Se analiza la estabilidad de los modos hidrodinámicos y se relacionan los resultados con el origen de las gala - xias durante esta época.

ABSTRACT. We consider the universe during the recombination era to be a fluid composed of neutral hydrogen, plasma and radiation interacting via photorecombination, photoionization and Thompson scattering. We analyse the stability of the hydrodynamical cosmological model and the results are discussed in relation to the origin of galaxies.

*Key words:* HYDRODYNAMICS — COSMOLOGY

## I. INTRODUCTION

The present work is concerned with the static calculation of a 3-component hydrodynamical model consisting of radiation, plasma and neutral hydrogen, during the recombination era. Our investigation is also a significant improvement over the 2-component calculations (Nowotny 1980, 1981), because we do not consider the hydrogen gas with the same physical properties as the plasma. We do not assume the degree of ionization x to be fixed. By Saha's equation x is an exponential function of the temperature T. A disturbance of T heads to a significant disturbance of x. This effect is fully included in our calculation.

In our analysis we make three assumptions

i) The geometry of space-time may be described by the Robertson-Walker metric;

ii) The substratum is in a static, uniform equilibrium condition originally;

iii) Gravitational forces are only present.

To obtain the equations of motion we apply the relativistic theory of perturbations (Weinberg 1972).

In this work, the signature of the metric tensor is + 2. Repeated indices are added, greek indices have values from 0 to 3 and latin indices from 1 to 3. Partial derivatives will be denoted by a bar so that $f_{|\mu} = \frac{\partial f}{\partial x^\mu}$, four dimensional covariant derivatives will be denote by a semicolon, $f_{\mu;\nu}$.

2.- The Basic equations of a three-component fluid.

2.1. The geometrical quantities.

The early universe is assumed to be homogeneous and isotropic, and is therefore, according to Einstein's theory, described by the Robertson-Walker metric.

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \mathrm{sen}^2\theta \, d\phi^2)\right], \tag{2.1}$$

Where K= $\pm$ 1 or 0 for a closed, open, or flat universe, respectively.

$r, \Theta, \phi$ are comoving coordinates - they follow the expansion of the Universe and a(t) is the cosmic scale factor of the unperturbed metric and obeys the Friedmann's equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho . \tag{2.2}$$

The perturbations of the metric will be constrained in the following way

$$h_{io} = 0 \quad , h_{oo} = 0, \tag{2.3}$$

2.2. The energy-momentum tensor of the fluids.

The hydrogen gas and the plasma are described by the well-known energy momentum tensor

$$T^{\mu\nu} = (e+P) \, u^\mu u^\nu + P g^{\mu\nu} \tag{2.4}$$

The energy-density

$$e_\alpha = \rho_\alpha c^2 + \varepsilon_\alpha \;\; ^{1)} \tag{2.5}$$

contains the internal energy $\varepsilon_\alpha$. It has the values

$$\varepsilon_p = \frac{3}{2} P_p + \frac{\chi \rho_p}{m_H} \tag{2.6}$$

and

$$\varepsilon_g = \frac{3}{2} P_g \tag{2.7}$$

for the plasma and gas respectively. $\rho_\alpha$ is the mass density of the fluid, $\chi$ represents the ionization potential for the hydrogen atom. The pressure obeys the equation of state.

$$P_\alpha = \frac{N_\alpha k_B T}{V} , \tag{2.8}$$

Where $k_B$ is the Boltzmann's constant, T the temperature and N the number of particles of the component of the fluid.

1) The subindex will indicate plasma if $\alpha$=p and neutral hydrogen gas if $\alpha$ =g .

Also the equation of state for the hydrogen gas is

$$P_g = \frac{\rho_g}{m_H} k_B T_g \tag{2.9}$$

and for the plasma

$$P_p = 2\frac{\rho_p}{m_H} k_B T_p . \tag{2.10}$$

For the radiation we have

$$P_r = \frac{e_r}{3} = \frac{\rho_r}{3} c^2 , \quad e_r = \frac{4\sigma T_r^4}{c} , \tag{2.11}$$

here $\sigma$ is the Stefan-Boltzmann constant, c is the light velocity, $\rho_r$ the mass-density of the energy-density of the radiation.

## 2.3. Equation of motion

Following Weinberg's treatment of the relativistic hydrodynamical perturbations s (Weinberg 1972 ), Corona has deduced the equations of motion for the hydrogen, plasma and radiation gases (Corona 1984 ). In that work is also exhaustively discussed the obtention of the transfer of energy-momentum density between the gases as soon as the total change of the plasma and hydrogen mass density, due to the photorecombination and photoionization process and the Compton scattering. Because space-reasons we will only write the equations that govern our model:

Equations of continuity for the matter

$$\dot{\rho}_{p1} + 3\frac{\dot{a}}{a}\rho_{p1} + \overset{(o)}{\rho}_p v^i_{p1/i} + \frac{\overset{(o)}{\rho}_p}{2a^2}\dot{h}_{ii} - \frac{\dot{a}}{a}\frac{\overset{(o)}{\rho}_p}{a^2} h_{ii} = -M_p , \tag{2.12}$$

$$\dot{\rho}_{g1} + 3\frac{\dot{a}}{a}\rho_{g1} + \overset{(o)}{\rho}_g v^i_{g1/i} + \frac{\overset{(o)}{P}_g}{2a^2}\dot{h}_{ii} - \frac{\dot{a}}{a}\frac{\overset{(o)}{\rho}_g}{a^2} h_{ii} = M_p . \tag{2.13}$$

The superindex (0) indicates unperturbed values of the quantities, while the subindex 1 represents its perturbations to first order. $M_p$ means the total change of plasma mass density and hydrogen mass density due to ionization and recombination effects.

Momentum balance equations

$$\overset{(o)}{\rho}_p \dot{v}^i_{p1} + 2\frac{\dot{a}}{a}\overset{(o)}{\rho}_p v^i_{p1} + \frac{P_{p1/i}}{a^2} = C^i_p , \tag{2.14}$$

$$\overset{(o)}{\rho}_g \dot{v}^i_{g1} + 2\frac{\dot{a}}{a}\overset{(o)}{\rho}_g v^i_{g1} + \frac{P_{g1/i}}{a^2} = C^i_g , \tag{2.15}$$

$$\frac{4}{3}\overset{(o)}{\rho}_r \dot{v}^i_{r1} + \frac{4}{3}\frac{\dot{a}}{a}\overset{(o)}{\rho}_r v^i_{r1} + \frac{c_s^2}{a^2}\rho_{r1/i} = C^i_r . \qquad (2.16)$$

Energy balance equations

$$\frac{3}{2}\dot{P}_{p1} + \frac{15}{2}\frac{\dot{a}}{a} P_{p1} + (\overset{(o)}{\varepsilon}_p + \overset{(o)}{P}_p) v^i_{p1/i} + (\overset{(o)}{\varepsilon}_p + \overset{(o)}{P}_p)\frac{\dot{h}_{ii}}{2a^2} -$$

$$- (\overset{(o)}{\varepsilon}_p + \overset{(o)}{P}_p)\frac{\dot{a}}{a^3} h_{ii} + \frac{\chi}{m_H}\dot{\rho}_{p1} + 3\frac{\dot{a}}{a}\frac{\chi}{m_H}\rho_{p1} = E_p , \qquad (2.17)$$

$$\frac{3}{2}\dot{P}_{g1} + \frac{15}{2}\frac{\dot{a}}{a} P_{g1} + (\overset{(o)}{\varepsilon}_g + \overset{(o)}{P}_g) v^i_{g1/i} + (\overset{(o)}{\varepsilon}_g + \overset{(o)}{P}_g)\frac{\dot{h}_{ii}}{2a^2} -$$

$$- (\overset{(o)}{\varepsilon}_g + \overset{(o)}{P}_g)\frac{\dot{a}}{a^3} h_{ii} = Eg. \qquad (2.18)$$

$$\dot{\rho}_{r1} + 4\frac{\dot{a}}{a}\rho_{r1} + \frac{4}{3}\overset{(o)}{\rho}_r v^i_{r1/i} + \frac{2}{3}\frac{\overset{(o)}{\rho}_r}{a^2}\dot{h}_{ii} - \frac{4}{3}\overset{(o)}{\rho}_r \frac{\dot{a}}{a^3} h_{ii} = E_r . \qquad (2.19)$$

$E_\alpha$ are the energy-transfer density between the gases by the processes mentioned above.

Equation for the gravitational Field

$$\frac{1}{a^2}[\ \ddot{h}_{ii} - 2\frac{\dot{a}}{a}\dot{h}_{ii} + 2h_{ii}\left(\frac{\dot{a}^2}{a^2} - \frac{\ddot{a}}{a}\right)\ ] = - \frac{8\pi G}{c^2}\left(e_{p1} + 3P_{p1} + e_{g1} +\right.$$

$$\left. + 3P_{g1} + e_{r1} + 3P_{r1}\ \right) . \qquad (2.20)$$

## 3.- Calculations

With the aid of the equations of state it is possible to solve the system of equations (2.12) - (2.20), which characterizes our three-component model of the universe during the recombination era. By the usual procedure in linear fluid dynamics, we find three coupled wave equations for the amplitude of the fourier components $\rho_{p1}\,exp(i k_\alpha x^\alpha)$, $\rho_{g1}\,exp(i k_\alpha x^\alpha)$, $\rho_{r1}\,exp(i k_\alpha x^\alpha)$ for the plasma, hydrogen gas and radiation density perturbations respectively, when the frequencies $\omega$ are large compared to the expansion rate $\tau^{-1}_{exp}$ of the universe.

The details of the calculations are in reference (1). We will only point here that the dispersion relation of the system is a polynomial of the form

$$\Lambda_0\omega^8 + \Lambda_1\omega^7 + \Lambda_2\omega^6 + \Lambda_3\omega^5 + \Lambda_4\omega^4 + \Lambda_5\omega^3 + \Lambda_6\omega^2 + \Lambda_7\omega + \Lambda_8 = 0.$$

Where the $\Lambda$s are all functions of the coefficients of the primitive equations.

The numerical solutions of the dispersion relation are showed in the figure (1) for a closed universe.

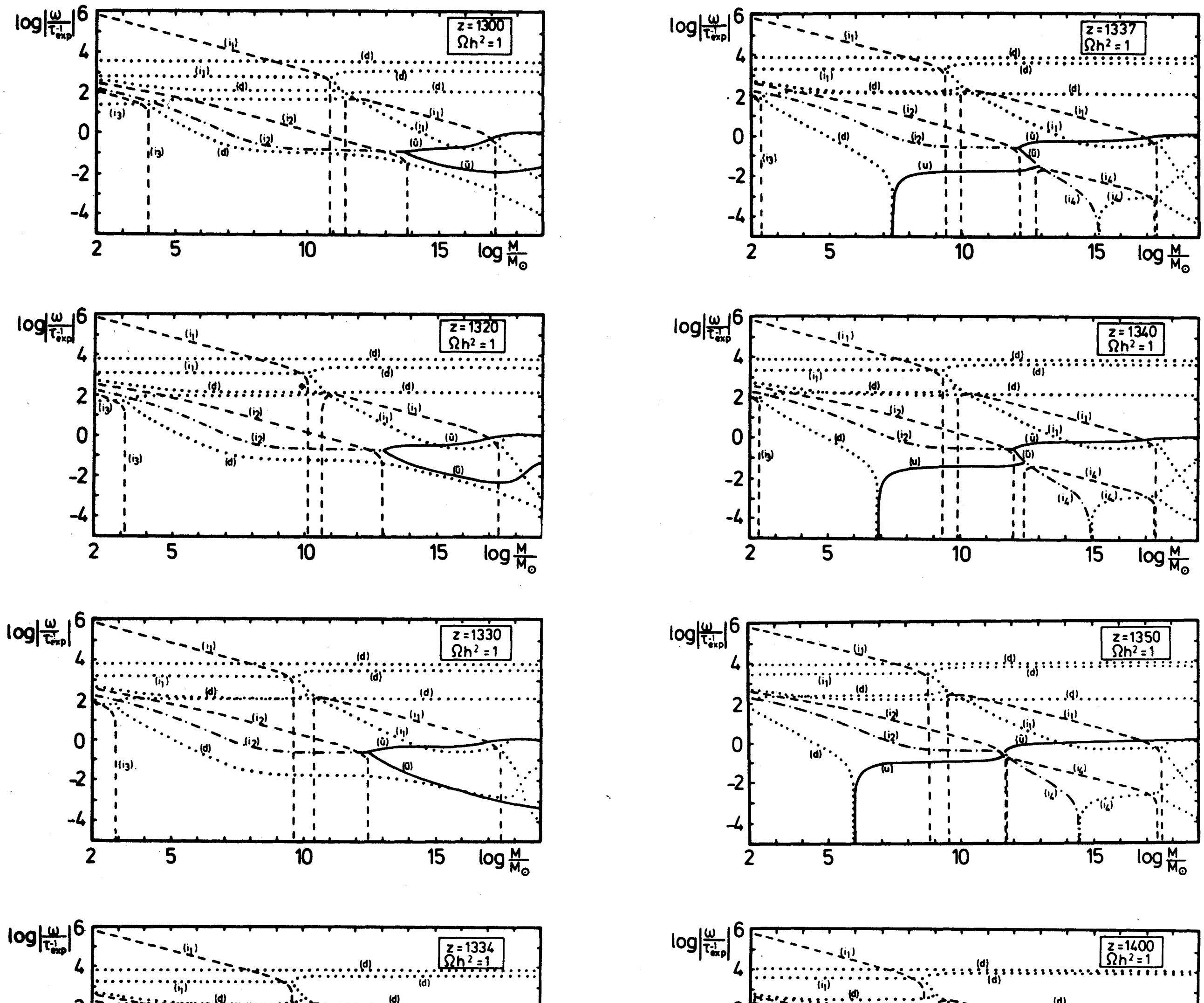


Fig. 1. The eigenfrequencies of the eight hydrodynamical modes at different redshifts z and for $\Omega h^2 = 1.0$ in dependence of the mass: (d) Represent the negative real part of a real frequency and indicate the damping rate of pure damped modes. (i) Represent the negative real part of a complex frequency and indicate the damping rate of the oscillating modes. (i) Represent the positive real part of a complex frequency. (i) Represents positive and negative imaginary parts of a complex frequency (usual frequency) and indicates oscillating modes. (u) (ũ) (u) Represent positive real part of a real frequency and indicate the rate of the unstable mode with vanishing imaginary part of $\omega$.

## 4.- Discussion of the results

Our analysis of the universe with high density as a three-component fluid shows the existence of two oscillating modes ($i_1$) - they differ only in the sign of the frequency. The oscillating character of both modes are damped for masses which lay between $10^8$ $M_\odot$ and after $10^{17} M_\odot$ respectively. The exact localization of the damped-range depends of the red-shift z. As we can see from the figures, one moves to the region of high masses and the other goes slightly down when z diminishes.

There are also three damped modes (d) for the whole range of masses.

Another damped mode (d) is also present but only for masses smaller than $10^7 M_\odot$, where the well-known Jeans' instability (u) appears. The situation of the Jeans' mode depends of the red-shift z as well. For high values of z (z=1400) the Jeans' instability arises by $10^5$ $M_\odot$ and, in proportion as the value of z goes down, it moves to the right in our figures (regions of greater masses) and finally it gets an oscillating character for red-shifts smaller than 1334.

Finally we have two "growing" oscillating modes ($i_2$) too. This "growing" oscillating character is present from the beginning of our masses range till $10^{11} M_\odot$ where two instabilities ($\hat{u}, \tilde{u}$) arise. The position of the origin of both of these instabilities also dependence on z, since for high values of z (z=1400) they appear very near to $10^{11}$ $M_\odot$ and it moves here after to the right in our figures when z decreases.

If we compare the results of our work with Nowotny's treatment of the universe during the recombination era (Nowotny 1980 , 1981 )we can see the similar behavior as well the oscillating modes as the damped modes. In the position of the Jeans' instability, there is no difference but well in the appearance of the oscillating character of this mode after z=1330.

The presence of both the "growing" oscillating modes and the two consequent instabilities are new completely.

The fact that, the three instabilities ($u, \hat{u}, \tilde{u}$) are present in the range of $10^{12} M_\odot$ -$10^{14} M_\odot$, shows a privileged region. It means, in our opinion, that the structure with very high masses were originated prior to the other cosmological structures, such as Zel'dovich asserts in his pancake theory.

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DISCUSSION

DOTTORI: ¿Qué temperatura tiene el gas para que sea notable la presencia de H neutro?

CORONA: Alrededor de 4500 K, esta relación depende del corrimiento al rojo, z.

IBAÑEZ: ¿En qué forma trata el problema del transporte de fotones?

CORONA: Obtengo el tensor de energía-momento, utilizando los momentos, y después derivo covariantemente.

PONCE: ¿Por qué no se han usado cantidades independientes del sistema de coordenadas ("gauge invariant") para caracterizar las perturbaciones? El uso de estas cantidades permite interpretar los resultados sin ambigüedades, sin "modos ficticios".

CORONA: Bueno, el problema es realmente encontrar esas "gauge" que sean invariantes y después introducir esas condiciones en la solución del problema hidrodinámico.

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