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COSMOLOGICAL MASS DENSITY AND THE REGGE LAW

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RESUMO. Em física de hadrions a propriedade de homogeneidade de potencial pode conduzir a "scalings" de spin-massa que são compatíveis com o comportamento predito pela lei de Regge. O "scaling" das grandezas fundamentais (tempo e espaço) pode levar, no caso gravitacional, através da homogeneidade de potencial, a "scalings" de quantidades físicas observáveis (massa, densidade, momentum angular) que, juntamente com a imposição de covariância do Lagrangiano, resulta na condição de Regge. Calculamos os momentos de inércia principais de uma distribuição tridimensional de massa rotante em um campo gravitacional homogêneo com simetria axial e mostramos que, sob certas condições, o momentum angular resultante é compatível com a lei de Regge. Esses resultados são usados para avaliar a massa e a densidade do universo.

ABSTRACT. The property of potential homogeneity may conduce in hadron physics to scalings for spin-mass wich are compatible with the behaviour predicted by the Regge law. Scaling of the fundamental quantities (length and time) may lead, in the gravitational case, through potential homogeneity, to scalings of some relevant observable physical quantities (mass, density, angular momentum) which, together with the imposition of covariance of the Lagragian, gives a Regge-like condition. We calculate the principal inertia moments of a three-dimensional mass distribution rotating in a self-gravitational homogeneous field with axial symmetry. We show that the angular momentum may, under certain conditions, be of the Regge-like type. These results are used to evaluate the mass and density of the universe.

Key words: COSMOLOGY

I. INTRODUCTION

Muradian (1980) in the analysis of observational data regarding intrinsic angular momentum J and mass M of galaxies and other celestial objects found that they may be accurately described by the generalized Regge-like

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dependence

$$J = (M / mp)^{1+1/n} \hbar \qquad (1)$$

where mp is the proton mass and $\hbar$ is the normalized Planck constant. In hadron physics the parameter n takes integer values (1,2 and 3) which characterize the spatial dimensionality of the hadrons. For celestial objects the values n=2 for galaxies, their clusters and superclusters, and n=3 for asteroids, planets and stars give angular momenta quantities in excellent agreement with the observable values, especially for heavier objects. These results are not in fact strictly new. Ozernoy (1967) had already shown that the virial relationship between gravitational and rotational energies for cosmic objects, which gives the condition of mechanical equilibrium, when applied to homogeneous spheroids rotating with constant angular velocity leads to a connection between mass and intrinsic angular momentum of the form (see also Carrasco et al. 1982)

$$J = (G\ Kr\ Kg)^{1/2}\ V^{-1/6}\ M^{3/2} \qquad (2)$$

where G is the gravitational constant, Kr is the relative gyration radius, Kg is a constant associated to the mass distribution and V is the volume of the spheroid. Expression (2) is a particular case of (1) with n=2.

These expressions present a surprising resemblance between the angular momentum-mass dependence of elementary particles and cosmic objects, bringing to (1) a universal character since, a priori, elementary particles and cosmic objects interact through so different mechanisms. As an example, asymptotic freedom and confinement, characteristics of quarks, are absent in the behaviour of cosmic objects which interact mainly through gravitational fields. We have demonstrated (Vasconcelos et al., 1985) that this universal character is a consequence of potential homogeneity. Based on this result we calculate the principal inertial moments of a mass distribution with axial symmetry and obtain its angular momentum. The final relation obtained for J and M are used to evaluate the mass and density of the universe.

II. MASS DISTRIBUTION AND THE REGGE LAW

We have considered a three-dimensional mass distribution rotating with angular velocity w in a self-gravitational homogeneous field with axial symmetry (see Prendergast and Tommer, 1970 and Van Albada, 1975)

$$@(r,\theta) = \sum_m @om(r/ro)^{a-m}\ Am(r,ro)\ Pm(\cos\theta) \qquad (3)$$

where Am(r,ro) is a a-dimensional and scale invariant (at least for $r > ro$) function defined so that $@(r,\theta) \longrightarrow$ constant as $r \longrightarrow 0$ and $@(r,\theta) \longrightarrow 0$ as $r \rightarrow \infty$. We have found (Vasconcellos et al. 1985) that scaling of length and

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time lead through potential homogeneity to scalings of mass and angular momentum which in turn with the imposition of covariance of the Lagrangian give a Regge-like condition for a-m:

$$a - m = n/2 - 3$$

$$= \begin{cases} -2 & \text{for } n = 2 \\ -2 + 1/2 & \text{for } n = 3 \,. \end{cases} \qquad (4)$$

These results show that two kinds of cosmological hadrons: namely, the hadrons which originate objects more massive than globular clusters would need an initial center-edge density decreasing as

$$\rho \sim r^{-2}$$

while those which were the predecessors of smaller systems would follow an initial density dependence

$$\rho \sim r^{-2 + 1/2} \,.$$

III. ANGULAR MOMENTUM AND MASS

We calculate the principal inertia moments of a rigid body with the above mass distribution for n = 2. We have obtained (Schmidt et al., 1986) for the modulus of the angular momentum vector

$$J = (4\pi/3)\, \rho_{oo}\, r_o^5\, W \int_0^{R/r_o} dy\, y^{1+n/2} \qquad (5)$$

where

$$W = [\, (2\, A0 + s\, A2)^2\, (W1^2 + W2^2) + 4\, (A0 - s\, A2)^2\, W3^2\,]^{1/2} \qquad (6)$$

W1,W2 and W3 being the angular velocity components and where we have assumed

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for simplicity

$$Am(y) = \begin{cases} Am & \text{for } r < R \\ 0 & \text{for } r > R \end{cases} ,$$

and s = @o2 / (5 @oo). The choice

$$@oo = (R^{3n/2}\ r^{n/2-3} / 8\pi)\ (W / 3h\ (n+4))^{n}\ (n\ mp\ /A0)^{1+n}$$

reproduces expression (1) for J and gives

$$M = (n\ W\ R^{2} / 3\hbar\ A0\ (n+4))^{n}\ mp^{1+n} \qquad (7)$$

Combining these expressions we have

$$J = (n\ W\ R^{2}\ mp / 3\hbar\ A0\ (n+4))^{1+n}\ \hbar\ .$$

IV. RESULTS

The above expressions may be used to evaluate some cosmological parameters as the mass and the average density. To calculate these parameters we consider the universe as a rotating system, since there are experimental results that give strong indications for at least a local rotating metagalaxy. Birch (1982) has found the angular velocity for the metagalaxy to be

$$W_{U} = 10^{-13}\ \text{rad yr}^{-1} .$$

The observed radius of the metagalaxy is expressed in terms of the Hubble constant H

$$R_{U} = c / H ,$$

(H = 50 - 100 Km/s/Mpc). We assume that the observed angular momentum is

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restricted to the 3-axis setting 2 A0 + s A2 = 0 in (6). The resulting mass of the metagalaxy is limited in the region

$$M = [2.2 - 0.14]\ 10^{55}\ g \quad ,$$

respectively for H = 50 and H = 100. The average density of the universe, according to this, must be

$$@ = [0.4 - 0.8]\ 10^{-30}\ g\ cm^{-3} \quad .$$

This result is in agreement with the usual accepted value for the average density of a closed universe (Sandage and Tammann, 1982)

$$@ = 10^{-30}\ g\ cm^{-3}$$

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