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A DYNAMICAL EXPLANATION FOR THE LOW FREQUENCY OF RING GALAXIES

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RESUMEN. Se investiga teóricamente una explicación para la baja frecuencia de galaxias anulares en base a la teoría colisional, de acuerdo a la cual la galaxia anular es el producto de una colisión entre una galaxia de disco y otra galaxia. Cálculos bajo la aproximación impulsiva indican que si la frecuencia esperada de las galaxias anulares se mide con respecto a las regiones densamente pobladas, usando el valor de la frecuencia colisional para tales regiones, entonces se encuentra que es del orden de 0.01% del de las galaxias espirales, compatible con la frecuencia observada de las galaxias anulares.

Sin embargo si consideramos regiones menos densamente pobladas por campos de galaxias, la frecuencia esperada disminuye unos cuantos órdenes de magnitud. Entonces los encuentros hiperbólicos azarosos no son suficientes para explicar la formación de las galaxias anulares. Esto indica que la mayoría de estos pares interactuantes deben haber sido previamente galaxias dobles ligadas, cuyas órbitas son tales que solamente hasta ahora - los encuentros "interpenetrantes" conllevan a la formación anular.

ABSTRACT. An explanation for the low frequency of ring galaxies is investigated theoretically on the basis of the collisional theory, according to which the ring galaxy is the aftermath of a collision between a disk and another galaxy. Impulsive approximation calculations indicate that if the expected frequency of ring galaxies is measured with respect to densely populated regions, using the value of the collisional frequency corresponding to such regions, then it comes out to be of the order of 0.01% of spirals, compatible with the observed frequency of ring galaxies.

But, however, if we consider sparse regions, populated by field galaxies, for frequency determinations, the expected frequency goes down by few orders of magnitude. Thus stray, hyperbolic encounters are too scarse to explain the formation of ring galaxies. This indicates that most of these interacting pairs must have already been bound doubles, whose orbits are such as to have brought about only now the interpenetrating encounters leading to ring formation.

Key words: GALAXIES-STRUCTURE

I. INTRODUCTION

Ring galaxies are a very rare specimen of interacting galaxies. Observational evidence indicates that they have a very low frequency (see Athanassoula and Bosma, 1985, for a review). According to the collisional theory, the ring galaxy is the aftermath of a collision between a disk and another galaxy (Lynds and Toomre, 1976; Theys and Spiegel, 1977). The present evidence strongly indicates that at least the majority of ring galaxies were formed by the collision process (see Thompson, 1977 and Theys Spiegel, 1976) involving a disk and a spherical galaxy. In a previous paper (Chatterjee 1984; hereafter referred to as Paper I), the conditions of ring formation during head-on collisions between disk and spherical galaxies was studied. Using the same method, in this paper we study both head-on, as well as off-center and non-vertical collisions, between disk and spherical galaxies and determine approximately the range of parameters favourable for ring formation. Thereafter we calculate approximately the expected frequency of fairly well defined ring galaxies on the basis of our results.

A detailed investigation on the frequency of various types of ring galaxies has been submitted to Astrophysics and Space Science (Chatterjee 1986). Here we deal only with the frequency of fairly well defined and prominent ring galaxies.

II. Theory

The density distribution of the disk galaxy is represented by (see Wyse and Mayall, 1942, or Freeman, 1970),

$$\sigma(r) = \sigma_c e^{-4r/RD} \tag{1}$$

Where σ_c is the central density given by,

$$\sigma_c = M_D/(2\pi R_D^2\ b) \tag{2}$$

where M_D and R_D denote, respectively, the mass and radius of the disk galaxy and b is a constant depending upon the density distribution. The spherical galaxy is modeled as a polytrope of index n=4 (see Limber 1961), having mass M_s and radius R_s. We assume as in Paper I, $R_s=R_D$, but due to the high central concentration of the spherical galaxy, effectively $R_s \sim \frac{1}{3} R_D$.

The theory is essentially the same as that of Paper I, except for the modification that here, in addition to head-on collisions, off-center and non-vertical collisions are also studied. Equations (10), (11), (12) and (13) of Paper I are modified. Using the impulsive approximation the change in velocity ($\Delta v'$) of the representative star in the disk galaxy, due to the collision (for the case $R_s = R_D = R$), is given by,

$$\Delta v' = -\ (GM_s)\ /\ (VR)\ \vec{I}, \tag{3}$$

Where, $$Ix = -x_D \int_{-\infty}^{\infty} (1/s'')\ (d\Phi_s/ds'')\ \left[s'\ /\ (s'^2 - p^2)^{\frac{1}{2}}\right] ds', \tag{4}$$

$$I_y = (p-y_D) \int_{\infty}^{\infty} (1/s'')\ (d\Phi_s\ /ds'')\ \left[\ s'/(s'^2-p^2)^{\frac{1}{2}}\right]\ ds'$$
$$-\ p \int_{\infty}^{\infty}(1/s'')\left(dX_{DS}\ /\ ds''\right)\ \left[s'/(s'^2 - p^2)^{\frac{1}{2}}\right]\ ds', \tag{5}$$

$$I_z = 0, \tag{6}$$

where the symbols have the same meaning as in Paper I, except for p, which is the distance of closest approach between the colliding galaxies measured in units of the radius of either galaxy, and χ_{DS}, which is a measure of the (mutual) potential due to the gravitational interaction between the disk and spherical galaxy and is defined as the mutual potential function for the disk-sphere pair (Ballabh, 1975).

Proceeding in the same way as in Paper I, we get the fractional change in internal energy of the disk galaxy due to the collision as,

$$\Delta U/|U| = \gamma_D \, \beta_D \qquad (7)$$

$$\text{where, } \gamma_D = (GM_s^2) \, / \, (V^2 M_D R) \qquad (8)$$

$$\text{and, } \beta_D = 1/(b \, U_{het}) \int_0^1 \langle I^2 (s) \rangle \, s e^{-4s} \, ds \qquad (9)$$

where U_{het} is a constant depending upon (σ), the density distribution of the disk (Ballabh 1973) and $\langle I^2(s) \rangle$ is the average value of I^2 at $s = r/R$.

$\vec{I}$, and hence $\vec{\Delta v}'$, is a function of both the distance of closest approach between the two galaxies, p, and the angle of inclination, i, of the disk to the direction of relative motion of the two galaxies. Using Equations (3) to (6), we calculate the velocity perturbation of the representative stars for different collisions, each collision being characterized by a value of p, i and γ_D, the last parameter determining the fractional change in internal energy of the victim disk due to the collision, $\Delta U/|U|$.

When then apply the Bottlinger method (Bottlinger, 1932, 1933), as discussed in Paper I, to study the post-collision orbits of the stars. As in Paper I, the orbits of the innermost and outermost stars are studied by numerical integrations, as the force law used in the Bottlinger method holds only within a limited range of distance in the disk galaxy. Using the same method (and units) as in Paper I, the density distribution of the disk after each collision is determined, approximately.

III. Numerical Results and Discussion

We increase p and decrease i (starting from i = 90°) in small steps and determine $\Delta v'$ for the representative stars for each collision and scale its value for different values of γ_D, which characterizes the fractional internal energy change of the victim disk, $\Delta U/|U|$. Then we determine the final density distribution of the disk for each collision approximately, and test whether a crowding of stars leading to ring formation is there or not.

We find that as p is increased, the rings become more asymmetrical and more intense on the side nearer to the perturber. Beyond $p \sim \frac{1}{3}$, well defined, prominent rings do not form. We denote this by p_{max}. Thus $p_{max} \lesssim \frac{1}{3}$.

The angle of inclination of the disk to the direction of relative motion of the two galaxies is denoted by i (see Paper I). For convenience we define the angle $\alpha = 90° - i$, so that for $\alpha = 0°$ we have a collision whose trajectory is normal to the plane of the disk. We find that as α is increased, the rings become fainter and their eccentricity increases. We find that up to $\alpha \sim 45°$ we still get acceptable rings. We denote this by α_{max}. Thus $\alpha_{max} \lesssim 45°$.

For ring formation the energy of the collision should also be suitable. We find that if the collisional energy is too low, then the victim disk is not much affected and only very faint structures are formed. If the collisional energy is too high, then disruption of the victim disk results for a comparatively massive intruder and tidal capture takes place for a comparatively light intruder. We have classified the collisions in terms of $\Delta U/|U|$ for the victim disk. We find that for fairly well defined rings $\Delta U/|U|$ should lie between $\frac{1}{2}$ and $1\frac{1}{2}$. We denote this energy range by E_R. Thus $E_R = 1\frac{1}{2} - \frac{1}{2} = 1$.

Now we proceed to calculate the corresponding probabilities.

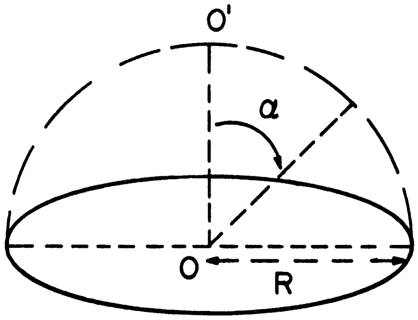
In regions of normal density, collisions between galaxies seldom occur; but in dense regions such encounters are relatively frequent. Moreover, observations of frequency determinations of colliding galaxies are likely to be carried out with respect to such regions where they will be comparatively numerous. Gallagher and Ostriker (1972) indicate that in dense regions, such as the Coma Cluster of Galaxies, there are about 3 interpenetrating collisions in a time of the order of the age of the Universe, i.e. 10^{10} years. We can take this as a typical value for dense regions. The typical lifetime of a ring galaxy is $\sim 10^{8}$ years (Theys and Spiegel, 1977; Lynds and Toomre, 1976; Freeman and de Vaucouleurs, 1974). hence in the lifetime of a ring galaxy the probability of an interpenetrating collision is 3/100. For a collision to be interpenetrating $p \leq 2$ and for it to be suitable for ring formation $p \lesssim p_{max} \lesssim \frac{1}{3}$. Hence in terms of impact parameter, the probability of the formation of a well defined ring galaxy is given by,

$$(\mathrm{Prob})_p = \frac{3}{100} \times \frac{\pi p_{max}^2}{\pi (p=2)^2} = 0.00083 \sim 0.001.$$

To study the favourable range of orientations of the collisions for ring formation, consider a sphere of radius R centered on the victim disk, and sorrounding it (Fig. 1). Let OO' define the axis of the disk normal to its plane and touching the sphere at O' Consider only the first quadrant of the sphere. Then the cone with vertical angle α defines the favourable trajectories of the collisions leading to ring formation, while the first quadrant of the sphere defines all possible trajectories of the collisions.

The area of this quadrant is proportional to all possible trajectories of the collisions, while the area of the base of the cone with vertical angle α and height R is proportional to the favourable trajectories of the collisions leading to ring formation. Hence the probability of ring formation, in terms of the orientation of the collision, is given by (see Fig. 2),

$$(\mathrm{Prob})_\alpha = \frac{\pi \{R \tan(\alpha/2)\}^2}{(1/4) \,.\, 4\pi R^2} = \tan^2 \frac{\alpha}{2}$$



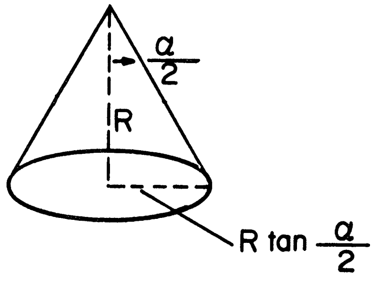


Fig. 1. Determination of the range of favourable angular orientations of the collisions for ring formation.

Fig. 2. The cone defining the favourable range of collisional trajectories leading to ring formation.

As $\alpha_{max} \lesssim 45°$, hence,

$$(\text{Prob})_{\alpha} = \tan^2 (\alpha_{max}/2) = 0.1715729 \sim 0.17$$

Theoretically the fractional change in internal energy of the victim disk $\Delta U/|U|$, can go from 0 to ∞, but collisions in which the internal energy change is greater than twice the initial internal energy seldom occer; so we can take $\Delta U/|U|$ to vary from 0 to 2 for practical cases, making its total range $E_T = 2$. Hence in terms of energy changes due to the collision, the probability of the formation of a ring galaxy is given by,

$$(\text{Prob.})_E = \frac{E_R}{E_T} = \frac{1}{2} = 0.5$$

Hence in terms of suitability of the impact parameter, orientation and energy changes pertaining to the collision, the compound probability of ring formation is given by,

$$\text{Prob.} = (\text{Prob})_p \times (\text{Prob})_{\alpha} \times (\text{Prob})_E$$
$$= 0.000085 \sim 0.0001.$$

Hence the expected frequency of fairly well defined and prominent ring galaxies is $\sim$ 0.01% of spirals.

We now compare our results with the observational values. Freeman and de Vaucouleurs (1974) find the frequency of ring galaxies to be,

$$\frac{N_R}{N_s} \times 100 \simeq \frac{7 \times 10^{-6} h^3 \text{ Mpc}^{-3}}{5 \times 10^{-2} h^3 \text{ Mpc}^{-3}} \times 100 \simeq 0.015\%, \text{ (of spirals)},$$

Where N_R and N_s denote the space densities of ring galaxies and giant bright spirals, respectively. This tallies quite well with our values.

Thompson (1977) finds $N_R \approx 2 \times 10^{-5}\ h^3\ \text{Mpc}^{-3}$. If we use the same value for N_S as Freeman and de Vaucouleurs (1979), we get the frequency of ring galaxies as 0.04% of spirals. But it should be noted that Thompson has included many ill defined and very faint rings in his survey and is hence expected to get a higher frequency than us. We deal with the frequency of such rings in another paper (Chatterjee 1986).

However, Arp and Madore (1977) and Dostal and Metlov (1978,1979) get a much higher frequency for ring galaxies. This is because their surveys include, in addition to ring galaxies, galaxies with outer rings, which do not have an origin related to the collision process. In fact such outer ring structures are observed in about 10% to 20% of late lenticulars and early type spirals (de Vaucouleurs, 1974). Hence these are obvious overestimates. Athanassoula and Bosma (1985) review these results.

Thus we find that our results are quite compatible with the observational values, if we exclude the overestimates.

IV. Conclusions

We find that the frequency of fairly well defined ring galaxies is ~0.01% of spirals, compatible with the observational results, provided the frequency determination is conducted with respect to densely populated regions where collisions are comparatively frequent, using the corresponding value of the collisional frequency in such regions. But, however if we consider a sparse region populated by field galaxies where the average distance between galaxies is ~100 R_a (where R_a denotes the mean radius of a galaxy) (see Mitton, 1977), and conduct frequency determinations with respect to such regions, then the expected frequency of ring galaxies goes down by few orders of magnitude. Theoretically the impact parameter will vary from 0 to ∞, but its mean value is of the order of this average distance between galaxies, and can be taken as, $p_\infty \lesssim 100\,R_a$ Hence in this case, as for ring formation $p_{max} \lesssim \frac{1}{3} R_a$, we get,

$$(\mathrm{Prob})_p = \frac{\pi p^2_{max}}{\pi p_\infty^{\;2}} = \frac{1}{9 \times 10^4} \sim 10^{-3}$$

Thus in this case the probability of ring formation, by the collision process, becomes,

$(\mathrm{Prob}) = (\mathrm{Prob})_p \times (\mathrm{Prob})_\alpha \times (\mathrm{Prob})_E \sim 10^{-6}$, and the expected frequency is $\sim 10^{-4}$% of spirals. Dostal and Metlov (1979) also find from theoretical calculations that the value of $N_R/N_S \sim 10^{-6}$ (or an expected frequency of 10^{-4}%) for regions of sparse density, populated by field galaxies, using the values of velocity distribution and densities corresponding to such regions. But for densely populated regions, like the Coma Cluster of Galaxies, they get a value of $N_R/N_S \gtrsim 10^{-4}$ (corresponding to an expected frequency $\gtrsim 10^{-2}$%), adopting the parameters of velocity distribution and densities given by Gregory (1975). This clearly indicates that stray, hyperbolic encounters taking place in sparse regions, populated by field galaxies, are too scarse by few orders of magnitude to explain the formation of ring galaxies. Hence most of these interacting pairs must have already been bound doubles, whose orbits are such as to have brought about only now the interpenetrating encounters leading to ring formation (see, Limber, 1965; Toomre, 1974). This is strongly supported by the fact that there are fields where ring galaxies are 3 to 5 times more frequent than the average (Dostal and Metlov, 1978, 1979).

It is necessary to conduct more observational and theoretical frequency determination of interacting galaxies in general and ring galaxies in particular.

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DISCUSSION

PIŞMIŞ: In the nuclei of some galaxies, in particular in those of early type barred spirals, there exist very tight spirals with dimensions around 1 to 2 kpc. Would your mechanism of the formation of outer rings in galaxies explain the inner ringlike form?

CHATERJEE: These inner rings are racially different from the prominent rings of ring galaxies. These rings do not have an origin related to the collision process, but are due to internal mechanisms in isolated galaxies.

AGUILAR: What are the differences or similarities between the results obtained here and those of the very extensive study by Toomre on this subject in the 70's?

CHATERJEE: The impulsive approximation method gives qualitative results. Hence, using this method, we can get general relations between the observed characteristics of the ring and the collision parameters, and predict the type of collision responsible for an observed ring and viceversa. The N-body simulations used by Toomre (1976) and Theys and Spiegel (1977) give particular results for a given collision, rather than general results.

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