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RADIATION HYDRODYNAMICS IN ASTROPHYSICS

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RESUMEN. A partir de las ecuaciones generales covariantes de la Hidrodinámica Radiativa (RH), se analizan sus formas particulares tanto en sistemas de referencia inerciales como en sistemas de referencia en comovimiento con el fluido. Se discute la importancia, en órdenes de magnitud, de los diferentes términos en dos regímenes límites de interés en astrofísica: (1) En el régimen de flujo radiativo libre (FSR) durante escalas temporales características del flujo radiativo y del flujo hidrodinámico. (2) En el régimen de difusión, estático (SDR) y dinámico (DDR).

Se clasifican los flujos radiantes en base a tres parámetros adimensionales: La razón entre las densidades de energía de la materia y del campo radiativo R, el número de Boltzmann Bo y el número de Reynolds radiativo Re_R.

Se discuten dos clases generales de soluciones: Lineales (propagación y estabilidad) y no-lineales. Se señalan algunos problemas de la RH no resueltos y de creciente interés en astrofísica.

ABSTRACT. Inertial and comoving - frame equations of Radiation Hydrodynamics (RH) are analized, starting from their general covariant form. The relative importance of their different terms, in two limit regimes of interest in astrophysics, is discussed: (1) in the free-streaming regime (FSR) during radiative and fluid-flow time scales; (2) in static (SDR) and dynamic (DDR) diffusion regimes.

The discussion is carried out with the help of three dimensionless parameters: the ratio between material and radiation field energy density R, the Boltzmann number Bo, and the radiative Reynolds number Re_R.

Two kinds of general solutions are discussed: linear solutions (stability and wave propagation), and non-linear ones. Several open questions of increasing interest in astrophysics are also examined.

Key words.: HYDRODYNAMICS

I. INTRODUCTION.

Generally speaking, radiation has been considered as playing two different roles in Astrophysics: The passive role (diagnostic), in which the physical structure of the emitting material is assumed to be known (atmospheres in hydrostatic, radiative and steady-state statistical equilibrium) and the radiative transfer equation is solved by means of well known analytical or/ and numerical techniques, according to the particular problem at hand (Chandrasekhar 1960; Mihalas 1978). In the active role, on the other hand, radiation is tied to the dynamics of the material, mainly because it transports energy and momentum, in such a way that one has to deal with radiating flows (Pomraning 1973; Mihalas and Mihalas 1984, hereafter MM; I.A.U. Colloquium 82, 1985).

The formal problem one has to deal with in radiation hydrodynamics (RH) is to determine the following parameters, as functions of position $\mathbf{r}$ and time t: the material density ρ, pressure p, temperature T, energy density per unit mass e and the three components of the velocity **v**; the radiation energy density per unit volume E, three components of the radiation flux **F**, and six independent component of the radiation pressure tensor P_{ij}. i.e., a total of seventeen unknowns. For the solution of the above problem, there are only eleven equations: the material equation of continuity, the energy equation, three components of the momentum equation, and two material constitutive relations (one equation of state and one caloric equation of state); the radiation energy equation and three components of the radiation momentum equation. In what follows, the notation used by Mihalas and Mihalas (MM) will be used with only minor changes. Usually, in order to complete the number of equations required to determine the seventeen unknowns, six closure relations relating P_{ij} with E have to be determined by an iterative procedure, or one has to give them as ad hoc assumptions. In addition, appropiate boundary and inicial conditions have to be given.

There is no hope of solving the problem analytically even in the "most simple" case of one dimensional radiating flow (eight unknowns related by eight relations, five of which are differential equations), and powerful numerical techniques have been developed (Winkler and Norman 1984; MM; and references therein) to work out particular problems. On the other hand, there are several approximate forms of the exact equations whose solutions are quasi-analytical. These solutions are sufficiently accurate in certain limit regimes, provide a great deal of physical insight and may be used as entry or testing solutions of numerical calculations. However, one must be aware of their exact scope and limitations in order to avoid misapplications which are frequently found in works on the subjet.

This talk will be concentrated on the discussion of the self-consistent radiation hydrodynamic equations (RHE), several useful approximate forms, and their solutions, in limit regimes of interest in Astrophysics. Several open questions for further research will also be indicated. The task of preparation has been inspired and guided by the Mihalas and Mihalas recent monograph (MM) which surely will soon become a classical work on the subject.

It is a pleasure to express my thanks to Professor D. Mihalas and Dr. B. Mihalas for providing me with a preliminar version of their outstanding book and other very interesting preprints, and for stimulating conversations and private communications.

II. RADIATING FLOWS LIMIT REGIMES.

If ℓ is the characteristic linear dimension through which the physical quantities defining the state of the fluid change appreciably, and λ_p is the characteristic photon mean free path, i.e. $\lambda_p = \chi^{-1}$, χ being the opacity, two well known limits for the photon transport may occur: the free streaming regime (FSR) if $\lambda_p/\ell \gtrsim 1$, and the diffusion regime (DR) if $\lambda_p/\ell << 1$.

The characteristic times associated with the above radiative transfer regimes are: the free streaming time t_s and the diffusion time t_d given by

$$t_s = \frac{\ell}{c} ; \quad t_d = (\chi\ell)\, t_s = \frac{\ell^2}{c\lambda_p} , \tag{1}$$

where c is the speed of light.

On the other hand, the characteristic fluid-flow time scale is given by

$$t_s = \frac{\ell}{v} . \tag{2}$$

From the above scale lengths and time scales, one may distinguish four different regimes: the FSR ($\lambda_p/\ell \gtrsim 1$) in a free streaming radiation time scale t_s and in a fluid-flow time scale t_f. The DR ($\lambda_p/\ell << 1$): the static diffusion regime (SDR) if $t_d << t_f$ (or $v/c << \lambda_p/\ell$), and the dynamic diffusion regime (DDR) if $t_d \gtrsim t_f$ (or $v/c \gtrsim \lambda_p/\ell$).

The relative importance of the radiation in determining the local properties of the radiating fluids is measured by the dimensionless parameter

$$R = \frac{\rho_0 e_0}{E_0} \tag{3}$$

and the relative importance of the material enthalpy (h_0) flux to the radiation flux F_0 is measured by the Boltzmann number Bo,

$$Bo = \frac{\rho_0 v h_0}{F_0} = \frac{\text{enthalpy flux}}{\text{radiation flux}} . \tag{4}$$

The subindex zero means that the respective quantity is measured in the comoving (proper) frame.

It is easy to verify that, to an order of magnitude accuracy [see equation (34) below]

$$Bo = \begin{cases} \left(\frac{v}{c}\right) R & \text{in FSR,} \\ \left(\frac{v}{c}\right)\left(\frac{\ell}{\lambda_p}\right) R & \text{in DR.} \end{cases} \tag{5}$$

Therefore, in FSR, even when $R >> 1$, Bo could be less than one if $v/c << 1$ (non relativistic flows), i.e., the radiation flux dominates the enthalpy flux even when the radiation energy density (E_0) is smaller than the material energy density ($\rho_0 e_0$). This conclusion also applies in SDR because there ($v\ell/c\lambda_p$) << 1. However, in DDR for which $v\ell/c\lambda_p$ could be >> 1, Bo < 1 if R << 1, i.e., the radiation flux dominates over the enthalpy flux only if the radiation energy density is greater than the material energy density.

Although the main interest in this talk is centered on the discussion of the RHE up to terms of the order v/c, some noteworthy remarks on radiative viscosity will also be made (a complete discussion on the subject can be found in Mihalas 1983 and MM). So, the radiative Reynolds number Re_R which measures the relative importance of the radiative viscosity affects with respect to the inertial forces is defined by

$$Re_R = \frac{\rho_0 v \ell}{\mu_R} \approx \left(\frac{t_f}{t_\lambda}\right) \frac{\rho_0 v^2}{E_0} \approx \left(\frac{\ell}{\lambda_p}\right)\left(\frac{c}{v}\right)\left(\frac{\rho_0 v^2}{E_0}\right) , \tag{6}$$

where $\mu_R = \lambda_p E_0/c$ is the coefficient of radiative viscosity, and $t_\lambda = \lambda_p/c$ is the photon flight time.

Typical values of R and Bo in several radiating fluids of interest in Astrophysics, are given in Table 1.

III. COVARIANT FORM OF THE RADIATION HYDRODYNAMIC EQUATIONS.

According to Mihalas and Mihalas (MM), the covariant form of the dynamical equations of the radiation field is

$$R^{\alpha\beta}_{;\beta} = -G^{\alpha} , \tag{7}$$

$R^{\alpha\beta}_{;\beta}$ being the covariant derivative of the radiation stress-energy tensor $R^{\alpha\beta}$ defined by the relation

$$R^{\alpha\beta} = \frac{1}{c}\int_0^\infty d\nu \oint d\omega \; I(\mathbf{n},\nu) n^\alpha n^\beta . \tag{8a}$$

More specifically,

$$\mathbb{R} = \begin{pmatrix} E & c^{-1}\mathbf{F} \\ c^{-1}\mathbf{F} & \mathbb{P} \end{pmatrix}, \tag{8b}$$

E, **F**, $\mathbb{P}$ being the radiation energy, radiation flux and radiation pressure tensors, respectively, which are given by

$$E = \frac{1}{c}\int_0^\infty d\nu \oint d\omega \; I(\mathbf{n},\nu) = \frac{4\pi}{c} J, \tag{9a}$$

TABLE 1

Order of Magnitude of R and Bo in Several Radiating Flows

Object		R	Bo
Sun	center	5×10^{2}	10^{-2}
	photosphere ($h \sim 0$)	6×10^{4}	2
	cromosphere ($h \sim 10^{3}$km)	10^{1}	4×10^{-4}
	corona ($h \sim 4\times10^{3}$km)	7×10^{-3}	3×10^{-6}
	flares	$10 \sim 10^{4}$	$10 \sim 10^{2}$
Accretion Flows on Protostellar Objects	in the shock wave	10^{-4}	10^{-3}
	infalling material	10^{-5}	10^{-8}
Jets (SS433)	at $\ell \sim 4\times10^{7}$cm	10^{-4}	10^{-5}
	at $\ell \sim 7\times10^{14}$ cm	7×10^{8}	$\lesssim 10^{8}$
Stellar Winds	solar at 1.A.U.	3×10^{-8}	10^{-10}
	massive winds in O stars	10^{-5}	5×10^{-8}
	massive cold winds in I-II LT stars.	1-10	3×10^{-4}
Supernova (type II)		$10^{-3}-10^{2}$	$10^{-5}-10$
Accretion Flows on Compact Objetc (Neutron stars, BH and White Dwarfs)		$10^{-1}\sim10^{-5}$	$1\sim10^{-5}$
Active Galactic Nuclei	on the accretion disk	10^{-4}	10^{-8}
	inner part of de a.d.	<< 1	<< 1
	outer part of de a.d.	>> 1	>> 1
	in the wind	10^{-9}	10^{-10}
Cooling Flows in Clusters of Galaxies		$10^{2}\sim10^{7}$	$10^{-2}\sim10^{4}$
Early Universe	$z \gtrsim 150$	$\lesssim 10^{-9}$	$\lesssim 10^{-14}$

$$\mathbf{F} = \int_0^\infty d\nu \oint d\omega\, \mathbf{n} I(\mathbf{n},\nu) \ , \tag{9b}$$

$$P^{ij} = \frac{1}{c}\int_0^\infty d\nu \oint d\omega\, n^i n^i\, I(\mathbf{n},\nu), \tag{9c}$$

where the dependence of all quantities on position $\mathbf{r}$ and time t is understood and has been dropped. When the respective quantities have not been integrated with respect to frequency ν they will be denoted by $E(\nu)$, $J(\nu)$, $\mathbf{F}(\nu)$, and so on. In the above relations, $I(\mathbf{n},\nu)$, $d\omega$ and n^α are: radiation intensity in the direction of the unit vector $\mathbf{n}$, element of solid angle, and the four vector $(1,\mathbf{n})$, respectively. G^α is the radiation four-force density defined by

$$G^0 = \frac{1}{c}\int_0^\infty d\nu \oint d\omega\, \left[\chi(\mathbf{n},\nu) I(\mathbf{n},\nu) - \eta(\mathbf{n},\nu)\right] \ , \tag{10a}$$

$$G^i = \frac{1}{c}\int_0^\infty d\nu \oint d\omega\, \left[\chi(\mathbf{n},\nu) I(\mathbf{n},\nu) - \eta(\mathbf{n},\nu)\right] n^i, \tag{10b}$$

where $\chi(\mathbf{n},\nu)$ and $\eta(\mathbf{n},\nu)$ are the macroscopic opacity and emission coefficients per unit volume, respectively. Hereafter the greek indices run as 0,1,2,3, and the latin indices as 1,2,3. A four-space with coordinates $x^\alpha = (ct,\mathbf{r})$ and signature $(-;+,+,+)$ is assumed.

On the other hand, the covariant form of the material dynamical equations is (see MM)

$$M^{\alpha\beta}_{;\beta} = f^\alpha \,, \tag{11}$$

$M^{\alpha\beta}_{;\beta}$ being the covariant derivative of the material stress-energy tensor $M^{\alpha\beta}$, and f^α the four-force density vector given by

$$f^\alpha = \gamma\left(\frac{\mathbf{f}\cdot\mathbf{v}+\hat{\varepsilon}}{c}, \mathbf{f}\right) \,, \tag{12}$$

where $\mathbf{f}$ is the ordinary force density, $\hat{\varepsilon}$ the rate of energy input from non-mechanical sources and

$$\gamma = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \,.$$

The stress-energy tensor $M^{\alpha\beta}$ for an ideal fluid is given by (Thomas 1930; see also Taub 1978 and MM)

$$M^{\alpha\beta} = \rho_0\left(1+\frac{h_0}{c^2}\right)V^\alpha V^\beta + p_0 g^{\alpha\beta} \,, \tag{13}$$

where

$$V^\alpha = \gamma(c, \mathbf{v}),$$

and $g^{\alpha\beta}$ is the metric tensor.

Explicit forms of $M^{\alpha\beta}$ for non-ideal fluids are given by Weinberg (1971), and Greenberg (1975).

A radiating flow can be considered as the material flow modified by the radiating four-force, i.e.

$$M^{\alpha\beta}_{;\beta} = f^\alpha + G^\alpha \,. \tag{14}$$

An equivalent expression derived from equation (7) is

$$(M^{\alpha\beta} + R^{\alpha\beta})_{;\beta} = f^\alpha \,. \tag{15}$$

i.e., a mixture of matter and radiation under the action of the external four-force f^α (MM).

In summary, the eleven equations which have to be solved for radiating flows are: the material continuity equation

$$(\rho_0 V^\alpha)_{;\alpha} = 0 \;; \tag{16}$$

the equation of state, for example

$$p_0 = p_0(\rho_0, T_0, \ldots) \;; \tag{17}$$

the caloric equation of state, for example

$$e_0 = e_0(\rho_0, T_0, \ldots) \;; \tag{18}$$

the energy ($\alpha = 0$) and momentum ($\alpha = i$) equations of the radiating fluid

$$(M^{\alpha\beta} + R^{\alpha\beta})_{;\beta} = f^\alpha, \quad \text{or} \quad M^{\alpha\beta}_{;\beta} = f^\alpha + G^\alpha \;; \tag{19}$$

the energy ($\alpha = 0$) and momentum ($\alpha = i$) equations of the radiation field

$$R^{\alpha\beta}_{;\beta} = -G^{\alpha} ; \quad (20)$$

and the closure relations

$$P^{ij} = Ef^{ij} , \quad (21)$$

where f^{ij} is the Eddington tensor.

a) Inertial Frame RHE.

In inertial frames the calculation of the covariant derivatives in equations (16), (19) and (20) in any geometry is direct. For example, in Cartesian coordinates the above equations become

$$(\gamma\rho_0)_{,t} + (\gamma\rho_0 v^i)_{,i} = 0 , \quad (22)$$

$$\rho_0 \left|\frac{De_0}{Dt} + p_0 \frac{D}{Dt}\left(\frac{1}{\rho_0}\right)\right| = \gamma\hat{\varepsilon} + cG^0 - v_i G^i , \quad (23)$$

$$\gamma^2\rho_0\left(1+\frac{h_0}{c^2}\right)\frac{Dv^i}{Dt} = f^i - \delta^{ij}p_{,j} - \frac{v^i}{c^2}(p_{,t} + v_j f^j + \hat{\varepsilon}) + G^i - \frac{v^i}{c} G^0 , \quad (24)$$

$$E_{,t} + F^i_{,i} = -cG^0 , \quad (25)$$

$$c^{-2}F^i_{,t} + P^{ij}_{,j} = -G^i , \quad (26)$$

where $\psi_{,x} \equiv \partial\psi/\partial x$.

For spherical geometry see MM. These authors also give the transformation equations, for the components of the tensor $R^{\alpha\beta}$ and the four-vector G^{α}, between the proper frame and any other inertial frame. For example, in the case of one-dimensional motion they are given by:

$$E = \gamma^2\left[E_0 + \frac{2v}{c^2} F_0 + \left(\frac{v}{c}\right)^2 P_0\right] , \quad (27)$$

$$F = \gamma^2\left[\left(1+\frac{v^2}{c^2}\right) F_0 + vE_0 + vP_0\right] , \quad (28)$$

$$P = \gamma^2\left[P_0 + \frac{2v}{c} F_0 + \left(\frac{v}{c}\right)^2 E_0\right] , \quad (29)$$

$$G^0 = \gamma\left(G^0_0 + \frac{v}{c} G^1_0\right) , \quad (30)$$

$$G^1 = \gamma\left(G^1_0 + \frac{v}{c} G^0_0\right) , \quad (31)$$

where

$$G^0_0 = \frac{1}{c}\int_0^{\infty} d\nu_0 \left[c\chi_0(\nu_0)E_0(\nu_0) - 4\pi\eta_0(\nu_0)\right] , \quad (32)$$

$$G^1_0 = \frac{1}{c}\int_0^{\infty} d\nu_0 \; \chi_0(\nu_0)F_0(\nu_0) . \quad (33)$$

The respective relations for the non-relativistic limit, are obtained taking $\gamma \to 1$ and $v^2/c^2 \to 0$ in the above equations.

From equations (27)-(31) it is seen that formulations of RHE neglecting terms of $O(v/c)$ (Pomraning 1973) do not distinguish between Eulerian and Lagrangian frames and therefore they are not correct; for further discussion on this point see MM.

It can be easily shown that the order of magnitude of the radiating flux F_0 and the absorption-emission term cG_0^0 become

$$F_0 \sim \begin{cases} cE_0 & \text{in FSR ,} \\ \left(\frac{\lambda_p}{\ell}\right)(cE_0) & \text{in DR.} \end{cases} \quad (34)$$

$$cG_0^0 \sim \begin{cases} \frac{cE_0}{\lambda_p} & \text{in FSR,} \\ \frac{c\lambda_p}{\ell^2} E_0 & \text{in SDR,} \\ \frac{v}{\ell} E_0 & \text{in DDR,} \\ 0 & \text{in radiative equilibrium.} \end{cases} \quad (35)$$

From relation (34) one concludes that if $(\lambda_p/\ell) > 1$, the flux F_0 becomes larger than the maximum radiating energy flux cE_0, error which one arrives at when diffusion approximations are applied to the FSR. The above error is partially avoided by introducing flux limiters, as will be discussed later.

On the other hand, due to the fact that the radiative force

$$f_R \sim \frac{\chi(\nu)F(\nu)}{c} \sim \begin{cases} \chi(\nu)E(\nu) & \text{in FSR ,} \\ \frac{E(\nu)}{\ell} & \text{in DR,} \end{cases}$$

f_R does depend on $\chi(\nu)$ in FSR but it becomes independet on $\chi(\nu)$ in DR; result of great importance in the study of radiation driven winds (Castor et al. 1975; Owocki and Rybicki 1984,1986).

b) Comoving Frame RHE.

There are several conceptual and practical reasons for handling radiating flows in comoving (lagrangian) frames (Mihalas et al. 1975, 1976; MM), although the radiative transfer equation becomes much more cumbersome in such frames. For example, in Cartesian coordinates equations (16), (19) and (20) can be written as

$$\frac{D\rho_0}{Dt} + \rho_0 v^i_{,i} = 0 \; ; \quad (36)$$

$$\rho_0\left[\frac{De_0}{Dt} + p_0\frac{D}{Dt}\left(\frac{1}{\rho_0}\right)\right] = \hat{\varepsilon}_0 + cG_0^0, \quad \text{or}$$

$$\frac{D}{Dt}\left(e_0 + \frac{E_0}{\rho_0}\right) + (p_0 + P_0)\frac{D}{Dt}\left(\frac{1}{\rho_0}\right) = \frac{\hat{\varepsilon}_0}{\rho_0} - \frac{1}{\rho_0} F^i_{0,i} \; ; \quad (37)$$

$$\rho_0\left(1 + \frac{h_0}{c^2}\right)\frac{Dv^i}{Dt} = f^i_0 - \delta^{ij} p_{0,j} + G^i_0 \;, \text{ or}$$

$$\rho_0\left(1 + \frac{h_0}{c^2}\right)\frac{Dv^i}{Dt} = f^i_0 - (\delta^{ij} p_0 + P_0^{ij})_{,j} - \frac{1}{c^2} F^i_{,t} \quad (38)$$

$$\rho_0 \frac{D}{Dt}\left(\frac{E_0}{\rho_0}\right) + F^i_{0,i} + P_0^{ij} v_{i,j} + \frac{2}{c^2} a_i F^i_0 + cG_0^0 = 0 \; ; \quad (39)$$

$$\frac{\rho_0}{c^2}\frac{D}{Dt}\left(\frac{F^i}{\rho_0}\right) + P^{ij}_{0,j} + \frac{1}{c^2} F^j v^i_{,j} + \frac{1}{c^2}(E_0 a^i + a_j P_0^{ij}) + G^i_0 = 0, \quad (40)$$

a_i being the accelaration Dv^i/Dt. The RHE in comoving frames and different geometries have been analyzed by Lundguist (1966), Castor (1972), Pomraning (1974), Buchler (1979, 1983), Mihalas (1980, 1981, MM), Munier and Weaver (1984), and Munier (1985).

For the one dimensional and non-relativistic case, the RHE (36)-(40) become

$$\frac{D\rho_0}{Dt} + \rho_0 \frac{\partial v}{\partial z} = 0 \tag{41}$$

$$\rho_0\left[\frac{De_0}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho_0}\right)\right] = \hat{\varepsilon}_0 + cG_0^0, \text{ or} \tag{42a}$$

$$\frac{D}{Dt}\left(e_0 + \frac{E_0}{\rho_0}\right) + (p + P_0)\frac{D}{Dt}\left(\frac{1}{\rho_0}\right) = \frac{\hat{\varepsilon}_0}{\rho_0} - \frac{1}{\rho_0}\frac{\partial F}{\partial z}\ ; \tag{42b}$$

$$\rho_0\frac{Dv}{Dt} = f_0 - \frac{\partial p}{\partial z} + G_0^1, \tag{43a}$$

$$\rho_0\frac{D}{Dt}\left(v + \frac{F_0}{c^2\rho_0}\right) = f_0 - \frac{\partial}{\partial z}(p + P_0) - \frac{2F}{c^2}\frac{\partial v}{\partial z}\ ; \tag{43b}$$

$$\frac{DE_0}{Dt} + \frac{\partial F_0}{\partial z} + (E_0 + P_0)\frac{\partial v}{\partial z} + cG_0^0 = 0\ ; \tag{44}$$

$$\frac{1}{c^2}\frac{DF_0}{Dt} + \frac{\partial P_0}{\partial z} + \frac{2F_0}{c^2}\frac{\partial v}{\partial z} + G_0^1 = 0\ . \tag{45}$$

The corresponding equations to (42b) and (43b) including dissipative affects in the DR were obtained by Weinberg (1971); see also (MM).

Equations (42) and (43) give the energetical and dynamical coupling between matter and radiation field, respectively. Equations (42a) and (43a) are more appropriate for use in FSR, and equations (42b) and (43b) in DR, respectively. In the extreme case of no matter-radiation coupling, $G_0^0 \to 0$ and $G_0^1 \to 0$. Hence, equations (42a) and (43a) reduce to the material energy equation with local heat input (or output), and the Euler equation, respectively.

From equation (42a) one obtains:

$$\left(\rho_0\frac{De_0}{Dt}\right):(cG_0^0) \sim \begin{cases} \left(\frac{\lambda_p}{\ell}\right)R & \text{in FSR and } t_s, \\ \left(\frac{v}{c}\right)\left(\frac{\lambda_p}{\ell}\right)R & \text{in FSR and } t_f, \\ R & \text{in DR.} \end{cases} \tag{46}$$

From the above relations, it is obvious that on the time scale t_f equation (42a) in FSR reduces to the quasi-static case, due to the fact that $t_s \ll t_f$ or $v/c \ll 1$. In DR the strongest energetic coupling between matter and radiation occurs for $R \sim 1$, but in FSR, R might reach small values, $R < 1$ if $\lambda_p/\ell > 1$ so that $(\rho_0\, De_0/Dt):(cG_0^0) \sim 1$.

From equation (43a), the ratio of the gas pressure force to the radiation force G_0^1 is

$$\left(\frac{\partial p}{\partial z}\right):(G_0^1) \sim \begin{cases} \left(\frac{\lambda_p}{\ell}\right)R & \text{in FSR} \\ R & \text{in DR.} \end{cases} \tag{47}$$

Therefore, in DR strong dynamical coupling occurs if $R \sim 1$ but in FSR, R could be < 1 (if $\lambda_p/\ell > 1$) so that radiation force can be comparable to the gas pressure.

The four terms of equation (44) scale as

$$
\begin{aligned}
&1 : 1 : \frac{v}{c} : \frac{\ell}{\lambda_p} \quad \text{in FSR and } t_s , \\
&\frac{v}{c} : 1 : \frac{v}{c} : \frac{\ell}{\lambda_p} \quad \text{in FSR and } t_f , \\
&\frac{v}{c}\frac{\ell}{\lambda_p} : 1 : \frac{v}{c}\frac{\ell}{\lambda_p} : 1 \quad \text{in SSR} , \\
&1 : \frac{c}{v}\frac{\lambda_p}{\ell} : 1 : 1 \quad \text{in DDR},
\end{aligned} \tag{48}
$$

and those of equation (45) scale as

$$
\begin{aligned}
&1 : 1 : \frac{v}{c} : \frac{\ell}{\lambda_p} \quad \text{in FSR and } t_s, \\
&\frac{v}{c} : 1 : \frac{v}{c} : \frac{\ell}{\lambda_p} \quad \text{in FSR and}, \\
&\frac{v}{c}\frac{\lambda_p}{\ell} : 1 : \frac{v}{c}\frac{\lambda_p}{\ell} : 1 \text{ in SDR and DDR.}
\end{aligned} \tag{49}
$$

Therefore, terms of $O(v/c)$ in equations (44) and (45) can be dropped (quasi-static limit) in FSR. If so, only first, second and fourth terms remain on a time scale t_s, and second and fourth on a time scale t_f. In DR the dominating terms are the second and fourth, but in DDR, first, third and fourth terms dominate in equation (44). However, in radiative equilibrium, $cG_0^0 \sim 0$, and all terms have to be retained in equation (44). The discussion for the corresponding equations in spherical geometry has been carried out by Mihalas and Mihalas (MM) and as emphasized by these authors, the inclusion of terms up to $O(v/c)$ in equation (44) ensures a correct transition between the two regimes (FSR and DR). Furthermore, the consistency between inertial and comoving descriptions is only guaranteed if terms up to $O(v/c)$ are retained in both descriptions of equation (44).

IV. LINEAR SOLUTIONS OF THE RHE.

There are efficent numerical techniques (Winkler 1984) which allow to solve the RHE under very different physical conditions. However, the classical problem of determining solutions to the RHE near to equilibrium states (linear solutions) continues to be a main problem to be treated, mainly for two reasons: (i) the above solutions are solutions in first approximation to the RHE; (ii) they describe the evolution and propagation of small disturbances.

The linear analysis proceeds as follows: One assumes that the solutions to RHE have the form

$$\psi_j(\mathbf{r},t) = \bar{\psi}_j(\mathbf{r}) + \tilde{\psi}_j(\mathbf{r},t) \ , \tag{50}$$

and

$$|\psi_j(\mathbf{r},t)| << |\bar{\psi}_j(\mathbf{r})| \ , \tag{51}$$

where $\bar{\psi}_j(\mathbf{r})$ are known stationary solutions and $\tilde{\psi}_j(\mathbf{r},t)$ are functions to be determined.

With condition (51), the system of RHE can be linearized and written in the form of an eigenvalue problem whose compatibility condition leaves a secular equation. For example, if

$$\tilde{\psi}_j(\mathbf{r},t) \sim \exp(nt + i\mathbf{k}\cdot\mathbf{r}), \tag{52}$$

n and $\mathbf{k}$ being the damping (or growth) rate and the wave number of the disturbance, respectively, the secular equation becomes a polynomial relation between n and $\mathbf{k}$,

$$n = n(\mathbf{k}), \tag{53}$$

(i) If one assumes $n = \theta + i\omega$ (θ and ω being real quantities) and $\mathbf{k}$ a real vector, one is dealing with the classical problem of stability (Chandrasekhar 1961). Therefore, the equilibrium is unstable if $\theta > 0$, stable if $\theta < 0$ and marginally stable if $\theta = 0$. The marginal state defines the critical wave number k_c which determines the scale length of structures that can be formed.

(ii) If one assumes $n = i\omega$ ($\theta = 0$) and $\mathbf{k} = \mathbf{k}_1 + i\mathbf{k}_2$ ($\mathbf{k}_1$ and $\mathbf{k}_2$ being real vectors), one is dealing with the propagation problem. The wave modes for which $k_2^j > 0$ are damped, and those with $k_2^j < 0$ are amplified; hence, they can originate nonlinear phenomena such as discontinuities (for example, shock waves).

a) Linear Solutions with Local Heat-Loss Function.

i) Local stability.

In Astrophysics there are many specific situations in which the radiative transport becomes irrelevant and the heat gain-losses can be represented by a function $L(\rho,T,...)$ depending only on local properties of matter. In this line of thought the local stability problem has been analyzed by Parker (1953), Zanstra (1955a, 1955b), Weymann (1960), Field (1965), Hunter (1966, 1969, 1970), Goldsmith et al. (1969), Field et al. (1969), de Jong (1977), Goldsmith (1970), Dofouw (1970), Schwartz et al. (1972), Yoneyama (1973), Glassgold and Langer (1976), Oppenheimer (1977), Sabano and Kannari (1978), Flannery and Press (1979), Lepp et al. (1985), and Gilden (1985).

Most of the cited work was mainly addressed to the problem of explaining the actual structure of the interstellar medium.

The role played by thermal instabilities as generators of primordial structures (galaxies, globular clusters and first generation stars) has been studied by Kondo (1970), Kondo et al. (1971), Gurevich and Chernin (1975), Zentsova and Chernin (1979), Shchekinov (1979), Zentsova and Urpin (1980), Shchekinov and Edel'man (1980), Suchkov et al. (1981), Silk (1982, 1983), Shchekinov and Éntél (1983), Zel'dovich and Novikov (1983), Ibáñez and Parravano (1983), and Fall and Rees (1985).

The origin of structures in different parts of the solar atmosphere has also been explained by thermal instabilities; see for example, Athay (1976), Antiochos (1979), Hood and Priest (1980), Priest (1981), Craig and McClymont (1981), and Cheng et al. (1983).

Thermal instabilities also seem to be the origin of clumps in cosmic jets (Königl 1984; Ferrari et al. 1985; Bodo 1985), and in active galactic nuclei (McCray 1979; Krölig et al. 1981; Shlosman et al. 1985; Mathews 1986;...) and of filaments in cooling flows in clusters of galaxies (Fabian and Nulsen 1977; Mathews and Bregman 1978; Cowie et al. 1980; Fabian et al. 1984).

ii) Wave Propagation.

The propagation of small disturbances in plasmas with local heat gain-loss function has been studied by Souffrin (1966), Flannery and Press (1979), Ibáñez (1985), Ibáñez and Mendoza (1986) and Ibáñez (1986a).

Attempts to extend the linear analysis to the non-linear regime have been carried out by McMillan et al. (1980), and Kritsuk (1985).

b) Linear Solutions to the RHE with Radiation Transfer.

In Astrophysics one also encounters situations in which the energy flux throughout the matter strongly depends on the bulk conditions of the region under consideration. Therefore, the radiative transfer effects become important, demanding radiation transfer calculations.

i) Stability.

The temporal behavior of linear disturbances, considering radiation transport, has been studied by Spiegel (1957a, 1957b, 1960, 1964), Schatzman (1958), Böhm (1963), Osaki (1966), Unno and Spiegel (1966), Field (1971), Delache and Froeschlé (1972), Le Guet (1972), Anderson (1973), Froeschlé (1973, 1977), Giaretta (1977), MM, and Ibáñez and Plachco (1986a).

Applications to the solar plasma have been carried out by Frisch (1970), McClymont and Canfield (1983a, 1983b), Canfield et al. (1983), An et al. (1983), and Fisher et al. (1985a, 1985b, 1985c); to the quasar gas by Mestel et al. (1976), Mathews (1976, 1986), Kippenhahn

(1977), and Beltrametti (1981); and to the gas sourrounding collapsed objects by Lánger et al. (1981), Egorenkov et al. (1983)...

During the last years renewed interest for the study of the temporal behaviour of small disturbances in radiating flows has arised because its connection to the problem of the origin and structure of the radiation driven winds (see for instance, Nelson and Hearn 1978; Martens 1979; MacGregor et al. 1979; Abbot 1980; Kahn 1981; Owocki and Rybicki 1984, 1986; MM).

ii) Propagation.

The propagation of the wave modes resulting from the roots of the equation $\mathbf{k} = \mathbf{k}(n)$ with $\theta \equiv 0$, has been studied by Pai (1966), Stein and Spiegel (1967), Hearn (1972, 1973), Kaneko et al. (1975), Bisnovatyi-Kogan and Blinnikov (1979), Mihalas and Mihalas (1983, MM), Lucy (1984), Gough (1984), Ibáñez and Plachco (1986b), Ibáñez (1986b), and Mihalas (1986). Here we will not consider the problem of stellar acoustics about which a vast number of work exists (see for instance Cox 1980; Mihalas 1984; Davis 1985;...).

c) Several Open Questions.

1. Which are the criteria for stability (instability) and damping (amplification) in the presence of gradients and under non stationary conditions?

2. The study of stability and wave propagation in confined radiating fluids with boundary conditions remains to be done.

3. More studies about disturbances with spherical and cylindrical symmetry are required.

4. Which are the general criteria determining time scales and scale lengths for amplification when unstable fluctuations or amplifying disturbances reach the non linear regime, i.e., $|\tilde{\psi}_j| / |\bar{\psi}_j| \gtrsim 1$?

5. Which are the criteria for stability (instability) and damping (amplification) in the turbulent regime?

6. What about the stability of relativistic flows? The need of an answer to this question arises from the fact that several relativistic fluids (for example, jets) seem to have clumpy structure. In models for those fluids, Field (1965) criteria for thermal instability have been applied (Bodo 1985).

V. NON-LINEAR SOLUTIONS TO THE RHE.

When $\lambda_p/\ell << 1$ it is possible to find explicit forms for the radiation energy-stress tensor $R^{\alpha\beta}$ in terms of series expansions of λ_p/ℓ and v/c which are assumed to be small.

In the equilibrium radiation diffusion approximation, the radiation field is assumed to be thermalized, $J = B$. The zero order approximation becomes independient on either λ_p/ℓ or v/c. Here, $E_0 = a_R T^4$, $P_0^{ij} = E_0 \delta^{ij}/3$ and $F_0^i = 0$. The first order approximation contains terms depending on λ_p/ℓ but not on v/c and the radiation flux obeys a diffusion relation $F^i \sim \delta ij (E_0)_{,j}/\chi_R$. This approximation does not contain dissipative terms which are of $O(v^2/c^2)$ and $O(\lambda_p v/\ell c)$, but they do appear in the approximation of order two (Thomas, 1930; Hazelhurst and Sargent 1959; Simon 1963; Hsieh and Spiegel 1976; Masaki 1981; Mihalas 1983). In the second order radiation diffusion approximation it is a assumed that therms of $O(\lambda_p v/\ell c)$ are more important than terms of $O(\lambda_p^2/\ell^2)$, but this assumption has not been justified. Approximations including terms of $O(\lambda_p^2/\ell)$ have not jet been carried out. A deep critical study of the diffusion approximations can be found in Mihalas and Mihalas (MM).

It is clear that the condition $\lambda_p/\ell << 1$ does not imply that $J_0(\nu) = B_0(\nu)$. Therefore, non-equilibrium diffusion approximations have been developed in which $J_0(\nu) \neq B_0(\nu)$. The above approximations are essentially first or second order diffusion approximations, $\mathbf{F}_0 \sim \nabla E_0$, (Zel'dovich and Raizer 1966; Castor 1972; Hsieh and Spiegel 1976) in which the energy equation is solved by assuming that the thermal behavior of matter and radiation field are determined by two-parametrical functions $B(T)$ and $B(T_r)$ respectively (Freeman 1965); or treating $E_0(\nu_0)$ with multigroup techniques (MM and references therein).

Due to the relative simplicity of the diffusion approximations, attempts have been made to extend them to the FSR; but there $|F| > cE$! and in addition, the speed of the diffusion of radiation becomes larger than the speed of light. To avoid these absurd results some authors have (ad hoc) introduced flux limiters (Winslow 1968; Alme and Wilson 1974; Levermore and Pomraning 1981). The reason why the diffusion approximations fail when $\lambda_p/\ell \gtrsim 1$, is the neglection of the term DF/Dt in the radiation momentum equation. Severe criticisms to the above types of approximations have been expressed by Mihalas and Mihalas (MM).

As a conclusion of this section one may say that in spite of the relative simplicity of the radiative diffusion approximations extreme precautions have to be taken if one wants to apply them in ranges other than $\lambda_p/\ell \ll 1$, or if radiative viscosity is considered.

VI. CONCLUSION.

In summary, one may conclude that a self-consistent and covariant formulation of the RHE has been completed. On this basis many astrophysical problems treated with the help of intuitive formulations wait for an appropriate restatement and solution, and several open question concerning the origin and structure of many cosmic objects will find the correct answers by its use.

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DISCUSSION

CARDONA: ¿Dónde se encontraría esta formulación respecto a los regímenes de la aproximación de Sobolev para vientos estelares y cuál es su validez?

IBAÑEZ: La aproximación de Sobolev permite calcular el espesor óptico efectivo de medios en expansión, el cual viene a ser proporcional a la razón entre la velocidad térmica y el gradiente de velocidad, lo cual a su vez, permite calcular la fuerza radiativa de líneas en vientos estelares. La anterior aproximación ha sido la base para el desarrollo de la teoría de vientos estelares originados por radiación, en particular, la teoría CAK y subsecuentes desarrollos.

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