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COMPARISON BETWEEN IMAGE PLANE PHASE RECONSTRUCTION METHODS IN OPTICAL INTERFEROMETRY

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RESUMEN. Hacemos una comparación cuantitativa y cualitativa de los métodos de reconstrucción de fase en el plano de la imagen.

ABSTRACT. We have carried out a quantitative and a qualitative analysis intercomparing image plane phase reconstruction methods in optical interferometry.

Key words: IMAGE PROCESSING — INTERFEROMETRY

I. INTRODUCTION

Several authors (Nisenson and Papaliolios 1983; Deron and Fontanella 1984; Wirnitzer 1985) have attempted to compute the error on the reconstructed phase from interferometric techniques. On the basis of the formalism developed by Goodman and Belsher (1976, 1977) we generalize the previous derivations and we compare the methods of speckle holography (Liu and Lohmann, 1973; Bates et al., 1973), Knox and Thompson,(1974), and bispectral analysis (Weigelt, 1977; Lohmann et al., 1983).

II. COMPARISON BETWEEN METHODS

The speckle holography method is the only one which directly provides the phase, but it requires the presence of a point source in the isoplanatic field near the object. The method is insensitive to aberrations, and since the phase of the transfer function is independently determined on the point source, it is free of speckle noise. On bright objects, the error on the determination of the phase tends to zero.

The Knox and Thompson method and bispectral analysis provide the phase gradient and the phase closures (Roddier, 1986), respectively. For normally distributed phases, the maximum likelihood estimate of the phase is obtained from a generalized least square fit, and it is not a simple issue to compute the formal expression of the error. We must point out that:

The Knox and Thompson method is sensitive to aberrations and is affected by speckle noise. On bright sources the error on the gradient tends to a nonzero value. With some simple hypothesis, one can easily estimate from partial gradients the error on the phase gradient for a distance $u_o(r_o/\lambda)$:

i) in presence of photon noise the error is proportional to a value $f(\Delta u/u_o)$ which depends on the ratio between the frequency sampling interval Δu and u_o. The function $f(\Delta u/u_o)$ decreases and reaches a minimum value of $2^{1/2}$ radians as Δu goes to zero.

ii) for faint sources in presence of additive noise, the error is minimum when $\Delta u/u_o=0.23$.

iii) for bright sources the error is independent of frequency and is equal to $f(\Delta u/u_o)$, this value is probably close to the final error on the phase.

The bispectral analysis is insensitive to aberrations, yet it is affected by speckle noise. The error on the phase closure is proportional to the square root of the number of speckles N_o. For bright sources the final error on the phase is probably independent on the number of speckles because there are $N_o/4$ more phase relations than unknowns. Finally we remark that for very faint sources, the bispectral analysis, being a third order treatment (cube of a given signal), is more sensitive to the noise than second order treatments (square of a signal) such as the methods of speckle holography and Knox and Thompson.

Table I summarizes for a point source the error at high frequency on the phase, the phase gradient, and the phase closure, in the photon noise and additive noise cases.

TABLE I

METHOD	ERROR (for 1 interferogram)		
		Photon noise	White additive gaussian noise (σ_o)
Speckle Holography	$\sigma^2 [\phi(u)] =$	$\frac{1+p_1T(u) + p_2T(u)}{2p_1p_2T^2(u)}$	$\frac{\sigma_o^4/K_1K_2+p_1T(u)\sigma_o^2/K_2+p_2T(u)\sigma_o^2/K_1}{2p_1p_2T^2(u)}$
Knox and Thompson	$\sigma^2 [\phi(u) - \phi(u+\Delta u)] =$	$\frac{e^\alpha-1}{2}\left(1+\frac{1}{pT(u)}\right)^2$	$\frac{e^\alpha-1}{2} + \frac{e^\alpha}{pT(u)}\frac{\sigma_o^2}{K} + \frac{e^\alpha}{2p^2T^2(u)}\frac{\sigma_o^4}{K^2}$
	$\sigma^2 [\phi(u) - \phi(u+u_o)] =$	$f^2\left(\frac{\Delta u}{u_o}\right)\left(1+\frac{1}{pT(u)}\right)^2$	$f^2\left(\frac{\Delta u}{u_o}\right) +$ cross term $+ \frac{u_o e^\alpha}{2\,\Delta u\, p^2T^2(u)}\frac{\sigma_o^4}{K^2}$
		with: $\lim_{\Delta u \to o} f^2\left(\frac{\Delta u}{u_o}\right) = 2$	
Bispectral Analysis	$\sigma^2 [\phi(u) + \phi(v) - \phi(u+v)] =$	$g(u,v,N_o)\left(1+\frac{1}{pT(u)}\right)\left(1+\frac{1}{pT(v)}\right)\left(1+\frac{1}{pT(u+v)}\right)$	$g(u,v,N_o)\left(1+\frac{\sigma_o^2/K}{pT(u)}\right)\left(1+\frac{\sigma_o^2/K}{pT(v)}\right)\left(1+\frac{\sigma_o^2/K}{pT(u+v)}\right)$
		with: $g(u,v,N_o) = \frac{0.28\, N_o\, T(u)\, T(v)\, T(u+v)}{8\, A^2(u,v)}$	

Notations: ϕ = phase; (u,v) = spatial frequencies; Δu: sampling frequency interval; u_o = atmospheric cut-off frequency; (K_1, K_2, K) = average number of photons per interferogram; (p_1, p_2, p) = average number of photons per speckle; N_o = average number of speckles; $A(u,v)$ = overlap area of three pupil images normalized to unity at the origin; $\alpha = 6.88 \left(\frac{\Delta u}{u_o}\right)^{5/3}$.

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DISCUSSION

LAVILLE: ¿Cómo eliminas el efecto de la atmósfera en el método de la pupila?

CHELLI: El efecto de la atmósfera es de doformar las franjas, pero no afecta el contraste de las franjas que es proporcional al módulo del espectro espacial del objeto $||O(f)||$.

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