Rev. Mexicana Astron. Astrof., **16**, 25-36 (1988)

THE GALACTIC ORBITS AND TIDAL RADII OF SELECTED STAR CLUSTERS

Christine Allen and Marco A. Martos

Instituto de Astronomía
Universidad Nacional Autónoma de México

Received 1988 January 22

RESUMEN

Mediante la integración numérica directa de las ecuaciones del movimiento, se han obtenido las órbitas galácticas de 10 cúmulos globulares y 3 galácticos. El modelo para el potencial galáctico empleado proporciona una buena representación de la curva de rotación galáctica observada en la región comprendida entre 1 y 15 kpc, y una curva plana hasta 100 kpc. Con las distancias pericéntricas resultantes de las órbitas numéricas se han calculado los radios de marea de los cúmulos. Como en trabajos previos, se encuentra que los radios de marea teóricos son en muchos casos considerablemente menores que los observados. Sin embargo, si se usan en el cálculo del radio de marea teórico las distancias pericéntricas correspondientes al último paso perigaláctico, se obtiene una buena concordancia con los radios de marea observados. Se discute brevemente la sensibilidad de estos resultados frente a errores en los datos observacionales de los cúmulos y frente a incertidumbres en el modelo teórico, y se examinan sus implicaciones respecto a la evolución futura del sistema de cúmulos globulares de nuestra Galaxia.

ABSTRACT

The galactic orbits of 10 globular and 3 galactic clusters have been obtained by direct numerical integration of the equations of motion; the model galactic potential that was used gives a good representation of the observed rotation curve in the region 1 – 15 kpc and a flat rotation curve out to 100 kpc. With the pericentric distances resulting from the computed orbits, cluster tidal radii have been obtained. Like in previous works, it is found that the calculated tidal radii are in some cases significantly smaller than the observed ones; however, if the theoretical tidal radii are calculated using the pericentric distance of the last perigalactic passage, substantial agreement with the observational values is found (within the uncertainties of the latter). The sensitivity of the results to errors in the observed cluster data and to model uncertainties is briefly discussed, and some implications for the fate of the globular cluster population of the Galaxy are pointed out.

Key words: **CLUSTERS-GLOBULAR – CLUSTERS-OPEN – STARS-DYNAMICS**

I. INTRODUCTION

A knowledge of the motions of globular clusters is a basic ingredient necessary for the understanding of fundamental problems related to the formation of individual clusters, to the origin and persistence of the cluster system in the Galaxy, and to the structure of individual clusters. One of the most important parameters that characterize the structure of a cluster is the tidal radius, which is directly dependent on the size and shape of its galactic orbit, particularly on the distance of closest approach to the galactic center. On the other hand, the apocentric distances reached by the clusters in the course of their orbits are presumably indicative of the galactic regions where the clusters were formed, and therefore they are likely to be related to observable quantities such as metal abundances. The radial motions of clusters have been used to estimate apocentric distances, which in turn provide relevant information on the extent and mass of the galactic halo and on the total mass of the Galaxy (Frenk and White 1980; White and Frenk 1983; Lynden-Bell, Cannon, and Godwin 1983; Peterson 1985). Indeed, reliable data on the dimensions and eccentricities of orbits of globular clusters and other extreme population II objects could prove as important for the understanding of the structure of the galactic halo and for our ideas about the formation and early dynamical evolution of the Galaxy as analogous data for nearby, high-velocity stars have been

Lack of sufficient information on the motions of star clusters has led many workers to attempt to infer some of their orbital parameters indirectly, from their observed tidal radii. Star clusters, particularly globulars, have well defined limiting radii, which are believed to be a result of interactions with the galactic tidal field occurring mostly during the cluster's perigalactic passage. Observed tidal radii of star clusters have been extensively used to derive information about the perigalactic distances of the cluster orbits, or else as probes of the galactic mass distribution, in spite of the fact that neither the dynamical processes that set the tidal radius nor the galactic tidal field are well known. A further problem inherent to this indirect approach is the fact that the observational determination of tidal radii is beset with great difficulties that render uncertain many of the published values.

In view of these problems, it is hardly surprising that the use of observed tidal radii to derive information on cluster orbits has led to confusing results. For instance, Innanen, Harris, and Webbink (1983) have used the measured tidal radii of a sample of well-observed globular clusters to estimate their perigalactic distances and the eccentricities of their orbits; they utilized a simple, spherically symmetric, form for the galactic potential. They obtained shapes for the cluster orbits that were too nearly circular to be compatible with the observed velocity dispersion of the cluster system. The values that they found for the perigalactic distances R_P turned out to be *larger* than the present galactocentric distances R for nearly a third of the clusters examined; when they changed the potential used to that of a point mass, almost half of the clusters studied turned out to have perigalactic distances larger than their present distances to the galactic center. These results obviously have no physical meaning; Innanen *et al.* 1983 attribute them to accidental errors in the observed distances and tidal radii, or else in the assumed M/L of the clusters. Such errors are undoubtedly present in the data, but the discrepancies found are so large that they cast doubt upon the whole idea of using the observed tidal radii as a starting point to derive information about cluster orbits or about the galactic mass distribution. Innanen *et al.* conclude, in fact, that estimates of cluster perigalactica from tidal radii are largely meaningless.

In view of the difficulties resulting from the indirect approach, we decided to start from the opposite end, and to regard the galactic mass distribution –as evidenced by the rotation curve and the perpendicular force– as the better known function, which can then be represented by means of appropriate interpolation formulae. Following this idea, we have developed a model for the galactic mass distribution (Allen and Martos 1986) that is well suited for the direct numerical integration of galactic orbits, yet gives a realistic representation of the rotation curve of our Galaxy, and of the force perpendicular to the plane. The model allows rapid and accurate computations of galactic orbits, and thus enables us to determine directly the orbital parameters (particularly the perigalactic distance) of those few clusters for which sufficient observational data exist. The orbital data can then be used to compute the theoretical tidal radii, which can be compared with the observed values. Although the number of clusters for which this is possible is at present small, because only few of them have well-determined transverse motions, we will nevertheless proceed, in the hope that any information on cluster orbits will be useful in the understanding of the processes that set the tidal radius of a cluster, and may throw light on the problems of the dynamics and origin of the cluster system.

II. THE GALACTIC ORBITS

For the mass distribution of our Galaxy we will use the model recently developed by Allen and Martos (1986), which represents well the observed data most directly related to the quantities that shape a galactic orbit. The model consists of three components: a central mass point, an ellipsoidal disk and a massive spherical halo. The resulting rotation curve is flat from about 17 kpc out to 100 kpc, and agrees well with the observed values in the range 1 to 17 kpc; it is illustrated in Figure 1. The run with z of the perpendicular force, K_z, computed from the model also agrees with observationally determined data. With an assumed $R_\odot$ = 8 kpc and $v_\odot$ = 225 km s^{-1}, the derived values for the rotation constants are A = 15.9 km s^{-1} kpc^{-1} and B = −12.2 km s^{-1} kpc^{-1}, in good concordance with modern observational determinations. The model potential is continuous everywhere and has continuous

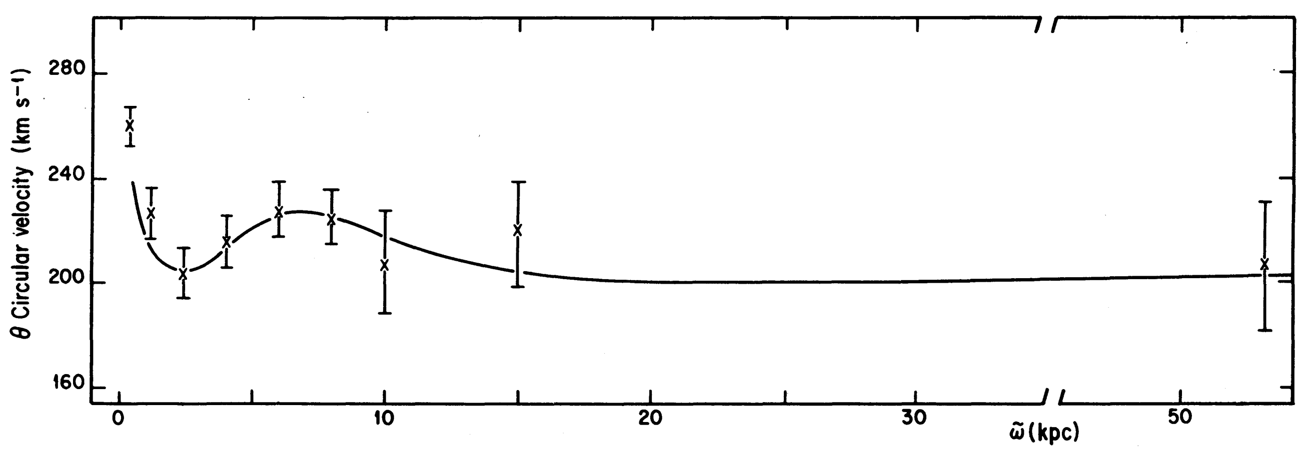

Fig. 1. The rotation curve resulting from the galactic mass model (Allen and Martos 1986).

TABLE 1

OBSERVED DATA FOR 13 STAR CLUSTERS

Cluster	α (1950) h	m	δ (1950) °	′	d kpc	$\mu_\alpha \cos\delta$ $'' \, y^{-1}$	μ_δ $'' \, y^{-1}$	v_r km s^{-1}	References
NGC 188	0	39.4	85	04	1.48	– 0.00385	– 0.0031	– 49	1,2
NGC 2420	7	35.5	21	41	1.9	– 0.0027	– 0.0011	115	4,16
M 67	8	47.7	12	00	0.8	– 0.0094	– 0.0070	33	5,2
NGC 4147	12	07.6	18	49	17.3	– 0.0027	0.0009	174	15
ω Cen	13	23.9	– 47	08	5.2	– 0.00224	– 0.0073	238	6,3
M 3	13	39.9	28	38	9.5	– 0.0053	0.0040	– 147	7,8
NGC 5466	14	03.2	28	46	14.5	– 0.0054	0.0006	115	10
M 5	15	16.0	2	16	7.6	0.0009	– 0.0127	49	9,17
M 13	16	39.9	36	33	6.3	– 0.0004	0.0076	– 241	11,3
M 92	17	15.6	43	12	8.1	– 0.0050	0.0018	– 118	13
M 22	18	33.3	–23	57	3.2	0.0087	– 0.0038	– 152	18
M 71	19	51.5	18	39	3.6	– 0.0022	– 0.0016	– 19	14
M 15	21	27.6	11	57	9.7	0.0022	– 0.0040	– 97	12

References to Table 1.

1. Upgren, Mesrobian and Kerridge 1972.
2. Eggen and Sandage 1969.
3. Arp 1965.
4. Cannon and Lloyd 1970.
5. Murray 1968.
6. Murray, Jones and Candy 1965.
7. Cudworth 1979*a*.
8. Gunn and Griffin 1979.
9. Cudworth 1979*b*.
10. Brosche *et al.*, 1983.
11. Cudworth and Monet 1979.
12. Cudworth 1976*a*.
13. Cudworth 1976*b*.
14. Cudworth 1985.
15. Brosche *et al.*, 1985.
16. Keenan and Innanen 1975.
17. Webbink 1985.
18. Cudworth 1986.

derivatives; its simple mathematical form makes it well suited to the direct numerical integration of galactic orbits.

A search of the literature yielded data on the absolute proper motions of 13 star clusters (10 globular and 3 galactic clusters). For these clusters, well determined radial velocities and distances are also available; these data, together with the cluster positions, are sufficient to compute their present galactocentric positions and velocities, which serve as "initial conditions" for the direct numerical integration of their orbits. It is, of course, regrettable that the observational data on absolute proper motions of clusters are so scarce; in this respect, the method recently employed by Brosche and collaborators (Brosche, Geffert, and Nincovic 1983; Brosche *et al.* 1985), in which the absolute proper motions of clusters are determined directly with reference to external galaxies, looks very promising; it is to be hoped that in the near future more and better data on cluster transverse motions will become available.

Table 1 gives the observational data for the ten globular clusters and three galactic clusters that comprise our sample, along with references to the sources for the absolute proper motions, distances, and radial velocities. Table 2 lists the calculated galactocentric positions and velocities, which are taken as "initial conditions" for the numerical integration of the orbits. The standard solar motion of 19.5 km s^{-1} in direction $\ell = 56°$, $b = +23°$ was assumed (Delhaye 1965).

For the numerical integration of the cluster orbits we used a 7th order Runge-Kutta-Fehlberg algorithm with automatic control of the step length (Fehlberg 1968). The errors in the total energy and in the z-component of the angular momentum accumulated at the end of the orbit computations were, typically, $\Delta E/E = 10^{-6}$ and $\Delta h/h = 10^{-7}$. The orbits were run backwards in time for 1.6×10^{10} years, a generous upper bound to the age of the globular clusters. Usually, the computation of an orbit took about 10 minutes on a Prime 550 computer.

Figures 2 to 17 show the orbits of the clusters as projected on the meridional plane, as well as two represen-

TABLE 2

COMPUTED INITIAL CONDITIONS FOR 13 STAR CLUSTERS

Cluster	$\tilde{\omega}$ kpc	z kpc	Π km s^{-1}	Z km s^{-1}	Θ km s^{-1}
NGC 188	8.82	0.57	– 23.25	– 30.82	225.56
NGC 2420	9.72	0.64	89.55	20.95	209.22
M 67	8.57	0.42	18.01	– 12.68	200.99
NGC 4147	9.84	16.87	152.63	153.35	237.07
ω Cen	6.22	1.35	– 77.21	– 95.78	– 95.75
M 3	6.74	9.32	340.98	– 91.83	152.66
NGC 5466	5.67	13.91	252.63	222.00	– 80.02
M 5	2.83	5.54	– 301.23	– 171.25	– 39.25
M 13	6.89	4.12	337.76	– 121.38	– 27.90
M 92	8.30	4.63	119.80	103.17	– 41.32
M 22	4.91	–0.42	181.31	–115.33	199.90
M 71	6.74	–0.29	57.54	27.27	196.79
M 15	8.95	–4.45	39.31	– 121.26	28.21

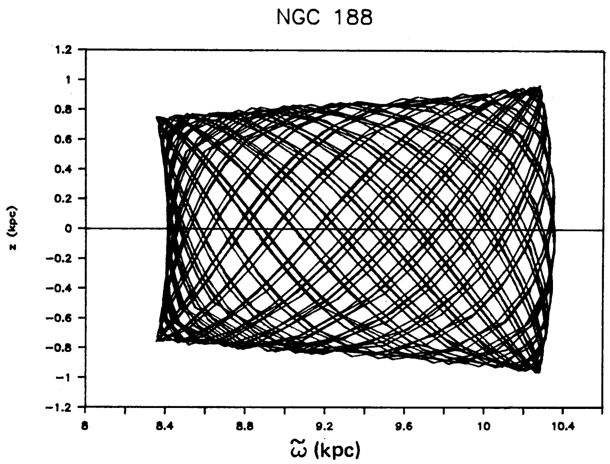

Fig. 2. The meridional orbit of NGC 188.

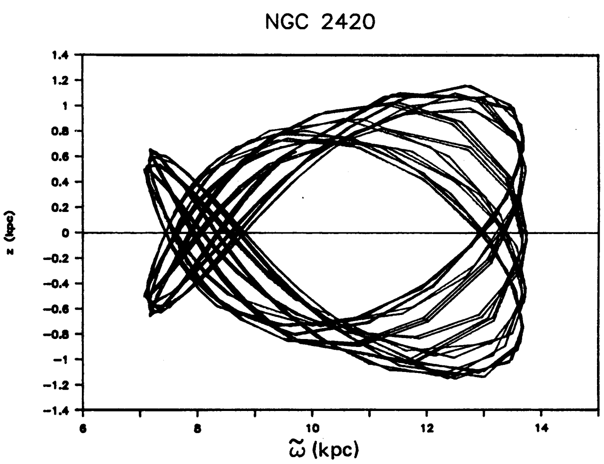

Fig. 3. The meridional orbit of NGC 2420.

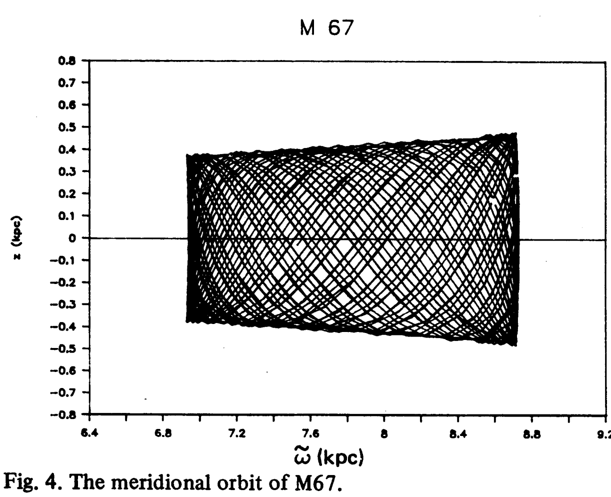

Fig. 4. The meridional orbit of M67.

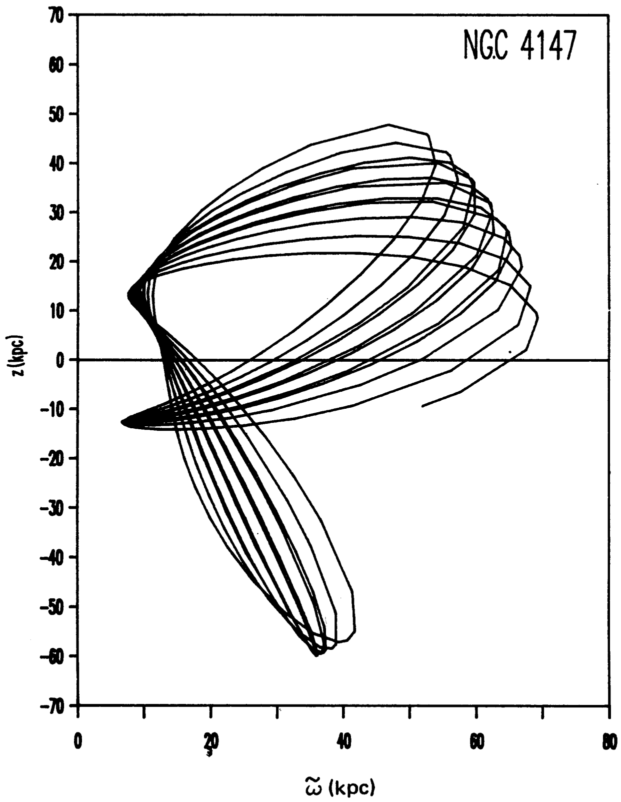

Fig. 5. The meridional orbit of NGC 4147.

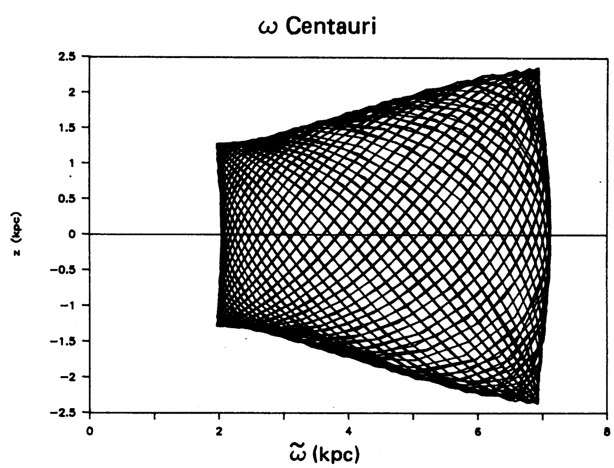

Fig. 6. The meridional orbit of ω Centauri.

tative surfaces of section. An inspection of the meridional orbits reveals at once many of the orbital characteristics of these systems. We can see, for instance, that the orbit of M71 resembles that of the old galactic cluster M67; they are both box-orbits of relatively low "eccentricity", qualitatively similar to the orbits of disk population objects. Among the globular clusters computed, the least eccentric orbit corresponds to M71; this clus-

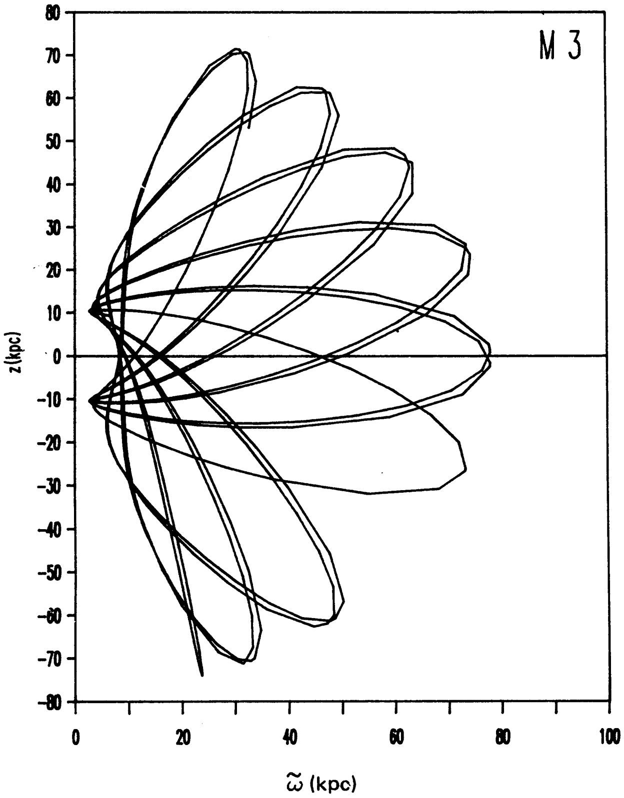

Fig. 7. The meridional orbit of M3.

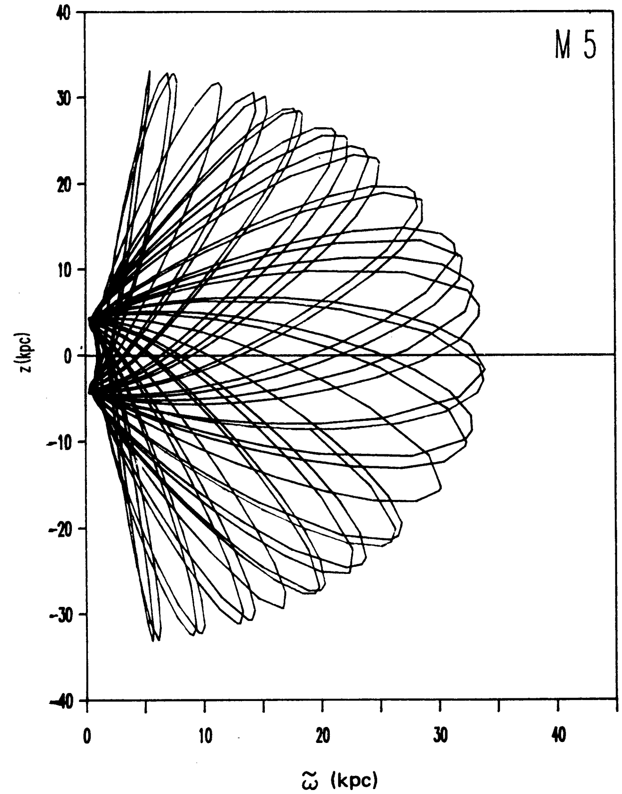

Fig. 9. The meridional orbit of M5.

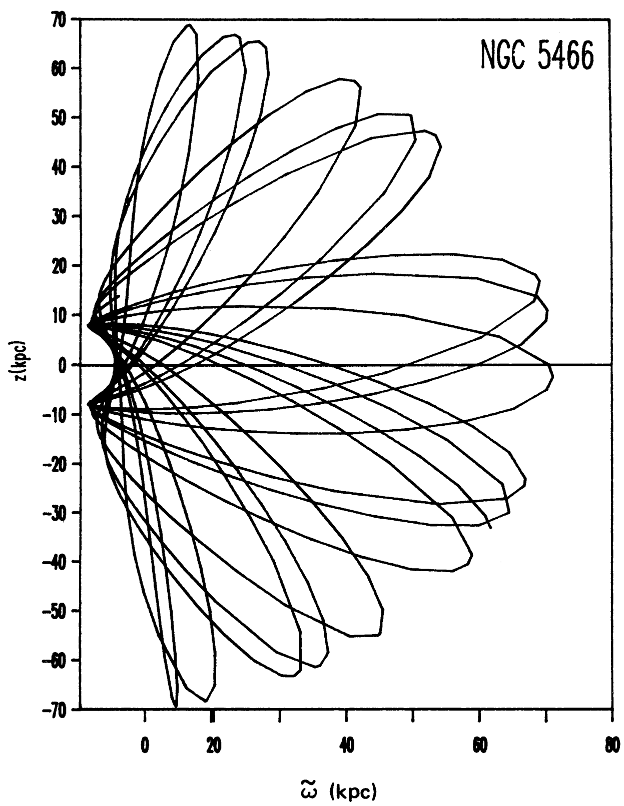

Fig. 8. The meridional orbit of NGC 5466.

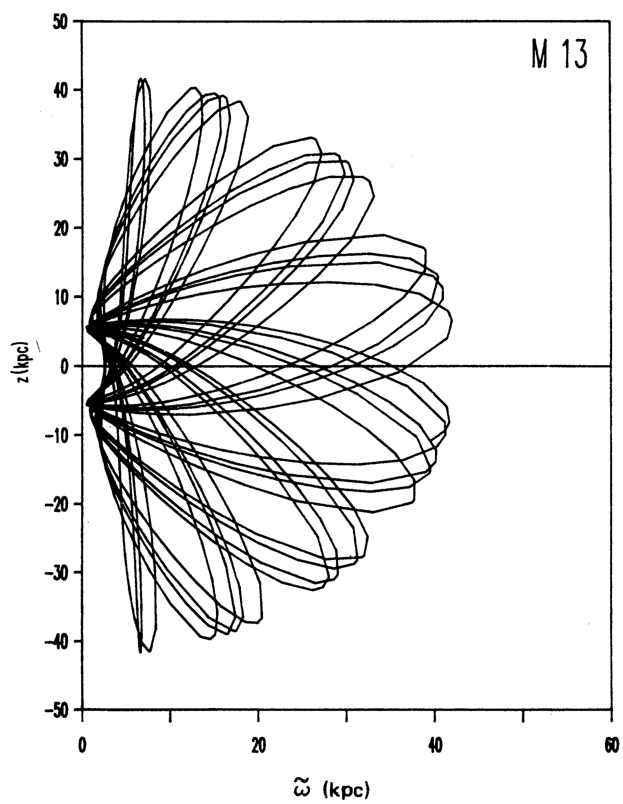

Fig. 10. The meridional orbit of M13.

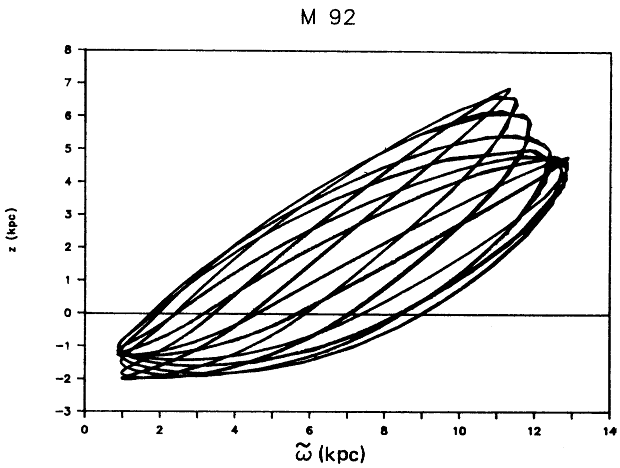

Fig. 11. The meridional orbit of M92.

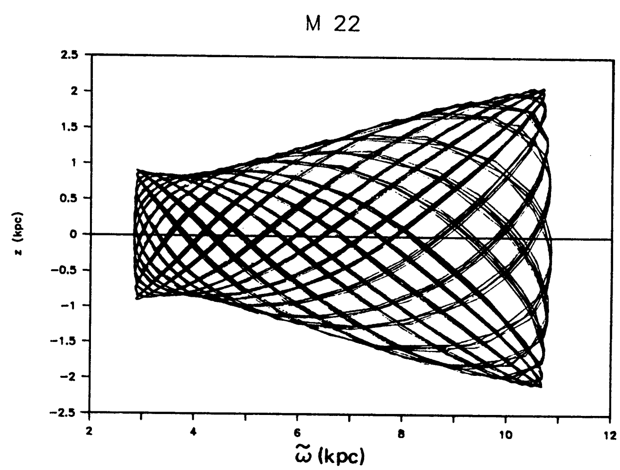

Fig. 12. The meridional orbit of M22.

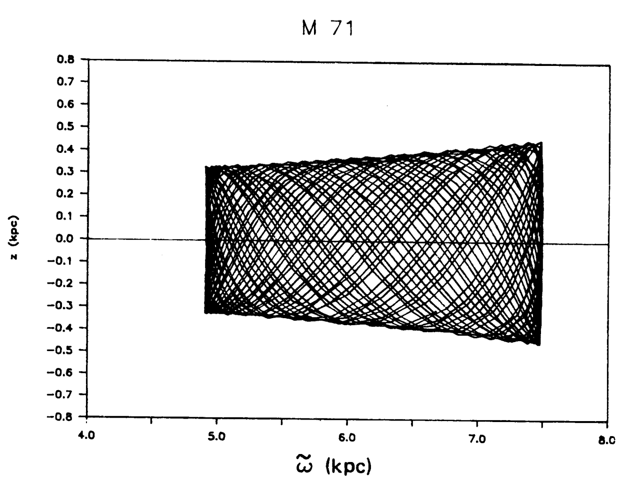

Fig. 13. The meridional orbit of M71.

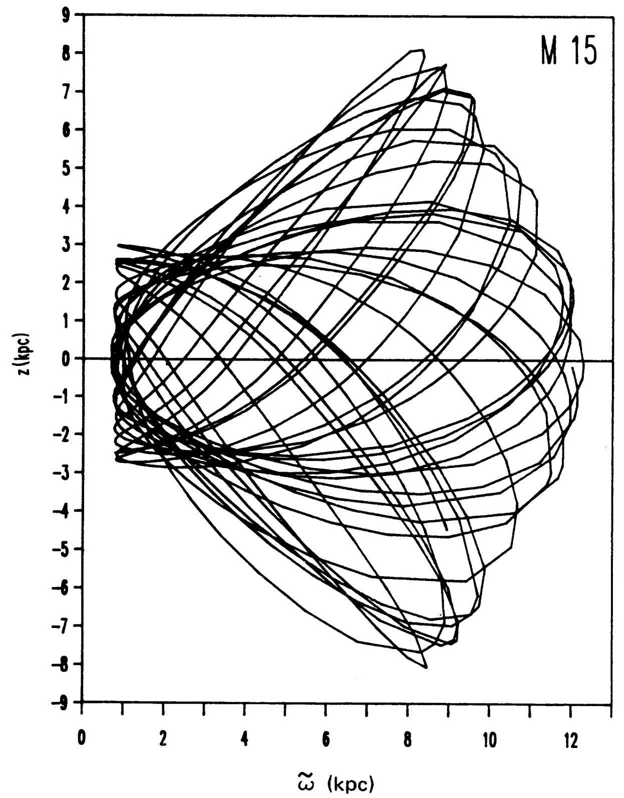

Fig. 14. The meridional orbit of M15.

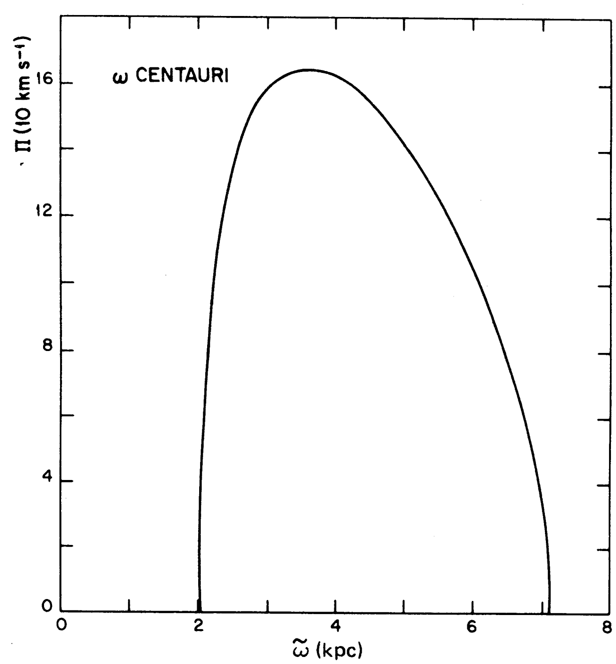

Fig. 15. The surface of section of ω Centauri.

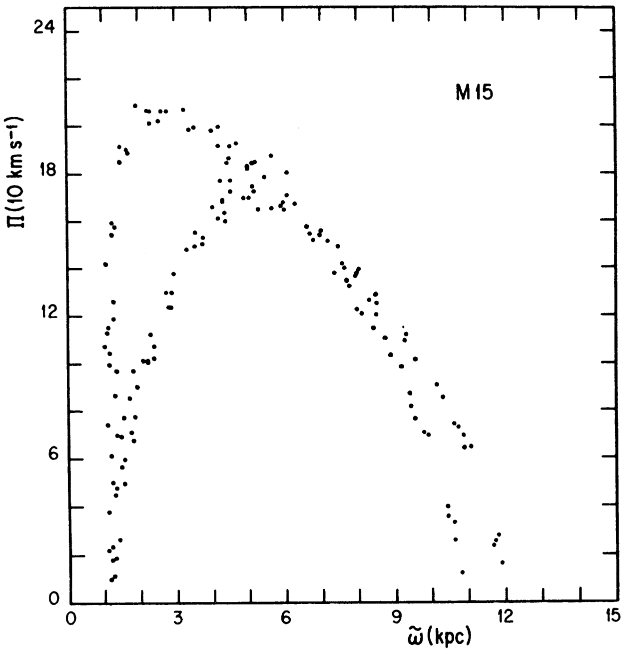

Fig. 16. The surface of section of M15.

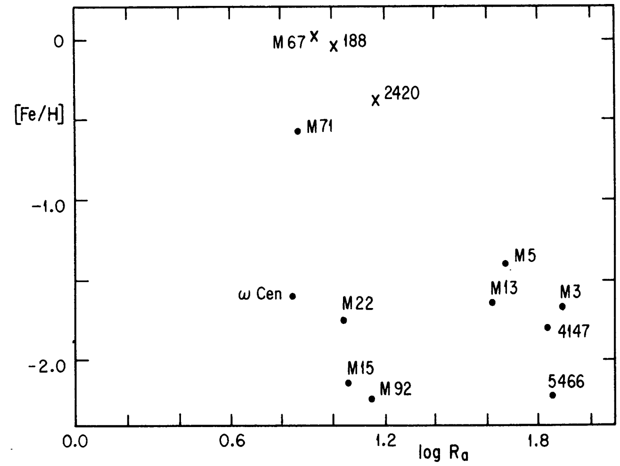

Fig. 17. The metal abundances of the globular clusters (dots) and galactic clusters (crosses) plotted against their apocentric distances.

ter reaches also the smallest maximum z-distance (in absolute value), very similar to the maximum z-distance reached by M67; not surprisingly, M71 is also the most metal-rich cluster among the globulars of our sample. In contrast, the orbits of M3, M5, M13, M15, and NGC 5466 are more typical of halo objects. The very old, extremely metal-poor, globular cluster M92 has a peculiar orbit, which remains asymmetric with respect to the galactic plane over the whole 1.6×10^{10} years; this orbit is qualitatively similar to that of the extreme-velocity wide binary LDS 519 (Allen, Poveda, and Martos 1986).

An examination of the surfaces of section of the cluster orbits indicates that, with the exception of M15 (shown in Figure 16), they are similar to that shown in Figure 15 –a typical surface of section for box orbits; thus, they correspond to quasi-periodic motion. The points on the surface of section of M15 cannot be joined by a smooth line; this is an indication of the semi-ergodic character of this orbit.

The only published galactic orbits for star clusters that we have found in the literature are those of M67, NGC 188 and ω Cen (Keenan, Innanen, and House 1973) and that of NGC 2420 (Keenan and Inannen 1975), all in Schmidt-type galactic mass models. Although the qualitative behaviour of these orbits is similar to the ones we have computed, there are important

TABLE 3

ORBITAL PARAMETERS FOR 13 STAR CLUSTERS

Cluster	E 100 km s^{-2}	h 10 kpc km s^{-1}	"e"	$\tilde{\omega}_{min}$ kpc	$\tilde{\omega}_{max}$ kpc	z_{min} kpc	z_{max} kpc	R_p (min) kpc	R_a (max) kpc	P_r 10^8 y
NGC 188	– 1172.71	198.85	0.11	8.35	10.36	– 0.97	0.97	8.39	10.36	2.06
NGC 2420	– 1127.66	203.30	0.32	7.07	13.75	– 1.16	1.16	7.09	13.75	2.32
M 67	– 1246.22	172.20	0.11	6.93	8.73	– 0.48	0.48	6.94	8.73	1.61
NGC 4147	– 546.81	233.26	0.69	6.59	69.28	– 59.88	47.82	12.71	69.85	9.30
ω Cen	– 1442.32	– 59.47	0.57	1.96	7.12	– 2.36	2.36	2.03	7.32	1.02
M 3	– 513.63	102.86	0.81	2.67	78.01	– 74.04	71.47	8.32	78.01	9.92
NGC 5466	– 555.77	– 45.39	0.86	1.14	71.17	– 69.38	68.89	5.25	71.21	8.94
M 5	– 854.64	– 11.11	0.94	0.29	33.92	– 33.12	33.14	1.02	33.96	4.37
M 13	– 764.40	– 19.21	0.89	0.51	42.00	– 41.66	41.64	2.59	42.53	5.45
M 92	– 1217.18	– 34.31	0.83	0.85	12.88	– 2.09	6.91	1.27	13.74	1.75
M 22	– 1286.52	98.05	0.59	2.83	10.84	– 2.09	2.09	2.83	10.88	1.50
M 71	– 1358.21	132.58	0.21	4.91	7.49	– 0.46	0.46	4.91	7.49	1.18
M 15	– 1250.98	25.24	0.90	0.58	12.42	– 8.29	8.22	0.67	12.88	1.59

quantitative differences that arise from the different mass model and galactic parameters we have used. By far the greatest differences are due to the inclusion of a massive halo in our galactic mass model, which largely determines the maximum galactocentric distance that a cluster will attain. In fact, as an inspection of Table 2 reveals, a number of clusters have total galactocentric velocities exceeding 315 km s^{-1}, the escape velocity at the Sun's position conventionally assumed for a Schmidt-type galactic mass model. In such a model, the clusters M3, NGC 5466, M5, M13, and probably also NGC 4147, would escape from the Galaxy.

Table 3 lists the main characteristics of the cluster orbits. We note that the apocentric distances reached by M3, NGC 4147, and NGC 5466 are remarkably large, despite the inclusion of a massive halo in the galactic mass model. With a less massive halo, these clusters would either reach even larger apogalactic distances, or else escape altogether; they also attain the largest (positive or negative) z-distances. We also note that although not all globular cluster orbits are of the extremely eccentric, "plunging" type, there is certainly no lack of high "eccentricities" among them, a fact clearly shown by the meridional trajectories. Indeed, a straight mean of the eccentricity parameters of 10 globular clusters gives the value $\langle e \rangle = 0.74$. It will be noticed that in some cases $|z_{min}| \neq z_{max}$ even when the orbit is clearly of box-type. This is due to the fact that not enough time has elapsed for the box to be completely delineated.

Although our sample of 10 globular and 3 galactic clusters is clearly too small to permit statistical inferences, it is illustrative to compare our results with those obtained by Innanen, Harris, and Webbink (1983). This comparison is contained in Table 4, where we list (columns 2 and 3) the mean values of R_P/R, the ratio of perigalactic distance to present galactocentric distance, computed by these authors for two assumptions about the mass distribution in the Galaxy (a point mass and a power law); also in Table 4 we compare the mean values of R_P/R with the values we obtain for R_P/R at three interesting points of the computed orbits: at the times of last and next passage through perigalacticon (columns 4 and 5), and at the time of minimum perigalactic distance (column 6). It is readily seen that the values computed by Innanen *et al.* 1983, tend to be larger than the ones we find. This, of course, is just another manifestation of the problem already remarked upon by these authors: the orbits they infer tend to be too nearly circular. In contrast, our numerically integrated galactic orbits show that all globular clusters except ω Cen and M71 have eccentricity parameters that exceed 0.67. This value corresponds approximately to $R_P/R = 0.33$, which is the minimum R_P/R found by Innanen *et al.* (1983) in their study. Thus, the great majority of our computed orbits for globulars have "eccentricities" larger than the maximum value found by these authors.

Another way of looking at the same problem is to take the average value of R_P/R for all our globular cluster orbits, and to compare it to that found by Innanen *et al.* (1983) for their sample of clusters, as well as to the average value computed by these authors for an isotropic distribution of cluster velocities with their two variants of the mass distribution in the Galaxy. We find $\langle \log (R_P/R) \rangle = -0.296$ for the last R_P, $\langle \log (R_P/R) \rangle = -0.367$ for the next R_P and $\langle \log (R_P/R) \rangle = -0.500$ for the minimum R_P ever reached by the cluster. These values are not too different from those theoretically expected for an isotropic velocity distribution (Innanen *et al.* 1983), namely $\langle \log (R_P/R) \rangle = -0.349$ (for a power law mass distribution) or $\langle \log (R_P/R) \rangle = -0.529$ (for a point mass galaxy); we thus conclude that the characteristics

TABLE 4

PERIGALACTIC DISTANCES

	$\langle R_p/R \rangle$ (IHW)		R_p/R (Present Work)		
Cluster	Power Law	Point Mass	Last	Next	Minimum
NGC 188	–	–	0.950	0.950	0.949
NGC 2420	–	–	0.728	0.728	0.728
M 67	–	–	0.809	0.809	0.809
NGC 4147	0.64	0.59	0.671	0.727	0.651
ω Cen	0.84	1.04	0.354	0.343	0.319
M 3	–	–	0.929	0.760	0.724
NGC 5466	–	–	0.396	0.538	0.349
M 5	1.06	1.29	0.650	0.453	0.163
M 13	0.82	0.94	0.696	0.630	0.323
M 92	0.58	0.68	0.211	0.232	0.134
M 22	0.61	0.85	0.580	0.559	0.544
M 71	0.53	0.69	0.729	0.729	0.728
M 15	0.66	0.75	0.308	0.116	0.067

of our cluster orbits are not incompatible with their being a random sample from an isotropic velocity distribution of the cluster system. However, our values for ⟨log (R_p/R)⟩ differ significantly from the values Innanen *et al.* deduce from the tidal radii of their sample of globular clusters, namely ⟨log (R_p/R)⟩ = –0.103 (for a power law mass distribution) or ⟨log (R_p/R)⟩ = –0.031 (for a point mass galaxy). We can only concur with Innanen *et al.* in their conclusion that observed tidal radii yield poor estimates of cluster perigalactica.

It is also interesting to ask the question of whether the computed orbits display any correlation with the cluster metal abundances, specifically, whether the radial abundance gradient suspected to be present in the globular cluster system (Zinn 1985) is more readily obvious when plotted as a function of, say, apocentric distance than it is when present-day galactocentric distances are taken. On theoretical grounds, we would expect cluster formation to take place preferentially either near apogalacticon, because at that location the gas was presumably quietest, or near perigalacticon, where the density was highest. Therefore, if abundance gradients were present in the Galaxy at the time of cluster formation, we might expect to see them more readily reflected in the cluster peri- or apogalactica, rather than in their present galactocentric distances, which is the way the data are usually displayed. Figures 17 to 19 are plots of the metal abundances [Fe/H] from Zinn (1985) as a function of eccentricity, apocentric, and pericentric distance. Admittedly, our sample is very small. Nonetheless, we can say that a correlation of metal abundance with pericentric distance [as claimed— with a sign opposite to the expected one— by Seitzer and Freeman (1982)] is probably excluded by Figure 18, which looks entirely like a scatter diagram; we can also say that a radial abundance gradient, if present in the sample, does not emerge more clearly in the plot as a function of apocentric distances (Figure 17) than it does when present-day galactocentric distances are used. However, a weak correlation of [Fe/H] with eccentricity does seem to be indicated by Figure 19. It should be noted that with the exception of M71, all our clusters belong to the metal-poor group ([Fe/H] $<$ –0.8). Note also that of the three clusters with the largest apogalactic and z-distances, only NGC 5466 would classify as extremely metal-poor.

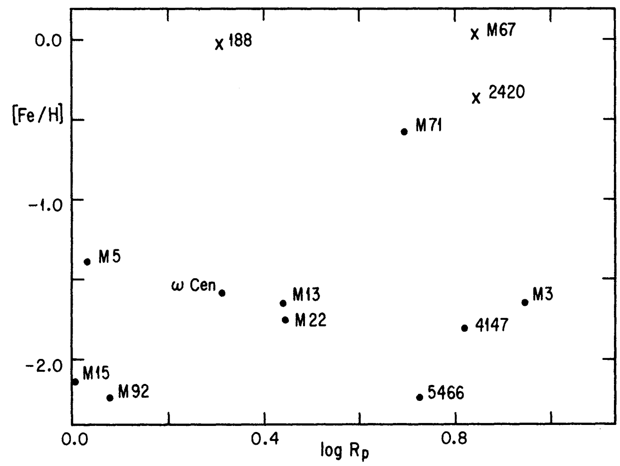

Fig. 18. The metal abundances of the globular clusters (dots) and galactic clusters (crosses) plotted against their pericentric distances.

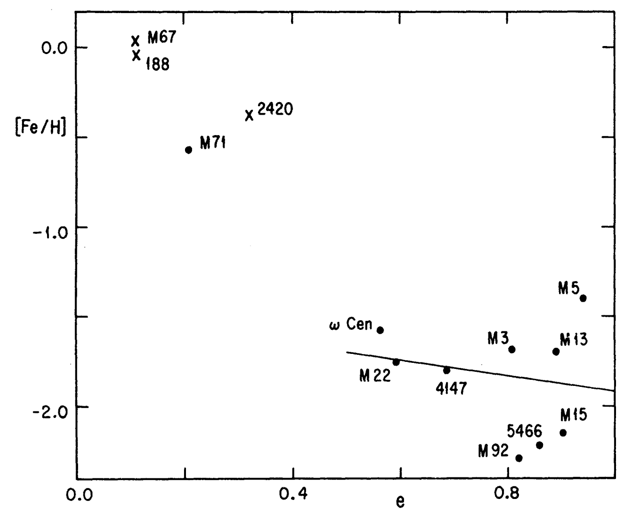

Fig. 19. The metal abundances of the globular clusters (dots) and galactic clusters (crosses) plotted against their orbital eccentricities. The line represents a least-squares fit to the metal-poor globular clusters (i.e. not considering M71).

III. THE TIDAL RADII

Table 5 displays the theoretical tidal radii and compares them with the observed values. Successive columns contain the observed limiting radii and the theoretical tidal radii computed by means of King's (1962) formula, using the perigalactic distances for the times of last and next perigalactic passage, as well as the minimum perigalactic distance ever reached by the cluster in the course of its galactic orbit; for M_G, the "effective" galactic mass, we used the equivalent point mass that produces the actual force felt by the cluster at the perigalactic point, as computed from the galactic mass model. An M/L ratio of 1.6 was assumed throughout (Illingworth 1976).

As discussed among others by Innanen (1979) and Keenan (1981*a,b*), it is not clear whether the classical King formula gives a good estimate of the actual limiting radius of a cluster. On theoretical grounds, it appears that the classical tidal radius is an overestimate, and should be reduced by a factor of 0.693 (Innanen, Harris, and Webbink 1983); but this tidal radius seems to correspond mainly to the direct orbits of the stars within the cluster; on the other hand, numerical work (Keenan 1981*a,b*; Innanen 1979) has shown that many stars situated at distances well beyond this theoretical tidal radius can remain bound to the cluster. In fact, the cluster is able to retain retrograde stars at distances of up to, and even over, twice the tidal radius for direct orbits for very long times (up to 50 cluster orbital periods). Now, the tidal radius for retrograde orbits is about twice that for direct orbits (Innanen 1979),which in turn, is just 0.693 the King radius. For this reason, we also show in Table 5 the tidal radii for direct and retrograde orbits of stars belonging to the cluster. If, as is customarily assumed, the tidal radius is set mostly at the times of perigalactic passage, then the presently observed limiting radii are more representative of the last perigalactic distance than of other perigalactica reached by the cluster, since the characteristic times of cluster core relaxation and presumably also envelope repopulation are of the same order of magnitude as their galactic periods of revolution.

It seems likely that the tidal radii observed at any given time are a result of both the complicated action upon the cluster of the galactic tidal field, and of the cluster's subsequent relaxation and dynamical evolution. Quite apart from the considerable observational uncertainties in the determination of cluster limiting radii (King *et al.* 1968) we have on theoretical grounds alone an uncertainty of at least a factor of two. In fact, all one can say at the present time is that, provided no significant repopulation of the envelope has taken place since the time of last perigalactic passage, the theoretical tidal radius of a cluster should lie between the values corresponding to the direct and the retrograde orbits at the time of last perigalacticon. For these reasons we have given in Table 5 the theoretical tidal radii for direct and retrograde orbits along with the King tidal radius. It is readily seen that nearly all observationally determined limiting radii are nicely bracketed by the theoretical values corresponding to direct and retrograde orbits. The exceptions are M71, which appears to be a factor of 2 smaller than calculated, and NGC 5466, which seems slightly larger; however, the observational limiting radius of this cluster is particularly uncertain. There is no tendency for the observed tidal radii to be systematically larger than the theoretical values, as has been the case in previous work. Consequently, there is no need to assume extremely high values of M/L to bring about an agreement between theory and observations, as Innanen, Harris, and Webbink (1983) suggest.

To examine the question of the sensitivity of the computed orbits, and particularly of the theoretical tidal radii, to errors stemming from observational uncertainties in the cluster distances or motions we performed the following experiment: the clusters M13, M22, and M71 were selected as having representative orbit types, and their galactic orbits were computed for two extreme cases; in the first case, we added to the distances and motions the quoted standard observational error; in the second case, we subtracted it. The results of our experiment are shown in Table 6. We see that although the numerical values for the orbital parameters differ somewhat, the computed tidal radii are very similar, well within the observational errors of the limiting radii. Moreover, the morphology of the orbits does not ap-

TABLE 5

OBSERVED AND COMPUTED TIDAL RADII

Cluster	r_t(observed) pc	m_c $M_\odot$	r_t(last) pc	r_t (next) pc	r_t (min) pc	r_t (last)$_d$ pc	r_t (last)$_r$ pc
NGC 188	11.6	9.00×10^2	12.1	12.1	12.1	8.4	16.8
NGC 2420	–	1.00×10^3	13.2	10.7	10.7	9.2	18.3
M 67	14.0	1.35×10^3	11.9	11.9	11.9	8.3	16.5
NGC 4147	40.0	3.40×10^4	56.6	60.9	53.6	39.2	78.5
ω Cen	66.0	3.25×10^6	73.1	70.5	72.0	50.7	101.4
M 3	107.5	6.08×10^5	131.8	108.7	99.5	91.4	182.8
NGC 5466	66.0	8.71×10^4	43.7	57.1	36.2	30.3	60.6
M 5	63.7	4.45×10^5	59.9	44.6	22.0	41.5	83.1
M 13	49.3	3.87×10^5	72.7	47.3	38.8	50.4	100.8
M 92	39.1	2.31×10^5	28.6	30.9	19.9	19.8	39.7
M 22	30.8	3.41×10^5	41.0	39.6	42.3	28.4	56.9
M 71	14.5	2.81×10^4	25.2	25.2	26.6	17.5	34.9
M 15	59.0	6.55×10^5	56.2	27.6	17.0	39.0	77.9

Notes to Table 5.

1. Globular cluster masses computed from the total visible light from the tabulation by Webbink (1985). An M/L ratio of 1.6 is assumed (Illingworth 1976). For ω Cen the mass calculated by Poveda and Allen (1975) was adopted.
2. Observed tidal radii from Webbink (1985) using the distances from Table 1. For NGC 4147, and NGC 5466 the values adopted by Brosche *et al.* (1983, 1985) are tabulated. For NGC 188 and M 67 the values found by Keennan *et al.* (1973) are used.

TABLE 6

TIDAL RADII COMPUTED WITH OBSERVATIONAL ERRORS IN CLUSTER DISTANCES AND MOTIONS.

Cluster	r_t (last) pc	r_t (next) pc	r_t (min) pc
M 13	72.7	47.3	38.8
M 13 + Δ	78.2	52.3	38.2
M 13 − Δ	78.2	50.0	41.1
M 22	41.0	39.6	42.3
M 22 + Δ	45.3	43.3	43.1
M 22 − Δ	41.5	40.9	40.7
M 71	25.2	25.2	26.6
M 71 + Δ	28.4	28.6	29.7
M 71 − Δ	26.5	26.5	27.4

preciably change. We conclude that our numerical results do not appear to be very sensitive to errors in the observed cluster parameters, at least not to errors of a size comparable to the standard errors given by the observers. On the other hand, we have already remarked that cluster perigalactica are not too sensitive to differences in the details of the galactic mass model used; in any case, in the regions typically visited by the clusters near perigalacticon the mass distribution is fairly well determined, and well represented by many models. The previously published orbits of ω Cen, NGC 2420, M67, and NGC 188 have perigalactic distances that differ by less than 10% from the values we obtain (after scaling to compensate for different galactic parameters used). Furthermore we have computed orbits for about half of our clusters in a Schmidt-type potential (Allen and Moreno 1981) and found perigalactic distances very similar to the ones presented here. For these reasons, we do not believe that our results will be significantly altered by choosing a different mass model for the Galaxy.

We still have to discuss the reasons why previous work has given discrepant values for the theoretical tidal radii as compared with the observed limiting radii, in the sense of the theoretical values being generally much too small. We believe the answer lies in the complicated nature of the cluster orbits. In the absence of the information provided by direct numerical integration, all one can do is to compute a perigalactic distance from the orbital angular momentum of the cluster, plus some approximation to the galactic potential. The perigalactic distances estimated in this way correspond most closely to the minimum perigalactic distances ever reached by the clusters, which in some cases differ significantly from the last perigalactic distances, as Table 5 shows. In fact, if we use the minimum perigalactic distances of our computations to estimate cluster limiting radii, the values we get are in most cases considerably smaller than the observed values. Especially striking are the cases of M5 and M15 (see Table 5), whose envelopes appear to have been severely cut back during very close perigalactic passages that occurred shortly after the formation of M15, and about 3×10^9 years ago in the case of M5. Both clusters have probably lost a significant fraction of their original stars; the times elapsed since their envelopes were cut back seem long enough for significant repopulation of the envelopes to have occurred. In fact, M15 seems to be heading towards another episode of severe tidal stripping of its envelope, which will occur at the time of its next perigalactic passage, some 1.1×10^8 years from now. A similar, though perhaps less drastic case is that of M92. It would be interesting –but clearly beyond the scope of this study– to assess the structural changes brought about in a cluster by one or several episodes of substantial tidal stripping of its envelope.

IV. SUMMARY AND CONCLUSIONS

We have presented the galactic orbits of ten globular and three galactic clusters. The orbits were obtained by direct numerical integration of the equations of motion, assuming a realistic mass model for the Galaxy, and using the observed values for the radial and transverse velocities of the clusters as well as their distances, to compute "initial conditions". The theoretical tidal radii calculated with the perigalactic distances obtained from the numerically determined orbits agree well with the observed values, provided they are assumed to be set essentially at the time of last perigalactic passage. The theoretical values are shown to be not very sensitive either to uncertainties in the galactic mass model or to errors in the computed galactic orbits arising from uncertainties in the observed distances or motions.

This study has clear consequences pertaining to the initial numbers of stars belonging to clusters, as well as to the total cluster population in the Galaxy. It seems most likely that the number of globular clusters that were present in the young Galaxy must have been many times larger than the number we now observe: because of the severe disrupting effect of the tidal force upon eccentric orbits only clusters with very moderate eccentricities and/or very high central concentration can survive. Also, many clusters must have lost a significant fraction of their member stars by the same effect of tidal stripping of their envelopes. Our results support the suggestion (Fall and Rees 1977) that the galactic halo population consists largely of the remains of disrupted clusters, but in our view they were destroyed not so much by internal dynamical evolution but rather by the effect of the galactic tidal force. Perhaps galaxies like M87, that manage to retain rich systems of globular clusters do so because of the extreme central concentration of the original clusters, or because orbits of large angular momentum were abundant.

The authors are grateful to A. Poveda and E. Moreno for useful discussions.

REFERENCES

Allen, C. and Martos, M.A. 1986, *Rev. Mexicana Astron. Astrof.*, **13**, 137.

Allen, C. and Moreno, E. 1981, unpublished.

Allen, C., Poveda, A., and Martos, M.A. 1986, *Rev. Mexicana Astron. Astrof.*, **14**, 213.

Arp, H.C. 1965, in *Galactic Structure*, (eds.) A. Blaauw and M. Schmidt (Chicago: University of Chicago Press), p. 401.

Brosche, P., Geffert, M., and Nincovic, S. 1983, *Publ. Astr. Inst. Czechoslovakia. Ac. Sci.*, **56**, 145.

Brosche, P., Geffert, M., Klemola, A.R., and Nincovic, S. 1985, *A.J.*, **90**, 2033.

Cannon, R.D. and Lloyd, C. 1970, *M.N.R.A.S.*, **150**, 279.

Cudworth, K.M. 1976*a*, *A.J.*, **81**, 519.

Cudworth, K.M. 1976*b*, *A.J.*, **81**, 975.

Cudworth, K.M. 1979*a*, *A.J.*, **84**, 1312.

Cudworth, K.M. 1979*b*, *A.J.*, **84**, 1866.

Cudworth, K.M. 1985, *A.J.*, **90**, 65.

Cudworth, K.M. 1986, A.J.,

Cudworth, K.M. 1986, *A.J.*, **92**, 348.

Cudworth, K.M. and Monet, D.G. 1979, *A.J.*, **84**, 774.

Delhaye, J. 1965, in *Galactic Structure*, (eds.) A. Blaauw and M. Schmidt (Chicago: University of Chicago Press), p. 61.

Eggen, O.J. and Sandage, A.R. 1969, *Ap. J.*, **158**, 669.

Fall, S.M. and Rees, M.J. 1977, *M.N.R.A.S.*, **181**, 37p.

Fehlberg, E. 1968, *NASA TR R-287*.

Frenk, C.S. and White, S.D. 1980, *M.N.R.A.S.*, **193**, 295.

Gunn, J.E. and Griffin, R.F. 1979, *A.J.*, **84**, 752.

Innanen, K.A. 1979, *A.J.*, **84**, 960.

Innanen, K.A., Harris, W., and Webbink, R.F. 1983, *A.J.*, **88**, 338.

Illingworth, G. 1976, *Ap. J.*, **204**, 73.

Keenan, D.W. 1981*a*, *Astr. and Ap.*, **95**, 334.

Keenan, D.W. 1981*b*, *Astr. and Ap.*, **95**, 340.

Keenan, D.W., Innanen, K.A., and House, F.C. 1973, *A.J.*, **78**, 173.

Keenan, D.W. and Inannen, K.A. 1975, *A.J.*, **80**, 290.

King, I.R. 1962, *A.J.*, **67**, 471.

King, I.R., Hedemann, E., Hodge, S.M., and White, R.E. 1968, *A.J.*, **73**, 456.

Lynden-Bell, D., Cannon, R.D., and Godwin, P.J. 1983, *M.N.R.A.S.*, **193**, 295.

Murray, C.A. 1968, *Royal Obs. Bull., No. 141.*

Murray, C.A., Jones, D.H.P., and Candy, M.P. 1965, *Royal Obs. Bull., No. 100.*

Peterson, R.C. 1985, *Ap. J.*, **297**, 309.

Poveda, A. and Allen, C. 1975, *Ap.J.*, **197**, 155.

Seitzer, P. and Freeman, K.C. 1982, in *Astrophysical Parameters for Globular Clusters*, (eds.) A.G.D. Philip and D.S. Hayes (New York: Davis), p. 185.

Upgren, A.R., Mesrobian, W.S., and Kerridge, S.J. 1972, *A.J.*, **77**, 74.

Webbink, R.F. 1985, in *Dynamics of Star Clusters*, (eds.) J. Goodman and P. Hut (Dordrecht: D. Reidel), p. 541.

White, S.D. and Frenk, C.S. 1983, in *Kinematics, Dynamics and Structure of the Milky Way*, (ed.) W.L.H. Shuter (Dordrecht: D. Reidel), p. 343.

Zinn, R. 1985, *Ap. J.*, **293**, 424.

Christine Allen and Marco A. Martos: Instituto de Astronomía, UNAM, Apartado Postal 70-264. 04510 México, D.F., México.