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AN ANALYTICAL MODEL FOR THE EVOLUTION OF PRIMARY ELEMENTS IN THE GALAXY

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RESUMEN. Se desarrolla un modelo analítico de evolución química autoconsistente para el halo galáctico y el disco en la vecindad solar, usando el modelo simple de Hartwick (1976), modificado para el halo, y el formalismo de influjo de Clayton para el disco (incluyendo un enriquecimiento modesto inicial del halo). Ambos formalismos fueron modificados para permitir un retraso temporal entre la producción de elementos "rápidos "y la de los elementos "retrasados ".

El modelo cumple satisfactoriamente las restricciones tradicionales, como el comportamiento de la tasa de formación estelar y el problema de las enanas G. Las predicciones también coinciden, (tanto como es de esperarse, en vista de las incertidumbres en los datos) con las tendencias observadas en Ba/Eu y en elementos O y α relativos al hierro, suponiendo rendimientos constantes en todo tiempo. Esto sugiere que el enriquecimiento como función del tiempo, es verdaderamente un efecto importante y que el proceso-s es "primario ". El modelo también se usa para probar algunas ideas respecto al origen de los gradientes de abundancias.

ABSTRACT. A self-consistent analytical chemical evolution model is developed for the Galactic halo and the disk in the solar neighbourhood using Hartwick's (1976) modified Simple model for the halo and Clayton's analytical inflow formalism for the disk (including a modest initial enrichment from the halo), both formalisms being modified to allow for a time delay between the production of "prompt "elements such as oxygen, α-elements, r-process elements and one component of iron and that of "delayed elements "such as s-process products (from intermediate-mass stars) and another component of iron (from Type Ia supernovae).

The model conforms fairly well to traditional constraints like the behaviour of star formation rates and the G dwarf problem, and its predictions also agree as well as can be expected in view of uncertainties in the data with observed trends in Ba relative to Eu and in O and α elements relative to iron, assuming constant yields throughout, which suggests that enrichment as a function of time is indeed an important effect and that the s-process is "primary ". The model is also used as a test of some ideas concerning the origin of abundance gradients.

Key words: **INTERSTELLAR-ABUNDANCES — NUCLEOSYNTHESIS**

I. INTRODUCTION

An attempt to provide a coherent Galactic chemical evolution (GCE) model designed to cover both the halo and disk and to account for the G dwarf metallicity distribution and abundance gradients was made by Tinsley and Larson (1978), and various aspects of this problem have been discussed in many papers. More recently there have been significant developments calling for a fresh discussion, mainly related to the differential behaviour of individual elements in metal-deficient stars, notably

1. Increasing attention to the iron-oxygen "anomaly" (see reviews by Wheeler, Sneden & Truran 1989, Lambert 1989; and discussions by Tinsley 1979, Matteucci and Greggio 1986, Matteucci 1988), related to the α/Fe anomaly which has actually been known for even longer (Wallerstein 1962; Pagel 1970).

2. Clarification of the roles played by r and s processes in the production of heavy elements in metal-poor stars. Following the systematic spectroscopic survey by Spite and Spite (1978), Truran (1981) made the seminal suggestion that all heavy elements in the most metal-poor stars come from the r-process, even in cases like that of Ba of which 88 per cent in the Solar System comes from the s-process (Cameron 1982), and this suggestion has been amply confirmed by subsequent work, culminating in that of Gilroy et al. (1988) which shows an abundance pattern in close agreement with that of r-process products in the Solar System and a two-slope logarithmic relation between Ba and Eu which has been extended by Lambert (1989) to reveal a quite well-defined cross-over from a pure r-process proportionality to a proportionality similar to that of the Solar System which holds in most stars of the disk (Butcher 1975). The relation between Eu and Ba is rather more coherent than that between either element and iron and it strongly suggests two things: (1) that the progress of time is at least an important factor influencing the growth of abundances (this is also supported by the O/Fe and α/Fe relationships); and (2) that the s-process is primary (assuming the r-process to be so). This latter result immediately raises a problem because although with the ^{13}C neutron source instead of ^{22}Ne it is easy to disbelieve the traditional view of the s-process as being secondary (i.e. with a yield that increases with metallicity), as has been recently shown by Malaney & Fowler (1989), these authors need a certain amount of fine tuning to get the yield exactly constant relative to those of the more classically primary elements.

Both the O-α-Fe trend with [Fe/H] and the Eu-Ba trend with [Eu/H] can be explained rather simply by just postulating a time delay due to the evolutionary time scale of the progenitors. This fact was noted in the first case by Tinsley (1979) and discussed further by Matteucci and her collaborators and in the second case by Gilroy et al. What I want to do in this paper is to present a very simple basic model of the evolution of the Galaxy, which incorporates these effects as well as traditional constraints like the G dwarf problem, and try to see what are the minimum assumptions that provide an explanation of the abundance ratios. In accordance with my usual (or rather unusual) practice, the models will be analytical, taking advantage of the very convenient and useful formalism for treating inflow invented by Clayton (1985, 1988).

II. CONSTRAINTS

Our model will aim to satisfy the following constraints:-

1. To provide a consistent story of the development of the halo and the disk, taking into account initial enrichment of the disk by prior activity in the halo (cf. Kumai et al. 1988).

2. Relative numbers of disk and halo stars in the solar cylinder.

3. The present-day gas fraction in the disk (Kulkarni and Heiles 1987).

4. The ratio of present star formation rate to average past SFR (Scalo 1986).

5. The age-metallicity relation in the disk.

6. The metallicity distribution functions in the halo and disk (G-dwarf problem).

7. Variations in relative abundances of primary elements (assuming s-process, r-process both primary).

8. To explain the presence of abundance gradients.

A further constraint, to wit data bearing on nuclear cosmochronology, will not be considered in this paper because the inherent uncertainties are too great (cf. Clayton 1988). It could, however, turn out that cosmochronology data discredit both conventional age-dating based on HR diagrams and the simple linear star formation law that I assume (following Clayton) in order to apply analytical models (Butcher 1987; Pagel 1989).

III. A MODEL FOR THE GALACTIC HALO

Hartwick (1976) and Searle & Zinn (1978) noted that the metallicity distribution of field stars and globular clusters in the halo was fairly consistent with a modified Simple model, the modification being that the effective yield is reduced by continuous loss of gas from the system. Zinn (1985) showed further that there is a distinct break between halo clusters distributed in a spheroid and having metallicity [Fe/H] < -1 or so and clusters distributed in a thick disk with [Fe/H] > -1. Pagel (1989) discussed the distribution of oxygen abundance in the globular clusters, on grounds that oxygen is a better candidate than iron for applying the instantaneous recycling approximation. Assuming a relation between oxygen and iron abundances based on the results of Clegg, Lambert & Tomkin (1981), Pagel found the halo clusters to fit Hartwick's modified Simple model with a mass loss parameter (defined below) $1 + \Lambda \approx 10$ except for the absence of any clusters with [Fe/H] < -2.5 or [O/H] < -2.0 when 10 or so would have been expected. For field stars the results of Beers, Preston & Shectman (1986) and Beers (1987) suggest that the corresponding very low-metallicity field stars with [Fe/H] ≈ -3 are in fact present in roughly the expected numbers compared to more moderately low-metallicity stars with [Fe/H] ≈ -2 so that it seems reasonable to use Hartwick's modified Simple model for the halo despite the shortage of very low-metallicity globular clusters which could be due either to a small amount of self-enrichment in the clusters (cf. Cayrel 1986) or to an absence of cluster stars among the first generations in the halo. The underlying physical assumption, that metallicity increased in the halo more or less as a function of time, is considered in some quarters to be appallingly naive - on grounds that mixing processes could not act fast enough on the time scale of collapse of the halo (S M Fall, private communication) - but this time scale is uncertain and our simple-minded hypothesis is supported by the differential abundance data.

Following Hartwick (1976) I assume a modified Simple model (Pagel & Patchett 1975; Tinsley 1980) in which gas is expelled from the system at a rate Λ times the rate at which mass is locked up in stars (or compact remnants), Λ being a constant = 9 from the globular cluster oxygen abundance distribution. The formation of oxygen, α-elements and r-process products, and mass loss from stars, are all treated in the instantaneous recycling approximation, but for the s-process I assume that production takes place at a fixed time interval Δ after star formation. (For iron I take $\Delta = 0$ in the halo, i.e. instantaneous recycling, but will introduce a time-delayed component in the subsequent disk phase corresponding to the contribution of Type Ia supernovae.) Taking the initial mass of the halo as the unit and a linear law of star formation,

$$\frac{ds}{dt} = \omega g(t) \tag{1}$$

where s is the mass locked in stars, g(t) is the mass of gas and ω is a constant, we have for the total mass

$$m(t) = g(t) + s(t) = 1 - \Lambda s(t), \tag{2}$$

for the gas

$$g(t) = 1 - (1+\Lambda)s(t) = e^{-(1+\Lambda)\omega t}. \tag{3}$$

for the stars (and remnants)

$$s(t) = [1 - e^{-(1+\Lambda)\omega t}] \;/\; (1+\Lambda) \tag{4}$$

and for the abundance of some element in the gas

$$Z(t) = p e^{(1+\Lambda)\omega\Delta}\omega(t-\Delta); \qquad t > \Delta, \tag{5}$$

where p is the true yield. When the halo phase comes to an end (taken to occur at a time t_1 when [Fe/H] = -1 or [O/H] = -0.5), the total amount of gas expelled is

$$G(t_1) = g(t_1) + \Lambda s(t_1) = \Lambda s_1 + g_1, \tag{6}$$

the total amount of any element produced is

$$A_1 = p s_1 \tag{7}$$

of which an amount

$$\overline{Z}s_1 = p(1+\Lambda)^{-2}\left[1 - \{1 + (1+\Lambda)\omega(t_1-\Delta)\} e^{-(1+\Lambda)\omega(t_1-\Delta)}\right] \tag{8}$$

is locked up in stars and the remainder provides an abundance

$$Z_o = (p - \overline{Z})s_1/(g_1 + \Lambda s_1) \approx (p - \overline{Z})/\Lambda \approx p/(1+\Lambda) \approx \overline{Z} \tag{9}$$

in the gas, which is assumed to fall into the disk with an initial abundance Z_o. (Δ is neglected in eq. (7) because s-process production from the halo is assumed to have time to be completed before star formation gets under way in the disk.) Unlike what happens in some other models, Z_o is considerably less than the final abundance Z_1 in the last stars to be formed in the halo because I have assumed the gas expulsion to be a continuous process, and this initial metallicity will be further diluted by inflow of altogether about 10 times as much mass of unprocessed gas during the evolution of the disk; the resulting contribution of long-lived stars of the halo to the present-day population in the Solar cylinder therefore ends up at about 1 per cent. Table 1 gives the yields and time delays assumed for the elements that I wish to consider; the same parameters apply to the halo and disk apart from an additional component of iron that has a time delay exceeding the duration of the halo phase. All yields are expressed in units of the total solar abundance of the corresponding element and all time delays expressed as $\omega\Delta$.

TABLE 1. Assumed yields and time delays

Element	Yield p [a]	Normalised Time delay $\omega\Delta$
Oxygen and α-elements	0.8	0.0
Prompt iron	0.25	0.0
Delayed iron (disk only)	0.9 $e^{-\omega\Delta^{b}}$	0.5 [b]
Europium (r-process only)	0.8	0.0
Ba (r-process)	0.1	0.0
Ba (s-process)	1.33 $e^{-(1+\Lambda)\omega\Delta^{c}}$	0.06 [c]

[a] In units of the total solar abundance of the same element.
[b] For whatever value of ω holds in the solar neighbourhood.
[c] For whatever value of ω holds in the halo and the solar neighbourhood, assuming the same in both.

The yields have been chosen <u>ad hoc</u> to give solar abundances at a reasonable gas fraction in the disk (as described below) together with [O/Fe] = [α/Fe] = 0.5 in the halo (a reasonable caricature of all the observational results) and a ratio of 12/88 for r and s process contributions to barium in the Solar System (Cameron 1982; Lambert 1989). The resulting chemical evolution of the halo is described in Table 2.

TABLE 2. Chemical evolution of the halo

ωt	s	g	[O/H][a]	[Fe/H]	Ba(r)[b]	Ba(s)[b]	[Ba/H]
.01	.01	.90	-2.10	-2.60	10^{-3}	0.0	-3.0
.06	.045	.55	-1.32	-1.82	.006	0.0	-2.22
.065	.05	.52	-1.28	-1.78	.0065	.0067	-1.88
.07	.05	.50	-1.25	-1.75	.007	.013	-1.70
.08	.055	.45	-1.19	-1.69	.008	.027	-1.46
.10	.06	.37	-1.10	-1.60	.01	.053	-1.20
.20	.09	.14	- .80	-1.30	.02	.19	- .68
.30	.095	.05	- .62	-1.12	.03	.32	- .46
.40	.098	.018	- .50	-1.00	.04	.45	- .31
$G(t_1)$		.90					
$p\ s_1$[b]			.078	.025	.010	.072	.082
$\bar{Z}\ s_1$[b]			.007	.002	.001	.007	.008
Z_o[b]			.08	.025	.010	.07	.08

[a] [O/H] = [α/H] = [Eu/H] according to the assumptions made.
[b] Abundance in solar units.

Apart from providing initial abundances Z_o for the evolution of the disk, the main interest of Table 2 is that it puts some additional flesh on to the relationship between Eu and Ba abundances discovered by Gilroy <u>et al</u>. in the form given by Lambert (See Fig. 1). The only <u>ad hoc</u> parameter that has an important effect on this diagram is the time delay for the s-process, $\omega\Delta = 0.06$, since the relative yields for r and s process are fixed by data in the Solar System. This choice leads to the curve shown in the Figure, which gives quite a reasonable fit to the data. The actual time delay depends on ω, which is generally considered to be about 0.3 Gyr^{-1} in the disk (Clayton 1988) and could be the same or perhaps greater in the halo. This value of ω gives 2 x 10^8 yrs for the time delay, corresponding to a stellar initial mass for s-process progenitors of about 3.5 $M_\odot$ which is not unreasonable, and a collapse time for the halo of 1.3 Gyrs, which is also not unreasonable. Thus ω could (although it need not necessarily) be identical in the halo and the disk: no dramatic burst of star formation here.

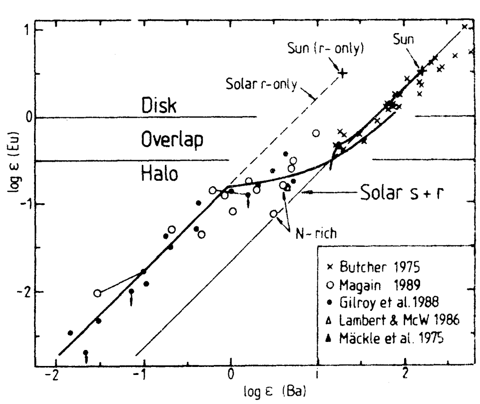

Fig. 1. Relation between stellar Eu and Ba abundances after Lambert (1989), showing a constant ratio corresponding to pure r-process at the lower abundances with a transition to the solar ratio at higher abundances. The thick lines are the representation of these data by the chemical evolution models for the halo and disk in Tables 2, 3.

IV. CHEMICAL EVOLUTION OF THE DISK IN THE SOLAR NEIGHBOURHOOD

Clayton (1985, 1988) has developed a set of analytical infall models which are exceedingly useful in discussing various problems in GCE and nuclear cosmochronology. In a detailed discussion of the G dwarf problem, Pagel (1989) pointed out the excellent fit to the oxygen abundance distribution in the Solar cylinder that is provided by Lynden-Bell's (1975) model of decaying inflow and the somewhat less good fit of Clayton's more tractable "standard" models with k = 4; the latter fit is, however, slightly improved when the initial enrichment from the halo is taken into account (see Fig. 2) and for present purposes I shall deem the fitting to be adequate and accordingly use that model, with allowance for time delays similar to the one made for the halo. In this model the inflow rate is given by

$$\frac{dm}{dt} = \frac{k}{t+t_o} g(t) \tag{10}$$

and I arbitrarily choose $\omega t_o = 2$ so as to obtain an ultimate mass multiplication factor M = 10.5 as explained above. (The precise value makes little difference.) The equations for the development of s and g, which will now be expressed in units of the initial mass of the disk, and the resulting gas fraction μ, are just those given by Clayton (see also Pagel 1989), but I have derived a new equation for the abundance in the gas:-

$$Z(t) = (1+t/t_o)^{-k}\left[Z_o + \frac{\omega t_o}{k+1} p e^{\omega\Delta}\left\{\left(1+\frac{t-\Delta}{t_o}\right)^{k+1} - 1\right\}\right]; \quad t > \Delta \tag{11}$$

$$= Z_o\left(1+\frac{t}{t_o}\right)^{-k}; \quad t < \Delta \tag{12}$$

(This differs slightly from Clayton's corresponding equation even when $\Delta = 0$; the difference apparently arises because I assume the infalling gas to be unprocessed.) The resulting distribution function of stellar abundances (with $\Delta = 0$) is given by

$$\frac{ds}{dz} = (k+1)\left(1+\frac{t}{t_o}\right)^k e^{-\omega t}\left[1 + k\left(1+\frac{t}{t_o}\right)^{-(k+1)}\left\{1 - \frac{z_o}{\omega t_o}(k+1)\right\}\right]^{-1} \tag{13}$$

where $z \equiv Z/p$.

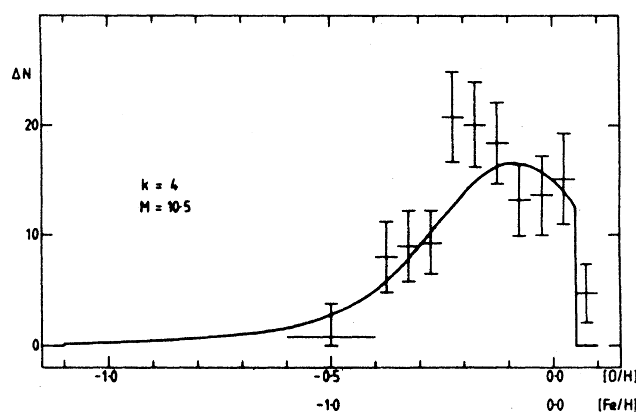

Fig. 2. The oxygen abundance distribution of 132 G dwarfs in the Solar cylinder (assuming [O/H] = 0.5 [Fe/H]) after Pagel (1989) and the theoretical distribution after Clayton's model used in Table 3 with k = 4, M = 10.5, Z_o = 0.08 $Z_\odot$, $\mu_1 = 0.10$, s_1 = 8.6, normalised to 132 stars. ΔN is the number of stars predicted in a bin of 0.05 in [O/H] or observed in a bin of 0.1 in [Fe/H] after correction for scatter. Vertical error bars are $\pm\sqrt{\Delta N}$ (uncorrected) and horizontal bars denote the extent of the bins.

The evolution of the disk is summarised in Table 3, where I have assumed the Solar System to have formed at $\omega t = 4$ and terminated the calculation at $\omega t = 5$, after which time the instantaneous recycling approximation is evidently becoming quite poor. Evolution is neglected from that time up to the present which is assumed to correspond to $\omega t = 5.5$.

TABLE 3. Evolution of the Galactic Disk in the Solar Neighbourhood

ωt	g(t)	m(t)	μ(t)	z(t)	ds/dz	[O/H][a]	f_1[b]	f_2[c]	[Fe/H]	Ba(r)[d]	Ba(s)[d]	[Ba/H]
0.0	1.00	1.00	1.00	0.10	1.25	-1.1	.025	.00	-1.6	.010	.072	-1.04
0.1	1.10	1.20	.91	.173	1.64	-.86	.044		-1.36	.018	.086	-.98
0.2	1.20	1.42	.85	.235	2.09	-.73	.059		-1.23	.024	.134	-.80
0.4	1.39	1.87	.74	.335	3.15	-.57	.085		-1.07	.034	.213	-.61
0.5	1.48	2.10	.70	.377	3.73	-.52	.095	.00	-1.02	.038	.246	-.55
0.6	1.57	2.34	.67	.415	4.33	-.48	.105	.035	-.85	.041	.275	-.50
0.7	1.65	2.59	.64	.450	4.94	-.44	.115	.066	-.74	.045	.302	-.46
1.0	1.86	3.33	.56	.541	6.67	-.36	.138	.146	-.55	.054	.373	-.37
1.5	2.09	4.55	.46	.668	8.85	-.27	.170	.253	-.37	.068	.472	-.27
2.0	2.17	5.70	.38	.781	9.90	-.20	.200	.347	-.26	.078	.56	-.20
3.0	1.94	7.56	.26	.992	9.43	-.10	.251	.522	-.11	.099	.72	-.08
4.0	1.48	8.82	.17	1.20	7.33	-.02	.302	.695	.00	.12	.88	.00
5.0	1.01	9.59	.105	1.40	5.03	.05	.355	.87	.09	.14	1.04	.07
5.5	Inst. recycling no longer a good approx.											

[a] [O/H] = [α/H] = [Eu/H] according to assumptions.
[b] f_1 = abundance of iron (in solar units) made instantaneously.
[c] f_2 = abundance of iron (in solar units) made after a time delay.
[d] Abundance of barium (in solar units) due to r and s process, respectively.

How does this model cope with the constraints? The final gas fraction of 1/10 and ratio of present to average past star formation rates ($\omega t_1\ g_1/s_1$) of 0.6 are quite reasonable, and the age of the disk is 3.67 times that of the Solar System, i.e. 17 Gyrs. The star formation rate peaks near $\omega t = 2$, i.e. half-way from the beginning of disk formation to the formation time of the Solar System. This is not in too bad disaccord with the results of Barry (1988) when the latter are corrected for scale heights, apart from the fact that our smooth model cannot allow for the bursts and gaps that he deduces from the chromospheric age distribution.

Oxygen and iron abundance as a function of time are shown in Fig. 3, where the oxygen abundance is compared to the corresponding data of Nissen et al (1985). The agreement is as good as can be expected, but the uncertainties in age-abundance relations are so great that this is not really a very strong constraint. Note the substantial overlap between halo and disk abundances predicted by this model, but the relative number of disk stars involved is fortunately quite small (see Fig. 2). The G dwarf abundance distribution is not perfect, but it does have the right sort of qualitative behaviour, making this model quite a useful one wherein to consider effects of stellar evolution as such. In the present paper I only consider primary elements.

Fig. 1 shows the predicted behaviour of Ba relative to Eu, already discussed in connection with the halo. The fit is very satisfactory, which is basically another way of repeating the conclusion that the s-process is primary if the r-process is. An implication of our simple model is that r-process and s-process both behave more like oxygen and α-elements for [Fe/H] > -1 or so, than like iron itself, and this could be a problem (cf. Edvardsson, Gustafsson & Nissen 1984); but Ba and Eu seem to be somewhat decoupled from Fe and O in any case.

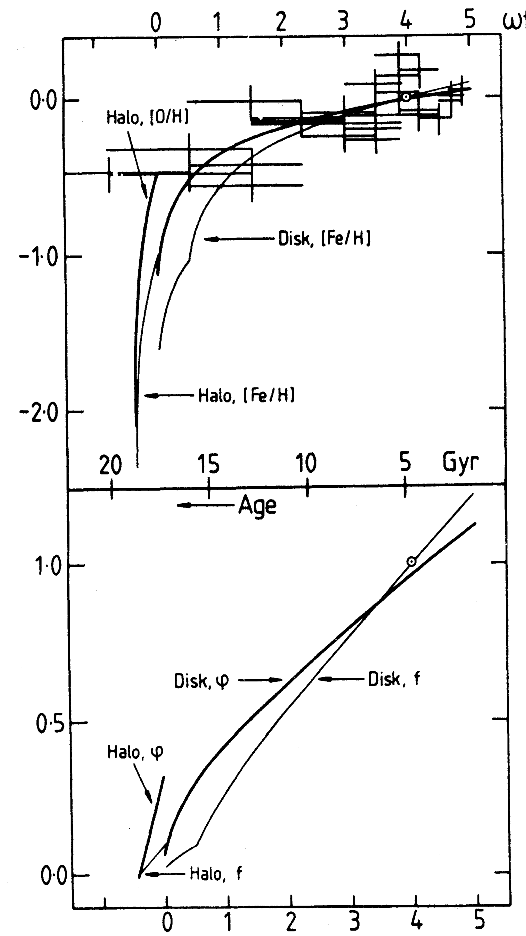

Fig. 3. Age-metallicity and age-oxygen abundance relations for the models of Tables 2, 3, compared to the age-oxygen data of Nissen, Edvardsson & Gustafsson (1985), assuming $\omega t = 5.5 - \omega \times$ age with $\omega = 0.32$ Gyr^{-1} and errors of $\pm$ 0.1 dex in both coordinates. ϕ and f are oxygen and iron abundances respectively, relative to solar.

Our combined halo-disk model makes predictions about the abundances of oxygen and α-elements relative to iron, which are confronted with observational data in Figs. 4, 5. Fig. 4, after Wheeler, Sneden and Truran (1989), shows the data for oxygen while Fig. 5 after Lambert (1989) shows magnesium, a typical α-element. The same theoretical curve - which is my pauper's analytical version of the numerical results of Matteucci (1988) - is shown in both Figures and we could get a better fit for oxygen by shifting the turnover point a bit to the right (i.e. by lengthening the relative time-delay $\omega\Delta$ for iron) and a better fit for α-elements by shifting it a bit to the left, i.e. by shortening $\omega\Delta$, which implies that the α-elements themselves should experience a small time delay relative to oxygen. Also the horizontal part of the curve looks a little too high in both panels. There is a problem with the oxygen abundances, however, in that people who measure permitted OI lines in dwarf stars tend to find higher O/Fe ratios than people who measure forbidden [OI] lines in giants, and this diagram does not include the results of Abia and Rebolo (1989) who find [O/Fe] going up continuously towards low metallicities. The sharp turnover for Mg, which comes directly out of the model, corresponds to point c in the inset of Lambert's diagram (Fig. 5), but Lambert's interpretation of the data involves an additional turning point marked b for which I have no theoretical counterpart. One could probably be invented by postulating an additional source of magnesium with an extra time delay; this would probably entail a small slope in the line segment ab, which I do not think is excluded by the data. For sulphur, Lambert gives a similar diagram in which my curve looks low, but he argues that the points from François (1987, 1988) are about 0.2 dex too high because of oscillator strength problems; if this is so, then the fit of my simple theory is well-nigh perfect for sulphur.

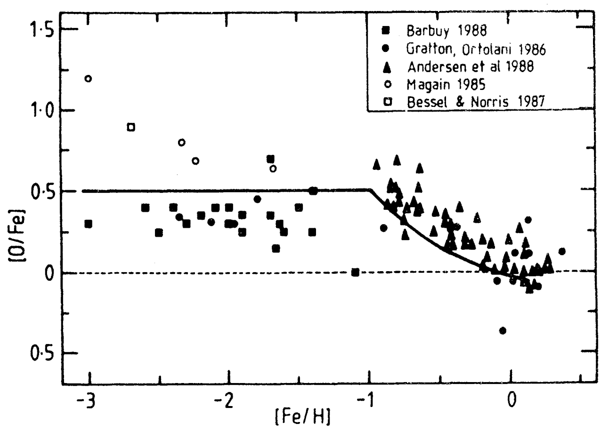

Fig. 4. Relations between iron and oxygen abundances after Wheeler, Sneden & Truran (1989). The thick line and curve give the prediction of the halo and disk models in Tables 2, 3.

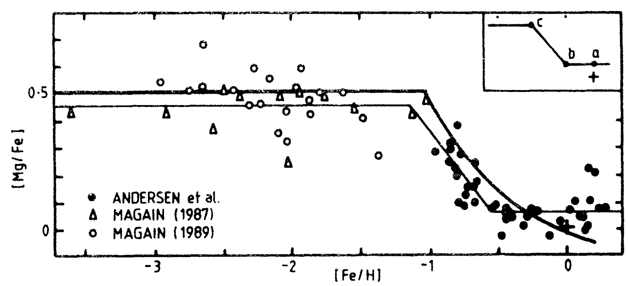

Fig. 5. Relations between iron and magnesium abundances after Lambert (1989). The thick line and curve give the prediction of the halo and disk models in Tables 2, 3.

V. ABUNDANCE GRADIENTS

The problem in understanding abundance gradients is that there are far too many possible hypotheses to account for them. These include the following:-

1. Variation in the true yield due to the IMF (Güsten and Mezger 1982). If the IMF is bimodal and is cut off or scaled down below 1 solar mass or so, then all the yields can be enhanced by the same factor. With a higher cutoff, they can be enhanced by different factors and you can get more or less what you like according to what you assume. This does not mean that such effects are not significant in reality, however.

2. Variation in effective yield due to continuous or sporadic ejection of hot gas (Pantelaki and Clayton 1987). The alternative idea of a terminal galactic wind, in which the interstellar medium is mixed and gradually heated up until it all escapes (Tinsley and Larson 1979; Arimoto and Yoshii 1987), hardly seems applicable to the solar neighbourhood.

3. Variation in gas fraction, other things being equal. This is the original idea of the "Simple" closed model (Searle and Sargent 1972) and we can examine its consequences by changing ω, leaving k, ωt_o (which fix the final mass multiplication factor) and Δ unaltered, and evolving the model for a fixed time of 15 Gyrs.

4. Variation in ratio of star formation rate to inflow rate (Diaz and Tosi 1984). We can investigate this again, in our model, by reducing k and/or increasing t_o, and evolving to the same gas fraction after 15 Gyr.

5. Inward gas flows caused by inflow of material with low angular momentum (Mayor and Vigroux 1981; Lacey and Fall 1985; Pitts and Tayler 1989).

6. Gas flows caused by viscous transfer of angular momentum across the disk (Clarke 1989; Sommer-Larsen and Yoshii 1989). These can be both outward and inward at different times and the consequences depend on the viscosity law which is assumed ad hoc and justified by its success in explaining the surface density and angular momentum distributions of galactic disks. In Clarke's model the variation in gas fraction also plays a significant role.

The sort of model considered in this work is not up to the task of evaluating all these different possibilities, but it can throw some light on the consequences of effects 3 (the μ effect) and 4 (the k effect), which will now be considered in turn.

Effect of changes in the gas fraction

The "other things" besides gas fraction in Clayton's model are the two inflow parameters k and ωt_o which are to be held fixed, while ω can be assumed to decrease outwards with galactocentric distance because of the influence of parameters such as total surface density on the star formation rate (Talbot and Arnett 1975; Dopita 1987). We accordingly choose 3 regions, a solar zone with ω = 1/3, an inner zone with ω = 0.4 and an outer zone with ω = 0.22 and evolve them for 15 Gyrs with k = 4, ωt_o = 2 in each case. Δ is kept at 1.5 Gyr for the delayed component of iron. Corresponding galactocentric distances $R - R_\odot$ are derived on the assumption of exponential scale lengths of 4 and 8 kpc for total and gas surface density respectively. The results are shown in Table 4.

TABLE 4. Results of gradient in gas fraction

	ωt	g	m	μ	[O/H]	f_1	f_2	[Fe/H]	[Fe/O]	$R-R_\odot$ (kpc)
Inner Galaxy ω = 0.4, t_o = 5	6.0	0.635	10.0	.063	.107	.40	1.08	.17	.06	-4.1
Solar ω = 0.33, t_o = 6	5.0	1.01	9.59	.105	.049	.35	.87	.09	.04	0.0
Outer Galaxy ω = 0.22, t_o = 9	3.3	1.82	8.00	.228	-.074	.26	.58	-.075	.00	6.2

With this model, the variation in gas fraction provides gradients of only .018 and .024 dex kpc^{-1} in oxygen and iron respectively, compared to the observed gradient of the order of .07 dex kpc^{-1} (Shaver et al. 1983). Simple closed models with the same gas fractions give .026 dex kpc^{-1} . The main uncertainty in these numbers comes from poor knowledge of the actual change in gas fraction with galactocentric distance; our assumptions correspond to a reasonable average of the limits adopted by Lacey and Fall (1985), and changes in the assumptions could alter these gradients by up to ± 100 per cent. In any case the change in [Fe/O] is quite small, only 1/4 of that in [Fe/H].

Effect of variation in star formation rate relative to inflow rate

The simplest way to investigate this is just to assume that the innermost part of the disk has evolved as a closed system reaching the same gas fraction as the solar neighbourhood after 15 Gyr. Then

$$\mu = g = e^{-\omega t} \Rightarrow \omega = 0.15 \quad Gyr^{-1} \tag{14}$$

and

$$Z = pe^{\omega\Delta}\omega(t-\Delta) + Z_o \tag{15}$$

giving [O/H] = 0.28, f_1 = 0.58, f_2 = 1.39, [Fe/H] = 0.30, [Fe/O] = .02.

This does not amount to a calculation of the gradients, but again it predicts little change in the Fe/O ratio, i.e. the gradients in iron and oxygen are virtually identical, and this may have some bearing on the absence of any O/Fe excess (rather the contrary) in stars of the Magellanic Clouds (Spite et al. 1986; Russell, Bessell and Dopita 1988). If the above state of affairs holds in the inner Galaxy 8 kpc from the Sun, it provides a gradient of .035 dex kpc^{-1} which, when added to the gas fraction effect, gives a total gradient of .05 ± .02 dex kpc^{-1} for oxygen or iron. This is within striking distance of the observed oxygen gradient of 0.07 dex kpc^{-1}, but leaves room for any one or more of the additional effects noted above like changes in the true yield and radial flows of gas.

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