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FURTHER GALACTIC ORBITS AND TIDAL RADII OF GLOBULAR CLUSTERS

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RESUMEN

Se calculan los movimientos espaciales y se integran numéricamente las órbitas galácticas de los cúmulos globulares M2 y NGC 6712, usando determinaciones recientes de sus movimientos propios absolutos. Para M2 se utilizan diferentes valores del apex solar y se estudian las diferencias en los parámetros orbitales resultantes; la órbita de NGC 6712 no cambia significativamente con el apex solar. Con los resultados de las órbitas galácticas se determinan los radios de marea teóricos para estos cúmulos y se comparan con los valores observados. Se evalúan los efectos de las incertidumbres en los datos de observación sobre las órbitas calculadas, así como sobre los radios de marea.

ABSTRACT

Using recently determined absolute proper motions for the globular clusters M2 and NGC 6712 their space velocities are calculated and their galactic orbits are numerically integrated. In the case of M2 several assumptions are made about the solar apex, and the resulting differences in the parameters of the galactic orbits are discussed. The orbit of NGC 6712 is insensitive to the assumed solar apex. With the computed orbital parameters the theoretical tidal radii are determined and compared with the observed values. The effects of observational uncertainties on the computed orbits and on the tidal radii are assessed.

Key words: **CLUSTERS-GLOBULAR – DYNAMICS – GALAXY-STRUCTURE**

I. INTRODUCTION

Knowledge about the space motions of star clusters is notoriously difficult to obtain, mainly because of the paucity of proper motion data. In a recent paper (Allen and Martos 1988; hereinafter Paper I) we used the observationally determined space motions of 13 clusters to compute their galactic orbits in a realistic model for the galactic potential; with the orbital parameters obtained from the computed orbits, theoretical tidal radii were calculated for these clusters and they were found to be compatible with the observational values. After the material for Paper I had been completed, two more absolute proper motions became available, that of M2 (Cudworth and Rauscher 1987), and that of NGC 6712 (Cudworth 1988). The unusually high galactocentric velocity found for M2 (370 to 440 km s^{-1}, depending on the adopted solar apex), and the very low angular momentum of NGC 6712 lead to the suspicion that these two clusters may represent rather extreme examples of the cluster population: M2 is likely to reach very large apogalactic distances, and NGC 6712 will penetrate deeply into the inner region of the Galaxy. For this reason, the galactic orbits of these clusters merit prompt discussion.

A cause for concern about the validity of the space motions of clusters has always been the conversion from relative to absolute proper motions. Normally, this is accomplished by modelling the velocity field of the reference stars, under some assumption about the solar motion. The great majority of authors have used the standard solar motion (Delhaye 1965); only rarely have the results from the Lick pilot program of proper motions been considered (Klemola and Vasilevskis 1971). It turns out, however, that the space motions of some clusters are quite sensitive to different assumptions about the size and the apex of the solar motion. Such is the case of M2, as we saw above; NGC 6712, in contrast, is insensitive to such assumptions. In this study, three orbits for M2 will be presented; they were computed using the standard solar motion, the Lick pilot program apex, as well as the newly determined Lick apex (Hanson 1987); their comparison will enable us to begin to assess the effects of these changes upon the galactic orbits. A full study of this matter, however, will have to await the publication of the complete new Lick results, since no values for the secular parallaxes are yet available. Other sources of uncertainty are observational errors in the proper motions, radial velocities and distances; their effects on the computed orbital parameters will also be discussed.

II. THE GALACTIC ORBITS

We have developed a model for the galactic potential (Allen and Martos 1986) that gives a realistic representation of both the rotation curve of our Galaxy and the force perpendicular to the plane, yet is well suited for the direct numerical integration of galactic orbits. The model allows rapid and accurate computations of galactic orbits; thus, the orbital parameters (particularly the perigalactic distance) of clusters with sufficient observational data can be directly determined. As discussed in Paper 1, a direct determination of the orbital parameters and tidal radii from the cluster orbits obtained by numerical integration in a realistic galactic potential model yields more reliable results than the customary approach of using observed tidal radii to derive information on cluster orbits.

Table 1 shows the observational data for the two clusters. Note that for M2 several values for the absolute proper motion are given. The first is obtained by assuming the standard value for the solar motion, namely 19 km s^{-1} in direction ℓ = 56°, b = −23° (Delhaye 1965). The second results from using the recent determination of the solar apex based on the new Lick proper motion survey (Hanson 1987). The third is obtained from the old results of the Lick pilot program of proper motions (Klemola and Vasilevskis 1971). The new Lick data have shown that the apex is dependent upon the galactic latitude of the stars with respect to which it is determined. Thus, if one wants to use the new apex for M2, one should take the value that most closely corresponds to the galactic latitude of this cluster, namely b = −35.8°. As can be seen in Figure 1 of Hanson's paper, this value is approximately ℓ_{apex} = 68.5°, b_{apex} = 14.0°. Unfortunately, no new values for the secular parallaxes as a function of magnitude are available yet, so an assumption has to be made. We can either take the value from the model of van Altena (1974) that corresponds to the standard conversion, or else use the Lick pilot program value. Neither procedure is really satisfactory; as remarked earlier, a proper study will have to await the availability of the full new Lick results. However, we will for the moment assume that van Altena's model is still approximately valid, and proceed to convert relative into absolute proper motions. This conversion can be carried out to first order in the standard way, and yields the results shown in Table 1.

The space velocity components, U, V, W, calculated from the data of Table 1 are listed in Table 2; the

TABLE 1

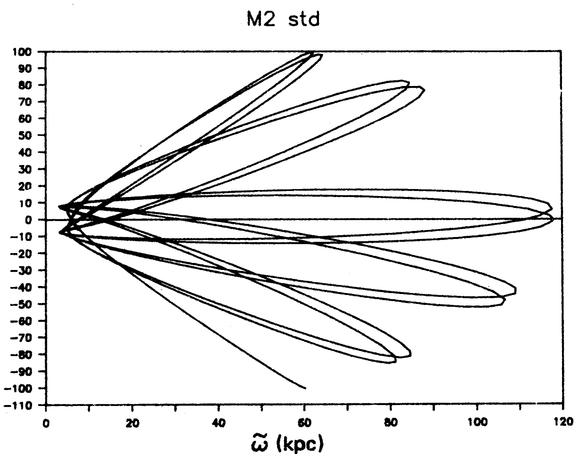
OBSERVATIONAL PARAMETERS FOR TWO GLOBULAR CLUSTERS

Cluster	Apex	α	δ	d kpc	$\mu_\alpha \cos\delta$ $'' y^{-1}$	μ_δ $'' y^{-1}$	v_r km s^{-1}
M2	Std	$21^h 30^m 52.7^s$	−1° 02′ 44″	10.76	0.0086	−0.0043	−5.2
M2	Lick 1	21 30 52.7	−1 02 44	10.76	0.0053	−0.0081	−5.2
M2	Lick 2	21 30 52.7	−1 02 44	10.76	0.0073	−0.0060	− 5.2
NGC 6712	Std	18 50 20.3	−8 45 57	6.05	0.44	−0.11	−107.5

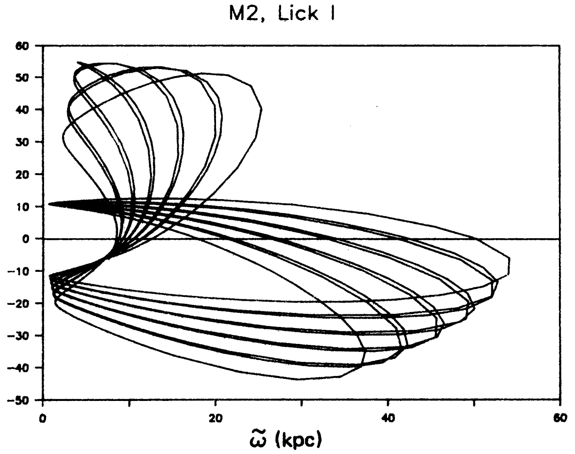
TABLE 2

INITIAL CONDITIONS FOR TWO GLOBULAR CLUSTERS

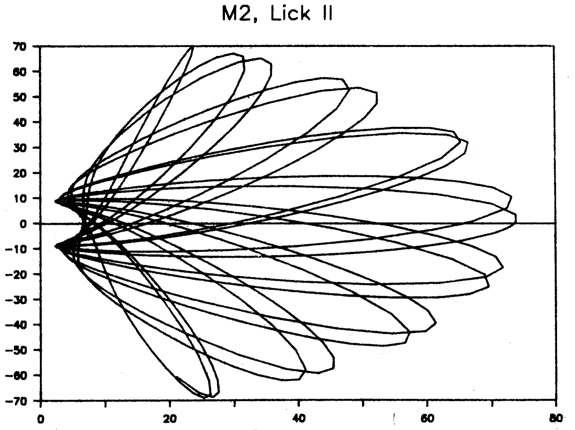
Cluster	Apex	U km s^{-1}	V km s^{-1}	W km s^{-1}	$\tilde{\omega}$ kpc	z kpc	Π km s^{-1}	Z km s^{-1}	Θ km s^{-1}
M2	Std	−220.7	−195.5	−391.6	7.54	−6.29	119.19	−383.98	−179.29
M2	Lick 1	− 6.6	−330.1	−364.3	7.54	−6.29	−85.08	−356.65	− 30.25
M2	Lick 2	−131.4	−255.5	−384.6	7.54	−6.29	−30.38	−376.92	−118.58
NGC 6712	Std	−119.6	− 21.8	−127.6	3.51	−0.49	239.60	−119.93	46.51


M2 std

ȝ. 1. The meridional orbit of M2 computed with the ındard solar apex.


M2, Lick I

g. 2. The meridional orbit of M2 computed with the lar apex as determined in the Lick pilot program of oper motions (see text).


M2, Lick II

g. 3. The meridional orbit of M2 computed with the lar apex as determined in the new Lick program (see xt).

galactocentric velocity components are also given. Note that our V component for M2 differs in sign from the one tabulated by Cudworth and Rauscher (1987).

In order to assess the sensitivity of the resulting orbital parameters to such relatively drastic changes in the solar motion three orbits were computed for M2, corresponding to the three space motions listed in Table 2.

Our second cluster, NGC 6712, turns out to be quite insensitive to the solar apex assumed, as already remarked by Cudworth (1988). Thus, the computation of its galactic orbit was done assuming that the standard solar motion (Delhaye 1965) is appropriate for this cluster. The space velocity and the galactocentric position and velocity components for this cluster are also listed in Table 2.

The numerical integrations were carried out by means of a 7th order Runge-Kutta-Fehlberg algorithm (Fehlberg 1968). The fractional errors in the constants of motion accumulated at the end of the runs were of the order of 10^{-6}. The orbits were run backwards in time over 1.6×10^{10} years.

Figures 1 to 4 show the meridional orbits of the clusters, Figures 5 and 6 two surfaces of section. The diagrams show at a glance that these clusters indeed represent rather extreme examples of the cluster population. In all computed cases, the orbit of M2 reaches the outermost confines of the Galaxy; this cluster attains apogalactic distances of almost 120 kpc (using the standard apex) or about 74 kpc (assuming the new Lick apex). In contrast, NGC 6712 is remarkable in that it penetrates deep into the central kiloparsec of the Galaxy; its pericentric distance attains values as small as 0.3 kpc. A look at the surfaces of section shows that the orbit of M2 is always a box (regardless of the assumed solar motion), whereas that of NGC 6712

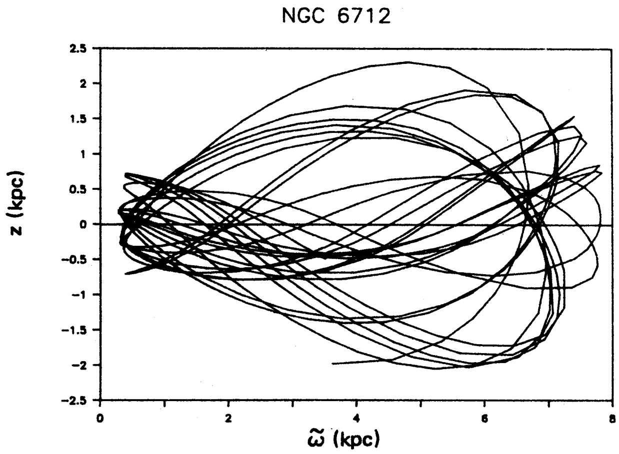

NGC 6712

Fig. 4. The meridional orbit of NGC 6712.

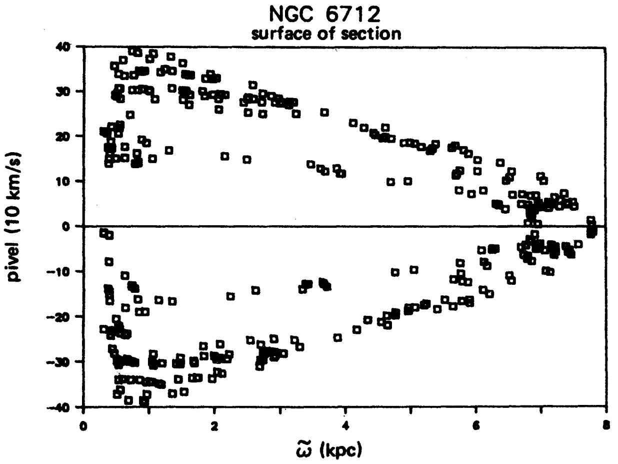


Fig. 5. The surface of section of the orbit of NGC 6712. The quasi-chaotic character of this orbit is readily apparent.

verges on chaotic; the behavior of the latter cluster can probably be attributed to its repeated close approaches to the central spherical mass included in the galactic potential model (Aarseth 1966). The quasi-chaotic nature of the orbit of this cluster implies that energy conversion from plane into z-motion or viceversa is quite efficient; indeed, at the times when the orbit exhibits radial excursions c about 8 kpc, it shows z-excursions of less than + kpc; later, the motion in z reaches values of ± 5 kp while that in $\tilde{\omega}$ is reduced to 3 kpc.

Table 3 contains the orbital parameters computed for the two clusters. In all cases the eccentricit parameter turns out to be rather large. We note als that despite the marked differences shown by th orbit of M2 as computed with different values fo the solar motion, the apogalactic distances reache by this cluster, although different, are always ver large, as are the eccentricity parameters. The perigalactic distances, in contrast, are in all cases nc very different.

III. THE TIDAL RADII

In Table 4 the theoretical tidal radii are showi and compared with the observed values. Successiv columns contain R_t(observed),the observed limitin radius, and the theoretical tidal radii computed, a in Paper 1, by means of King's (1962) formula; i the computation of the theoretical tidal radii the perigalactic distances for the times of last and nex perigalactic passage, as well as the minimum perigalactic distance ever reached by the cluster in the course of its galactic orbit have been used; th resulting tidal radii are designated as R_t(last) R_t(next), and R_t(min), respectively. M_g, the "e

TABLE 3

ORBITAL PARAMETERS FOR TWO GLOBULAR CLUSTERS

Cluster	Apex	E	h	R_{min}	R_{max}	z_{min}	z_{max}	P_r	e
		100 km^2 s^{-2}	10 kpc km s^{-1}	kpc	kpc	kpc	kpc	10^8 yr	
M2	Std	− 354.20	−135.21	5.77	118.00	−100.57	99.52	15.05	0.950
M2	Lick 1	− 646.42	− 22.82	8.49	55.15	− 43.60	54.89	5.84	0.976
M2	Lick 2	− 537.92	− 89.43	6.69	73.92	− 69.09	69.42	9.35	0.940
NGC 6712	Std	− 1482.33	16.31	0.33	7.88	− 2.05	2.31	0.92	0.932

TABLE 4

OBSERVED AND COMPUTED TIDAL RADII

Cluster	Apex	r_t(observed)	m_c	r_t(last)	r_t(next)	r_t(min)	$r_t(\mathrm{last})_d$	$r_t(\mathrm{last})_r$
		pc	$M_\odot$	pc	pc	pc	pc	pc
M2	Std	50.8±10	8.8×10^5	85.3	98.6	83.9	59.1	118.2
M2	Lick 1	50.8±10	8.8×10^5	112.1	138.4	111.8	77.6	155.2
M2	Lick 2	50.8±10	8.8×10^5	96.4	110.8	92.2	63.9	127.8
NGC 6712	Std	20.7:	1.2×10^5	9.1	7.4	4.5	6.3	12.6

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tive" galactic mass, was set equal to the equiva- nt point mass that produces the actual force acting on the cluster at the perigalactic point, as compu- d from the galactic potential model. The mass of GC 6712 was obtained from its integrated magni- de (Webbink 1985) assuming an M/L ratio of 1.6 lingworth 1976). For M2 a mass of 9×10^5 solar sses was taken, as calculated by Pryor *et al.* (1986) m a dynamical model. The last two columns of ble 4 contain the theoretical tidal radii for direct d retrograde orbits of stars belonging to the clus- ; they are designated as $R_t(\mathrm{last})_d$ and $R_t(\mathrm{last})_r$, spectively; the meaning of these quantities will be plained below.

As discussed in Paper 1 it is doubtful whether the ssical King formula provides a good estimate of e actual limiting radius of a cluster. On theoretical ounds, the classical tidal radius overestimates the iiting cluster radius, and according to Innanen, arris and Webbink (1983) should be reduced by actor of 0.693; but the tidal radius so estimated rtains to the direct orbits of the stars within the ster; numerical work (Keenan 1981*a,b*; Innanen 79) has shown, however, that a cluster is able retain many retrograde stars at distances of the der of twice the tidal radius for direct orbits over nificant times (up to 50 orbital periods). But e tidal radius for retrograde orbits is about twice at for direct orbits (Innanen 1979), which in turn just 0.693 the King radius. If the tidal radius set mostly at the times of perigalactic passage, en the presently observed limiting radii shoud representative of the last perigalactic passage, ce the typical times of cluster core relaxation and velope repopulation are of the same order of agnitude as the galactic periods of revolution. In m, on theoretical grounds alone an uncertainty at least a factor of two exists for the tidal radii. fact, all we were able to say in Paper I was at, provided no significant repopulation of the velope has taken place since the time of last rigalactic passage, the theoretical tidal radius of a ster should lie between the values corresponding the direct and the retrograde orbits at the time of last perigalactic passage. For these reasons we have given in Table 4, along with the King tidal radius, the theoretical tidal radii for direct and retrograde orbits of stars belonging to the cluster; the agreement between theoretical and observed values is not particularly good. For M2 the best agreement corresponds to the orbit computed using the standard apex; but the uncertainties are so large that this agreement cannot be construed as an argument in favour of the standard apex. In the case of NGC 6712 the agreement between theroretical and computed tidal radius is poor; it should be noted, however, that the observed tidal radius for this cluster is particularly uncertain.

To examine the question of the sensitivity of the computed orbits (and of the inferred tidal radii) to errors arising from observational uncertainties in the cluster distances or motions we have computed two additional galactic orbits for each cluster; the initial conditions for these orbits were determined by adding to, and subtracting from, the galactocentric velocity components the uncertainties resulting from observational errors as calculated by Johnson and Soderblom (1987). For this experiment, only the standard apex was considered for M2. The results are shown in Table 5. We see that the largest differences occur for the apogalactic distances. The computed tidal radii are quite similar, the differences being well within the observational errors of the limiting radius. This is a direct consequence of the relatively minor variations of the perigalactic distances that result from taking slightly different initial conditions. Furthermore, the form, dimensions and character of the orbits do not significantly change as a result of observational errors. For M2, the changes that do occur are much smaller than those resulting from taking different solar apices.

IV. SUMMARY AND CONCLUSIONS

The galactic orbits for two additional globular clusters have been numerically integrated. The orbit of M2, particularly its apogalactic distance R_{max}, is quite sensitive to the adopted solar apex.

TABLE 5

ORBITAL PARAMETERS AND TIDAL RADII COMPUTED WITH OBSERVATIONAL UNCERTAINTIES

ster	E 100 km^2 s^{-2}	h 10 kpc km s^{-1}	R_{min} kpc	R_{max} kpc	z_{min} kpc	z_{max} kpc	e	r_t(last) pc	r_t(min) pc
2 + Δ	− 373.29	−169.12	4.97	112.22	−76.23	66.31	0.9362	75.5	75.5
2 − Δ	− 161.40	−101.53	7.12	259.51	−186.37	245.37	0.9824	98.8	97.4
GC 6712 + Δ	− 1469.71	17.30	0.54	7.87	−1.74	−1.73	0.9121	8.0	7.8
GC 6712 − Δ	− 1490.00	15.65	0.34	7.20	−4.57	2.25	0.9109	10.3	5.2

This cluster reaches apogalactic distances of more than 55 kpc and up to 118 kpc, according to the adopted solar apex. Notwithstanding these large differences, it can be concluded that M2 belongs to the kinematically most extreme galactic population. It should be noted that its metal abundance [m/H] = −1.81 (Webbink 1985) is not among the lowest determined, as could perhaps be surmised from the extremely large apogalactic distances reached by this cluster.

In contrast to M2, the orbit of NGC 6712 is insensitive to the adopted solar apex. This cluster moves on an orbit that verges on chaotic, a behavior that can be attributed to the fact that it repeatedly visits the inner kiloparsec of the galaxy; in fact, it reaches perigalactic distances R_{min} of less than 300 pc. The metallicity of NGC 6712, listed as [m/H] = −1.26 by Webbink (1985) does not place this cluster among the more metal-rich ones, as could perhaps be inferred from the character of its orbit.

By far the largest source of uncertainty in the case of the orbit M2 is the solar apex. For this cluster, the tidal radius calculated using the standard apex agrees moderately well with the observed value. The orbit of NGC 6712 turns out to be insensitive to the solar apex assumed. Neither the orbit of M2 nor that of NGC 6712 change in character as a result of observational uncertainties. The computed tidal radii of both clusters are found to be relatively insensitive to the slight variations in the initial conditions of the orbits that arise from observational uncertainties.

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