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NUMERICAL RESULTS FOR A POLYTROPIC COSMOLOGY INTERPRETED AS A DUST UNIVERSE PRODUCING GRAVITATIONAL WAVES

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RESUMEN. A nivel cosmológico pensamos que se ha estado produciendo radiación gravitacional en cantidades considerables dentro de las galaxias. Si los eventos productores de radiación gravitatoria han venido ocurriendo desde la época de la formación de las galaxias, cuando menos, sus efectos cosmológicos pueden ser tomados en cuenta con simplicidad y elegancia al representar la producción de radiación y, por consiguiente, su interacción con materia ordinaria fenomenológicamente a través de una ecuación de estado politrópica, como lo hemos mostrado en otros trabajos. Presentamos en este artículo resultados nunéricos de este modelo.

ABSTRACT. A common believe in cosmology is that gravitational radiation in considerable quantities is being produced within the galaxies. If gravitational radiation production has been running since the galaxy formation epoch, at least, its cosmological effects can be assesed with simplicity and elegance by representing the production of radiation and, therefore, its interaction with ordinary matter phenomenologically through a polytropic equation of state as shown already elsewhere. We present in this paper the numerical results of such a model.

Key words: **COSMOLOGY – GRAVITATION**

INTRODUCTION

Gravitational radiation is believed to be produced by various astrophysical events taking place within the laxies, for example, during supernova explosions. If this and other gravitational radiation producing events have en going on since early times, they may have contributed in some way to conform the present state of our universe.

The cosmological consequences of its existence has been studied, using different models, by several ithors (Rees 1971; Jackson 1972; Bertotti and Cavaliere 1971; Isaacson and Winicour 1973; Swinerd 1977b).

In this paper we present numerical results of a cosmological model that takes into account the production gravitational waves. Such a model can be assessed in a rather simple way through an isotropic cosmological model at contains matter producing gravitational radiation both considered as perfect fluids. In the geometrical optics nit of General Relativity (i.e. when the scale factor $R >> \lambda$, the gravitational radiation wavelenght), the gravitational diation can be treated as a perfect fluid with equation of state given by $p_r = \frac{1}{3}\rho_r c^2$ (Isaacson 1968a,b; Swinerd 1977a).

We have investigated (Chauvet, Cervantes-Cota and Klapp 1990a,b) the possibility of introducing of a olytropic equation of state to represent phenomenologically at a cosmological level, the interaction between matter id gravitational radiation that may give an adequate gravitational radiation time behaviour.

In this paper we present the range of model parameters giving reasonable model universes and its edictions.

II. PHYSICS BEHIND THE MODELS

To describe a cosmological model with matter and gravitational radiation, we have considered that tl gravitational radiation field $h_{\mu\nu}$, can be taken as perturbations to a cosmological background space-time $\gamma_{\mu\nu}$, given l a Robertson-Walker line element. The total metric $g_{\mu\nu}$ is then

$$g_{\mu\nu} = \gamma_{\mu\nu} + \epsilon h_{\mu\nu}, \qquad 0 < \epsilon \ll 1.$$ (

In the geometrical optics limit of General Relativity (high frecuency waves), when the total metric substituted in the Einstein field equations, and after expanding in power of ϵ, one obtains

$$R^{(o)}_{\mu\nu} - \frac{1}{2}\gamma_{\mu\nu}R^{(o)} = -8\pi G(T^{(o)}_{\mu\nu} + E^{eff}_{\mu\nu}),$$ (

where both, the matter-energy tensor $T^{(o)}_{\mu\nu}$ and the "effective" tensor $E^{eff}_{\mu\nu}$ associated with the gravitational field, c‹ be considered as perfect fluids. Especifically, the radiation field has an equation of state given by $p_r = \frac{1}{3}\rho_r c^2$ (Isaacs‹ 1968a,b; Swinerd 1977a).

From equations (2) the model dynamics is given by

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) - \frac{\kappa c^2}{R^2},$$ (

where ρ_m, ρ_r are the matter and gravitational radiation densities, κ is the curvature constant, c is the speed of light ar a dot denotes time derivative.

The energy conservation equation is

$$(\rho R^3)^{\cdot} + 3(ln R)^{\cdot}\,\frac{pR^3}{c^2} = 0,$$ (

where $\rho(= \rho_m + \rho_r)$ and $p(= p_m + p_r)$ are the total density and pressure, respectively.

We then have a two fluid model, gravitational radiation and matter, with equations of state given by

$$p_r = \frac{1}{3}\rho_r c^2 \quad (high\ frequency\ radiation)$$ (

and

$$p_m = 0 \quad (dust).$$ (

Due to the present ignorance of the precise mechanisms by which gravitational waves are generat‹ by matter we have introduced (Chauvet, Cervantes-Cota and Klapp 1990a,b) a polytropic equation of state given l $p = \beta\rho^{1+n}$, with β and n (the polytropic index) constants, leading to an adequate time profile for the densities of matt and radiation: ρ_m and ρ_r are decreasing and increasing functions of time, respectively. We have obtained that

$$\rho_m = \frac{F-3}{\beta^n F^{n+1}} \quad with \quad F \equiv \frac{\alpha}{\beta}R^{3/n} - 1$$ (

and

$$\rho_r = \frac{3}{\beta^n F^{n+1}}, \quad where\ \alpha\ is\ an\ integration\ constant.$$ (

Equation (8) fulfils the "galactic hypothesis" ansatz that implies that the gravitational radiation is postgalactic nature. This means that the polytropic index must be in the range $-1 < n < 0$. In this case $\rho_r \rightarrow$ when $R \rightarrow 0$, that is, there is no gravitational radiation at the beginning of the time.

III. NUMERICAL INTEGRATION

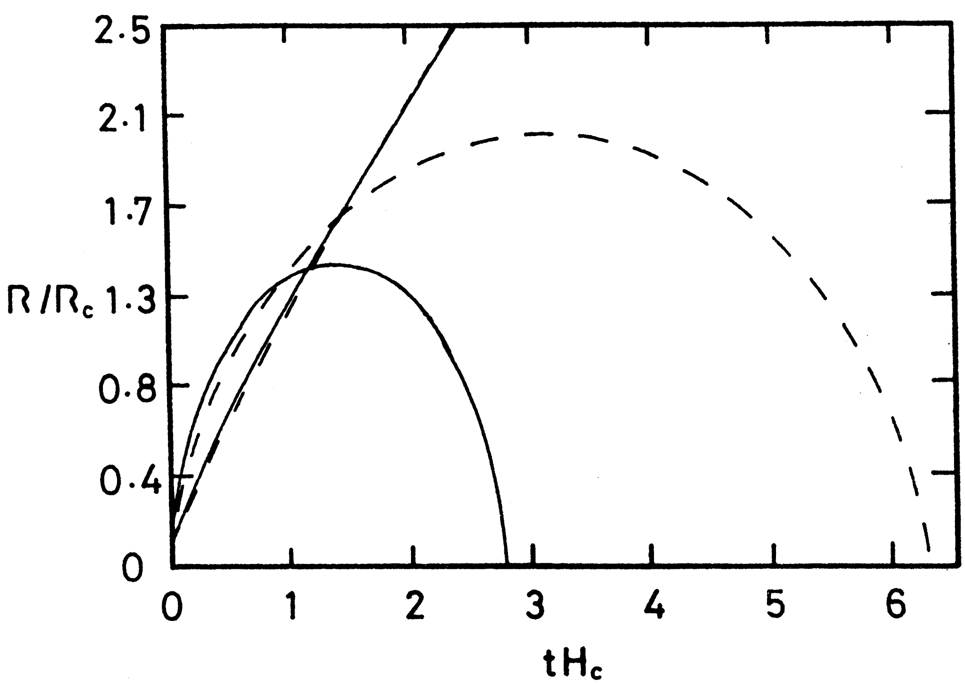
To obtain the dynamics of the model we have integrated equation (3) together with equations (7) ar (8). However, from equation (7) one can see that if $R \geq R_i \equiv (4\beta/\alpha)^{n/3}$ one gets a negative matter density, which unphysical. To avoid this condition we numerically integrate our equations up to ($t = t_c, R = R_c < R_i$) and then join a decoupled model of matter and gravitational radiation, that is, without interaction. Thus, the final model has to as: a "generating era" ($t \leq t_c$) and the decoupled era ($t > t_c$). To join the interacting model with the non interacting ie, we used the Lichnerowicz (1955) conditions (see Chauvet, Cervantes-Cota and Klapp 1990*b* for more details).

$$\frac{\rho_m}{\rho_{mc}} = \left(\frac{R_c}{R}\right)^3 \quad and \quad \frac{\rho_r}{\rho_{rc}} = \left(\frac{R_c}{R}\right)^4, \tag{9}$$

here ρ_{mc} and ρ_{rc} are appropriate constants determined by the junction conditions.

In some closed models the scale factor reaches its maximum before ρ_m becomes negative, still, it is ecessary to stop the radiation production. The reason is that, since the densities depend solely on the scale factor ıey invert their roles after that point, that is, instead of gravity waves production one would have absorption which is ɔt permitted, according to Hawking (1966).

One of the above models, including a standard dust Friedmann model for comparation is shown in igure (1).



g. 1. The sclale factor as a function of time for the parameters: $\epsilon = 0.1$, $n = -0.9$, $f_o = 10^{-10}$, $H_o = 50$ km per megaparsec. Open odels have $\sigma_{mo} = 0.1$. Closed models have $\sigma_{mo} = 1.0$. The standard Friedmann dust models are depicted with a dashed line.

V. NUMERICAL CALCULATION AND CONCLUSIONS

To carry out the numerical integrations and calculations we use the cosmological parameters

$$\sigma_{mo} = \frac{4\pi G}{3H_o^2}\rho_{mo}, \quad H_o = \frac{\dot{R}_o}{R_o} \quad and \quad f_o = -(ln\, \rho_m R^3)^{\cdot}|_o \tag{10}$$

here f_o is the present fractional convertion rate of matter into radiation. These parameters are in the range:KV

$$\begin{gathered} 30 \leq H_o \leq 100 \quad (km\ per\ megaparsec) \\ .1 \leq \sigma_{mo} \leq 1 \\ 10^{-12} \leq f_o \leq 10^{-8} \quad (year^{-1}), \end{gathered} \tag{11}$$

while polytropic constants are in the range

$$-1 < n < 0, \quad 0 < \epsilon < 1/3, \tag{12}$$

where ϵ is a constant defined in terms of α and β.

With these values we derive the range of other cosmological parameters such as the actual gravitational radiation energy density σ_{ro} and the actual dessaceleration parameter q_o:

$$\sigma_{ro} = \frac{4\pi G}{3H_o^2}\rho_{ro} \quad \textit{and} \quad q_o = -\frac{\ddot{R}_o R_o}{\dot{R}_o^2}, \tag{13}$$

that in terms of our quantities are given by

$$\sigma_{ro} = \frac{n\sigma_{mo}}{8}\left[1 - \frac{3}{n} - \frac{\mathbf{f}_o}{H_o} - \sqrt{\left(1 - \frac{3}{n} - \frac{\mathbf{f}_o}{H_o}\right)^2 - \frac{16}{n}\frac{\mathbf{f}_o}{H_o}}\,\right] \quad \textit{and} \tag{14}$$

$$q_o = \sigma_{mo} + 2\sigma_{ro} \tag{15}$$

Furthermore, to compare the magnitud of today quantities with those at the junction point, we have calculated the ratio of cosmological parameters evaluated at $t = t_o$ with those at $t = t_c$ (see Chauvet, Cervantes-Cota and Klapp 1990b).

From Table (1) one can see that the present density of gravitational radiation, σ_{ro}, for n fixed, is an increasing function of $\mathbf{f}_o$. On the other hand, σ_{ro} also increases when $n \to -1$. Since the gravitational radiation contribution to the dessaceleration parameter is twice that of matter (see equation 15), q_o increases when $\mathbf{f}_o$ increases or when $n \to -1$.

We demand $(R_o/R_c) < 1$ because we assume that nowadays gravitational waves are still being produced within the galaxies. We note that (R_o/R_c) disminishes with decreasing $\mathbf{f}_o$ or when $n \to -1$. As expected, from table (1)

Table 1. Cosmological parameters σ_{ro}, q_o, R_o/R_c and H_o/H_c as a function of n and $\mathbf{f}_o$ for $\epsilon = 0.1$, $\sigma_{mo} = 0.1$ and $H_o = 50$ km per megaparsec.

n	$\mathbf{f}_o$	σ_{ro}	q_o	R_o/R_c	H_o/H_c
-0.9	10E-09	4.30E-1	0.96	1.29	3.99
-0.9	10E-10	4.36E-2	0.19	1.00	0.99
-0.9	10E-11	4.39E-3	0.11	0.57	1.94
-0.9	10E-12	4.39E-4	0.10	0.29	3.86
-0.5	10E-09	2.47E-1	0.59	1.13	2.03
-0.5	10E-10	2.64E-2	0.15	0.95	1.09
-0.5	10E-11	2.71E-3	0.11	0.68	1.61
-0.5	10E-12	2.72E-4	0.10	0.46	2.39
-0.1	10E-09	5.56E-2	0.21	1.01	0.99
-0.1	10E-10	6.04E-3	0.11	0.95	1.08
-0.1	10E-11	6.13E-4	0.10	0.88	1.18
-0.1	10E-12	6.14E-5	0.10	0.82	1.29
-0.01	10E-08	5.65E-2	0.21	1.00	0.99
-0.01	10E-09	6.20E-3	0.11	0.99	1.01
-0.01	10E-10	6.31E-4	0.10	0.99	1.02
-0.01	10E-11	6.32E-5	0.10	0.98	1.03
-0.01	10E-12	6.32E-6	0.10	0.97	1.04

ur model predicts that the smaller the present density parameter σ_{ro} is, the later in time the matter and gravitational adiation can be decoupled.

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