Rev. Mexicana Astron. Astrof., **21**, 28-30 (1990)

INSTABILITIES IN A RELATIVISTIC VISCOUS FLUID

M. G. Corona-Galindo[1]

Centro de Investigación en Física, Universidad de Sonora

Jaime Klapp

Instituto Nacional de Investigaciones Nucleares and
Departamento de Física, Universidad Autónoma Metropolitana-Iztapalapa

A. Vázquez

Facultad de Ciencias, UNAM

RESUMEN. Las ecuaciones hidrodinámicas de un fluido imperfecto relativista son resueltas, y los modos hidrodinámicos son analizados con el propósito de establecer correlaciones con las estructuras cosmológicas.

ABSTRACT. The hydrodynamical equations of a relativistic imperfect fluid are solved, and the hydrodynamical modes are analysed with the aim to establish correlations with cosmological structures.

Key words: **COSMOLOGY – HYDRODYNAMICS – RELATIVITY**

The physics of the era before the H-recombination is very important for a relativistic treatment of the cosmological substratum, particularly since the main component of the Universe during that epoch was radiation. Some authors have analysed the hydrodynamical fluctuations in the radiation era in order to determine the instability conditions of hydrodynamical modes (Silk 1968; Weinberg 1972). This analysis has been done in a non-relativistic framework, overlooking dissipative effects. However it has been shown that dissipative phenomena play a very important role in galaxy formation (Simon 1970, 1971; Nowotny and Corona 1981; Zimdahl 1985; Corona-Galindo and Dehnen 1989). Connecting both ideas we construct a model of the Universe as a relativistic non-ideal hydrodynamical fluid, where the viscosity and heat conduction are taken into account. Radiation is the main constituent of this cosmological substratum in which the radiation transport coefficients - viscosity η and thermal conductivity κ - are determined by the interactions between photons and electrons via Thomson scattering. Their values are given by

$$\kappa = \frac{4}{3}\rho_r \frac{\tau c^4}{T}, \tag{1}$$

$$\eta = \frac{4}{15}\rho_r c^2 \tau, \tag{2}$$

$\lambda_\gamma = (n_0\sigma_{th})^{-1}$ being the mean free path and $\tau = \lambda_\gamma / c$, the time between collisions of the photons with free electrons in case of full ionization.

1. On sabbatical leave from Instituto Nacional de astrofísica, Optica y Electrónica, Puebla, México.

The energy density of the matter and the photons, and the pressure p which is due only to the radiation l are

$$\epsilon = \rho_m c^2 + aT^4, \quad \rho_r = \frac{aT^4}{c^2}, \quad p = \frac{1}{3}aT^4. \tag{3}$$

For a viscous fluid with heat conduction the momentum balance equation and the energy balance ation are derivable from the stress-energy tensor

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + pg^{\mu\nu} - 2\eta\sigma^{\mu\nu} - \varsigma u^\alpha{}_{;\alpha}h^{\mu\nu} + \frac{1}{c}(q^\mu u^\nu + q^\nu u^\mu), \tag{4}$$

:re $\eta \geq 0$ is the coefficient of dynamic viscosity, $\varsigma \geq 0$, the coefficient of bulk viscosity and $\sigma^{\mu\nu}$, $u^\alpha{}_{;\alpha}$ and $h^{\mu\nu}$ are the ar, expansion, and projection tensor of the fluid. q^μ is the heat-flux-4 vector

$$q^\alpha = -\kappa h^{\alpha\beta}(T_{,\beta} + Tu^\gamma u_{\beta;\gamma}). \tag{5}$$

The continuity equation is obtainable from

$$(\rho u^\mu)_{;\mu} = 0. \tag{6}$$

As gravitational equation we take the Poisson equation

$$\nabla^2\phi = 4\pi G\rho. \tag{7}$$

We assume at first approximation an Euclidean space as the framework for the solution of the equations notion. Likewise, solutions of the form $exp(\omega t + i\bar{k}\cdot\bar{r})$ are proposed so that the dispersion relation gives

$$\begin{aligned} &\frac{kT_0}{\rho_{0m}c^4}\omega^4 - \tilde{R}\omega^3 - \left[\frac{4\eta k^2c^2}{3\rho_{0m}c^2} + \frac{\kappa T_0}{\rho_{0r}c^4}\left[(1-4R)k^2c^2 - 4\pi G\rho_{0m}R\left(1+\frac{8R}{3}\right)\right]\right]\omega^2 \\ &-\left[\frac{16R}{3}k^2c^2 - 4\pi G\rho_{0m}\tilde{R}\left(1+\frac{8R}{3}\right) + \frac{kT_0}{\rho_{0r}c^4}\frac{4\eta k^2c^2}{3\rho_{0m}c^2}(k^2c^2 - 8\pi G\rho_{0m}R)\right]\omega + \\ &\frac{kT_0}{\rho_{0r}c^4}\cdot 4\pi G\rho_{0m}k^2c^2\left(1-\frac{8R}{3}\right) = 0. \end{aligned} \tag{8}$$

To know the behavior of the hydrodynamical modes, equation (8) must be solved numerically. This ution is in process and will be reported in a later paper.

In the case of $\kappa = 0$, $\eta = 0$ equation (8) yields to the following mass-values

$$M_1 = 7\times 10^{24}M_\odot(1+z)^{-6}(\Omega h^2)^{-2}, \tag{9}$$

$$M_2 = 3\times 10^{26}M_\odot(1+z)^{-3}(\Omega h^2). \tag{10}$$

The physical characterization of the two masses is, first, the range at which the gravitational influence is :he same order as the thermal pressure of the fluid (Jeans mode) and, second, the mass which is contained in the ume with dimension of the mean free path of the photons.

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M.G. Corona-Galindo: Centro de Investigación en Física, Universidad de Sonora, Apartado Postal A-088, 8319 Hermosillo, Son., México.

Jaime Klapp: Instituto Nacional de Investigaciones Nucleares and Departamento de Física, Universidad Autónom Metropolitana-Iztapalapa, Apartado Postal 55-532, 09340 México, D.F., México.

A. Vázquez: Facultad de Ciencias, UNAM, México, D.F., México.