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DISC INSTABILITY TO THE PULSATING MODE

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RESUMO: Muitas gálaxias têm curvas de velocidade que crescem ou decresceo suavemente e que podem ser representadas por leis de potência, $v = v_o(R/R_o)^{-\beta}$. Estes discos são auto-semelhantes se não tivereo fronteira. O uso desta propriedade permite que a solução seja analítica. Neste trabalho nós estudamos a estabilidade destes discos quando perturbados homologamente.

ABSTRACT: Many galaxies have slowly rising or decreasing velocity curves which can be well represented as weak power laws such as $v = v_o(R/R_o)^{-\beta}$. These discs are self-similar If they have no boundary. The use of self-similarity allows the mathematics to be carried out analytically. In this work we study the stability of these disks under a homologous transformation.

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It was shown by Mestel (1963) that discs with v = constant can be produced from the collapse of a primeval cloud. These discs were used by Lynden-Bell (1970) to provide a mechanism for the growth of giant black holes. Here we generalize the velocity law given ahove to

$$v = v_o(R/R_o)^{-\beta} \qquad (1)$$

where v_o and R_o are constants and the exponent β is in the range $-0.5 < \beta < 0.5$. These discs are self-similar if they have no boundary, i.e., the intrinsic properties at one radius are scaled versions of the properties at any other radius. This principle still holds for discs with velocity dispersion σ , provided it has the same radial dependence as in (1). We consider the disc as infinitesimally thin. The use of self-similarity allows the mathematics to be carried out analytically (for details see Lemos 1987; Lemos, Kalnajs and Lynden-Bell 1989).

Using a technique developed by Mestel (1963) where the disc is considered as the limiting case of a superposition of spheroids with maximum excentricity one finds that the surface density is given by

$$\Sigma(R) = [L(\beta)/2\pi G]\ [v^2 + (1 + 4\beta)\ \sigma^2 / R\ ] \qquad (2)$$

where G is the constant of gravitation,

$$L(\beta) = [\Gamma(1 - \beta)\ \Gamma(1/2 + \beta)]/\ [\Gamma(1 + \beta)\ \Gamma(1/2 - \beta)]$$

and $\sigma^2 \equiv p/\Sigma$ (p is the pressure).

We study axisymmetric stability through the zero frequency mode and use the logarithmic spiral representation (Kalnajs 1971) where the displacement vector $\xi \propto \exp i \alpha \ln R$. If in addition the perturbation is adiabatic, $\Delta p/p = \gamma \Delta\Sigma/\Sigma$ (γ is the adiabatic index), then we obtain the following dispersion relation (Lemos et al. 1989)

$$\gamma \sigma^2 \alpha^2 - [v^2 + (1 - 4 \beta) \sigma^2] L(\beta) K(\alpha) (\alpha^2 + 1/4) + A = 0 \qquad (3)$$

Where

$$K(\alpha) = \frac{1}{2} \frac{\Gamma(1/4 + i \alpha/2) \Gamma(1/4 - i \alpha/2)}{\Gamma(3/4 + i \alpha/2) \Gamma(3/4 - i \alpha/2)}$$

and

$$A = 2(1 - \beta) v^2 + 2\{[(1+\beta) (1+2\beta)+ 1/8] \gamma - (1+4\beta) (1+\beta) \} \sigma^2.$$

With equation (3) we can study stability to all wavelength perturbations. Here we are interested only in the pulsating mode.

It is possible to show that, in general, the displacement vector for this set of discs is given by $\xi \propto R^{1/2+2\beta} \exp i \alpha \ln R$. By definition the pulsating mode is a mode that suffers a homologous deformation, $\xi \propto R$. So we must have $\alpha = 0$ and $\beta = 1/4$. In fact, a formal analysis shows that only for the disc with $\beta = 1/4$ one has a zero frequency normal mode which obeys the proper boundary conditions (ξ/R finite at both ends). By substituting the above values for α and β in equation (3) we obtain

$$v^2/\sigma^2 = 4 (3/2 - \gamma) \qquad (4)$$

Thus for a flat system with no rotaticn ($v = 0$) $\gamma = 3/2$ is the equivalent to $\gamma = 4/3$ for a 3 - dimensional system, i.e., $\gamma = 3/2$ gives a pulsating mode. In addition, the marginally stable pulsating mode for $\beta = 1/4$ is given by equation (4). When $\gamma > 3/2$ the instability never sets in. If we crudely approximate a galaxy by a 2 - dimensional monoatomic ideal gas then $= 2$ which implies the galaxy does not pulsate.

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