Rev. Mexicana Astron. Astrof., 21, 255-258 (1990)

A CONTRIBUTION TO THE STUDY OF THE STRUCTURE OF THE GALAXY

Pere Cortés and Ferran Sala
Departament de Física de l'Atmosfera, Astronomia i Astrofísica
Universitat de Barcelona

RESUMEN. En el marco de las hipótesis de Chandrasekhar para la solución de la ecuación fundamental de la Dinámica Galáctica, se ha desarrollado una familia de modelos que determinan formas integrables de los potenciales de las galaxias. Se ha interpretado la distribución espacial de los cúmulos globulares en la Galaxia como una consecuencia dinámica de uno de los modelos estudiados.

ABSTRACT. In the frame of Chandrasekhar hypotheses for solving the fundamental equation of Galactic Dynamics, a family of models determining integrable forms of the potentials of galaxies has been developed. The space distribution of globular clusters in the Galaxy is interpreted as a dynamical consequence of one of the studied models.

Key words: GALAXY-DYNAMICS — GALAXY-STRUCTURE

I. INTRODUCTION

Wide regions of the stellar systems can be described by a distribution function $f(t,\mathbf{r},\mathbf{V})$ in phase space that gives the number of stars that at a time t have positions between $\mathbf{r}$ and $\mathbf{r} + d\mathbf{r}$ and velocities between $\mathbf{V}$ and $\mathbf{V} + d\mathbf{V}$. Chandrasekhar (1942) developed a theory for the study of the dynamics of the stellar systems based on three fundamental hypotheses:

1) At any point of the system it is possible to define a local standard of rest, whose velocity is $\mathbf{V}_0(t,\mathbf{r})$.

2) The distribution function f is an arbitrary function $f(Q + \sigma)$ where

$$Q = \mathbf{v}^t \cdot \mathbb{A} \cdot \mathbf{v}$$

is a quadratic form,

$$\mathbf{v} = \mathbf{V} - \mathbf{V}_0$$

is the peculiar velocity of a star, $\mathbb{A}(t,\mathbf{r})$ is a symmetric second order tensor and $\sigma(t,\mathbf{r})$ is a scalar function.

3) The motions of the stars are governed by a potential $U(t,\mathbf{r})$ per unit mass.

II. DYNAMICAL MODELS

The main features of the derivation of the solutions in the more general time depending case are found in Sala (1990). Particularly, when the stellar system is supposed to be independent of t and to have, at right angles, a plane (the galactic plane) and an axis of symmetry (the rotation axis), the form of the potencial U per unit mass can be determined. A detailed description of the process can be found in Chandrasekhar (1942) and Orús (1977). Two different cases are found:

1) The potential can be expressed in spherical coordinates (r,ϕ,θ):

$$U = U_1(r^2) + U_2(\theta)/r^2 \tag{2.1}$$

where U_1 and U_2 are arbitrary functions of their respective arguments.

2) The potential can be expressed in prolate spheroidal coordinates (λ,ϕ,ν):

$$U = \frac{F(\lambda) - F(\nu)}{\lambda - \nu} \tag{2.2}$$

where F is an arbitrary function of its different arguments.

In both cases the system has three independent isolating first integrals

$$\begin{aligned} I_1 &\equiv \Pi^2 + \Phi^2 + Z^2 + 2U = \text{constant} \\ I_2 &\equiv \varpi\Phi = \text{constant} \\ I_3 &\equiv (z\Pi - \varpi Z)^2 + z^2\Phi^2 - \frac{k_1-k_3}{k_4} Z^2 - \frac{2}{k_4}\left(k_1 U + \frac{1}{2}\chi\right) = \text{constant} \end{aligned} \tag{2.3}$$

where ϖ,ϕ,z are the cylindrical coordinates of a star, Π,Φ,Z are its physical cylindrical velocity components, k_1, k_3 and k_4 are constants, being $k_1 = k_3$ in the case 1) and determining the position of the foci of the coordinate surfaces (ϖ=0, $z=\pm((k_3-k_1)/k_4)^{1/2}$) in the case 2), and χ is a function that can be expressed

$$-\frac{1}{2}\chi = k_1 U_1(r^2) + \left(\frac{k_1}{r^2} + k_4\right)U_2(\theta) + \text{constant}$$

in the case 1), and:

$$-\frac{1}{2}\chi = k_4 \frac{\nu F(\lambda) - \lambda F(\nu)}{\lambda - \nu} + \text{constant}$$

in the case 2).

The motion of each local standard of rest is a circular rotation around the axis with velocity

$$\Phi_0 = \frac{-\beta\varpi}{k_1+k_2\varpi^2+k_4 z^2}$$

where β and k_2 are constants.

In both cases, the three first integrals (2.3) are in involution, so that the equations of motion are separable and they can be integrated as quadratures. The orbits of stars in the frame of Stäckel potentials, being (2.1) and (2.2) special cases of them, have been studied by de Zeeuw (1985). Bound angular momentum orbits are stable short axis tubes limited by coordinate surfaces.

On the other hand, the potential derived by Aguilar from the model of the Galaxy by Bahcall and Soneira (1980) has been recently approximated within a 3% error by an Eddington type potential of the form (2.2) (Dejonghe and de Zeeuw 1989). These authors have also found Eddington approximations of another derived galactic potentials.

III. THE DISTRIBUTION OF GLOBULAR CLUSTERS IN THE GALAXY

Showing globular clusters a strong concentration towards the centre of the Galaxy, they were used in the early determination of its position (Shapley 1930). Lists of globular clusters, containing positions and radial velocities among other data, have been published (Arp 1965, Woltjer 1975, Harris 1976, Zinn 1985).

Sasaki and Ishizawa (1978) in a study of the space distribution of globular clusters in Harris (1976) list found a cone of avoidance around the rotation axis of the Galaxy. These authors used this cone of avoidance, supposed to have been caused by tidal effects, to determine the distance to the galactic centre.

Being the globular clusters gravitationally bound to the Galaxy, their space distribution must show the fact that they are moving on short axis tubes. Any one of these short axis tubes is limited by two spheroids λ_{min} and λ_{max} and the two sheets of one hyperboloid of revolution ν_{max}, whose particular values can be determined from the first integrals (2.3) (Cortés and Sala 1990). If envelopes of the spheroids λ_{min} and the hyperboloids of revolution ν_{max} of the different tube orbits of the globular clusters of the system can be determined due to their similar kinematics, two galactocentric regions of avoidance will appear: a spheroid of avoidance λ_0, whose volume will increase with the average rotation of the system, and a hyperboloid of revolution of avoidance ν_1 whose volume will increase with the average flatness of the system, and that can be interpreted as a cone of avoidance because of its shape. In fact, both regions will be a sphere and a cone, respectively, if the galactic potential would have the form (2.1).

The heliocentric positions of the globular clusters in table 3 of Harris (1976) have been used to evaluate the galactocentric distance $\varpi_\odot$ of the Sun. With clusters at galactocentric distances $3 \leq r \leq 40$ *kpc*, trials for the determination of the spheroid λ_0 and the hyperboloid of revolution ν_1 of avoidance have been done for galactocentric distances $6.5 \leq \varpi_\odot \leq 10.5$ *kpc* in the direction of the galactic centre. A maximum λ_0 and a minimum ν_1 have been obtained for $\varpi_\odot = 8.9$ *kpc*. Taking this value as an estimation of $\varpi_\odot$, new spheroid $\lambda_0 = 3.080796$ and hyperboloid of revolution $\nu_1 = 0.678235$ of avoidance have been determined by considering now all the clusters with $r \leq 40$ *kpc*. The spheroid has obviously reduced its volume but the hyperboloid of revolution has remained the same. Nevertheless, two globular clusters are placed inside the zones of avoidance: NGC 6558 and NGC 6723. A plot of the ϖ, z coordinates of the clusters with $\varpi \leq 12$ *kpc* and $|z| \leq 4$ *kpc* showing the meridional sections of the spheroid λ_0 and the hyperboloid of revolution ν_1 of avoidance is given in figure 1. The clusters NGC 6558 and NGC 6723 should be placed inside the avoidance zones mainly due to their rotation lower than the average rotation of the rest of the system of globular clusters.

FIGURE 1. Space distribution of the clusters in Harris (1976) list with $\varpi \leq 12$ *kpc* and $|z| \leq 4$ *kpc*. The Sun is shown as a dotted circle at $\varpi_\odot = 8.9$ *kpc*. The meridional sections of the zones of avoidance are marked as solid curves, with the clusters NGC 6558 (+) and NGC 6723 (*) inside.

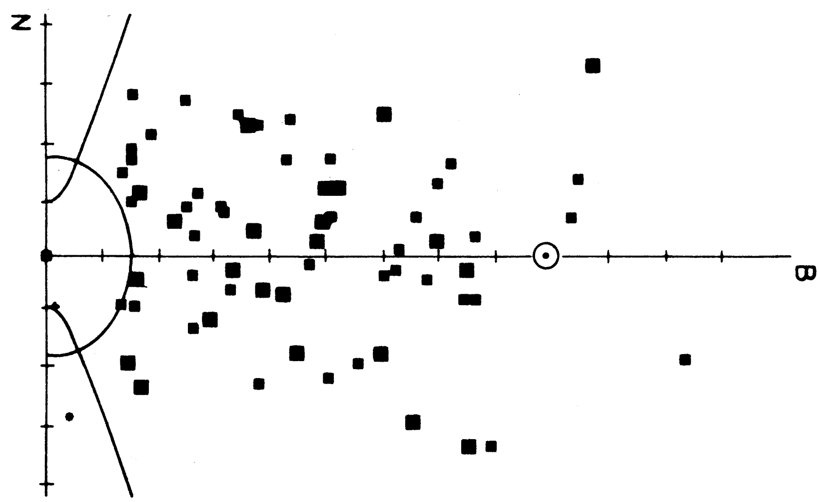
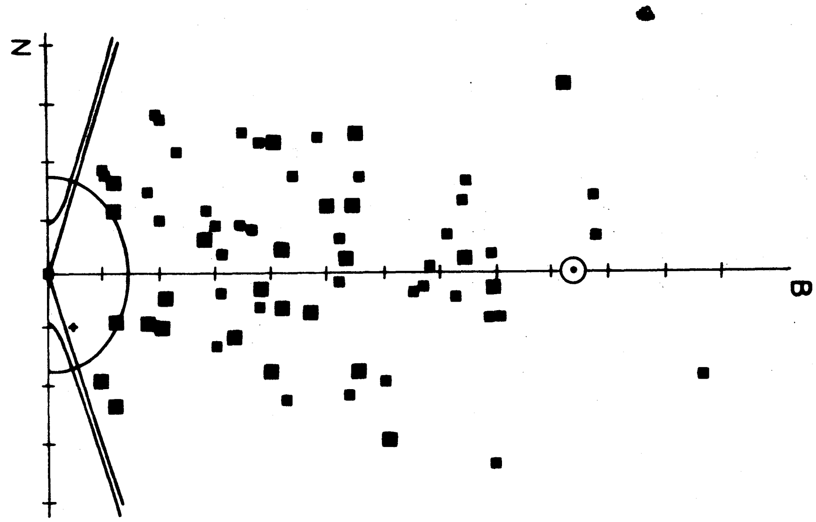


Figure 2 shows the ϖ, z distribution of the clusters with $\varpi \leq 12$ *kpc* and $|z| \leq 4$ *kpc* when $\varpi_\odot = 9.4$ *kpc* (Sasaki and Ishizawa 1978) is adopted. The meridional sections of a spheroid $\lambda_0 = 2.941128$ and a hyperboloid of revolution $\nu_1 = 0.705796$ obtained in the same way as described before are shown, being the cluster NGC 6558 placed inside the zones of avoidance. The asymptotes of the section of the hyperboloid are also shown and they determine the cone of avoidance given by Sasaki and Ishizawa (1978).

IV. SUMMARY AND CONCLUSIONS

The galactic potential has been recently approximated by an Eddington type potential, one of the axisymmetric stationary solutions found

FIGURE 2. Space distribution of the clusters in Harris (1976) list with $\varpi \leq 12$ *kpc* and $|z| \leq 4$ *kpc*. The Sun is shown as a dotted circle at $\varpi_{\odot} = 9.4$ *kpc* (Sasaki and Ishizawa 1978). The meridional sections of the zones of avoidance are marked as solid curves, with the cluster NGC 6558 (+) inside.



in the Chandrasekhar theory for the galactic potential. All of these solutions have three first integrals in involution so that the orbits can be integrated as quadratures, the bound angular momentum orbits being short axis tubes. The space distribution of globular clusters is described as a consequence of their orbits in the Galaxy. Two regions of avoidance, a galactocentric spheroid and a galactocentric hyperboloid of revolution, the latter previously identified as a cone, are likely to be found. These two regions are determined although two globular clusters are found inside them due to their special kinematics. The obtained results are finally compared with the cone of avoidance determined by Sasaki and Ishizawa (1978).

Acknowledgements: The authors wish to thank Dr. M. Sanromà for his collaboration in making possible this contribution to the VI Latin American Regional Meeting of the International Astronomical Union, and Ms. N. Cutillas for her advisoring with the english version of this paper.

REFERENCES

Arp, H.C. 1965, in *Stars and Stellar Systems*, vol. **5**, ed. A. Blaauw and M. Schmidt (Chicago: University of Chicago Press), p. 401
Bahcall, J.N. and Soneira, R.M. 1980, *Astrophys. J. Suppl.* **44**, 73
Chandrasekhar, S. 1942, *Principles of Stellar Dynamics* (Chicago: University of Chicago Press)
Cortés, P. and Sala, F. 1990, *Rev. Mex. Astron. Astrof.* this volume
Dejonghe, H. and de Zeeuw, T. 1989, *Astrophys. J.* **329**, 720
Harris, W.E. 1976, *Astron. J.* **81**, 1095
Orús, J.J. de 1977, *Apuntes de Dinámica Estelar* (Barcelona: University of Barcelona)
Sala F. 1990, *Rev. Mex. Astron. Astrof.* this volume
Sasaki T. and Ishizawa, T. 1978, *Astron. Astrophys.* **69**, 381
Shapley, H. 1930, *Star Clusters* (New York: McGraw-Hill)
Woltjer, L. 1975, *Astron. Astrophys.* **42**, 109
de Zeeuw, T. 1985, *Monthly Notices Roy. Astron. Soc.* **216**, 273
Zinn, R. 1985, *Astrophys. J.* **293**, 424

Pere Cortés and Ferran Sala: Departament de Física de l'Atmosfera, Astronomia i Astrofísica, Universitat de Barcelona, Av. Diagonal, 647, 08028 Barcelona, Spain.