1990RMxAA..21..259C

ev. Mexicana Astron. Astrof., **21**, 259-263 (1990)

ORBITS OF STARS IN A MODEL WITH THREE FIRST INTEGRALS

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RESUMEN. Se aplica una forma integrable del potencial de la Galaxia para describir la variación de los pericentros y apocentros de las órbitas de las estrellas de acuerdo con sus velocidades en el entorno solar. Como ejemplo, se presenta un estudio más detallado de la órbita del Sol.

A8STRACT. An integrable form of the potential of the Galaxy is applied to describe the variation of the pericentres and the apocentres of the orbits of stars in accordance with their velocities in the solar neighbourhood. As an example, a more detailed study of the orbit of the Sun is presented.

Key words: DYNAMICS -- GALAXY-STRUCTURE

. INTRODUCTION

Chandrasekhar (1942) developed a theory for the study of the ynamics of the stellar systems. Among its main consequences, stellar systems re described as integrable systems: the different forms of the galactic otential U per unit mass have sets of first integrals that are in involution. short description of the formulation of the prohlem and the derivation of tationary axisymmetric solutions is given in Cortes and Sala (1990) and a ore detailed description of time depending axisymmetric solutions in Sala 1990).

Dejonghe and de Zeeuw (1989a) have approximated within a 3% error the otential derived by Aguilar from the model of the Galaxy by Bahcall and oneira (1980) by an Eddington type potential

here λ,ϕ,ν are prolate spheroidal coordinates, being the foci of the pheroids are the hyperboloids of revolution placed on the z- axis at z = $(\gamma - \alpha)^{1/2}$ = ± 0.88 kpc with γ = - 10^{-6} and where F can be found in Dejonghe nd de Zeeuw (1989a).

I. THE ORBITS OF STARS

Having the galactic potential U the Eddington form (1.1) the orbits f stars have the three first integrals

$$I_1 \equiv \Pi^2 + \Phi^2 + Z^2 + 2\,\frac{F(\lambda) - F(\nu)}{\lambda - \nu} = \text{constant}$$
$$I_2 \equiv \varpi\Phi = \text{constant} \qquad (2.1)$$
$$I_3 \equiv (z\Pi - \varpi Z)^2 + z^2\Phi^2 + (\gamma - \alpha)Z^2 + \frac{\nu F(\lambda) - \lambda F(\nu)}{\lambda - \nu} = \text{constant}$$

here ϖ,ϕ,z are the cylindrical coordinates and Π,Φ,Z the physical cylindrical elocity components of a star.

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1990RMxAA..21..259C

The first integrals (2.1) are in involution, so that, if they are replaced by

$$H = \frac{I_1}{2} = \frac{\rho_\lambda^2}{2P^2} + \frac{\rho_\phi^2}{2\varpi^2} + \frac{\rho_\nu^2}{2R^2} + \frac{F(\lambda) - F(\nu)}{\lambda - \nu} = \text{constant}$$

$$I = I_2 = \rho_\phi = \text{constant}$$

$$J = \frac{I_3 + I_2^2 - \gamma I_1}{2} = \frac{\nu\rho_\lambda^2}{2P^2} + \frac{(\lambda+\nu+\alpha)\rho_\phi^2}{2\varpi^2} + \frac{\lambda\rho_\nu^2}{2R^2} + \frac{\nu F(\lambda) - \lambda F(\nu)}{\lambda - \nu} = \text{constant}$$

where $\rho_\lambda = P^2\lambda'$, $\rho_\phi = \varpi^2\phi'$, $\rho_\nu = R^2\nu'$ are the momenta conjugate to the spheroidal coordinates λ,ϕ,z, being its time derivatives denoted by λ', ϕ', ν' and being

$$P^2 = \frac{\lambda - \nu}{4(\lambda+\alpha)(\lambda+\gamma)} \qquad R^2 = \frac{\nu - \lambda}{4(\nu+\alpha)(\nu+\gamma)}$$

the equations of the motion of a star in a meridional plane can be expressed

$$\lambda' = \left[\frac{8(\lambda+\alpha)(\lambda+\gamma)}{(\lambda - \nu)^2}\left(\lambda H + \frac{\alpha - \gamma}{2(\lambda+\alpha)} I^2 - J - F(\lambda)\right)\right]^{1/2}$$

$$\nu' = \left[\frac{8(\nu+\alpha)(\nu+\gamma)}{(\nu - \lambda)^2}\left(\nu H + \frac{\alpha - \gamma}{2(\nu+\alpha)} I^2 - J - F(\nu)\right)\right]^{1/2} \qquad (2.2)$$

so that they are separable and can be integrated as quadratures (Lynden-Bell 1962). Finally, the motion around the rotation axis will be given by integrating $\phi' = 1/\varpi^2$ where $\varpi^2 = (\lambda+\alpha)(\nu+\alpha)/(\alpha-\gamma)$ has been obtained from the integration of (2.2). The obtained orbits with non-zero angular momentum are stable short axis tubes limited by spheroids and hyperboloids of revolution (Dejonghe and de Zeeuw 1989b) being both direct and retrograde orbits possible. These limiting coordinate surfaces are derived from the equations $\lambda' = 0$ and $\nu' = 0$ obtained from (2.2) so that a meridional section of a short axis tube is determined by the conditions $\lambda_{min} \le \lambda \le \lambda_{max}$ and $-\gamma \le \nu \le \nu_{max}$. A more graphical representation of the tube is obtained by giving the coordinates ϖ_{min} and ϖ_{max} of the points where λ_{min} and λ_{max} respectively intersect the ϖ-axis and the coordinates (ϖ_1,z_1) and (ϖ_2,z_2) where λ_{min} and

TABLE 1. Coordinates $(\varpi_{min},0)$, $(\varpi_{max},0)$, (ϖ_1,z_1) and (ϖ_2,z_2) in *kpc* for a tube orbit with $Z = 10$ *kms*$^{-1}$

kms^{-1} Π \ $\Phi-\Phi_0$	-80	-60	-40	-20	0	20	40
0	4.01, 0.00	4.92, 0.00	5.91, 0.00	7.03, 0.00	8.32, 0.00	8.50, 0.00	8.50, 0.00
	8.50, 0.00	8.50, 0.00	8.50, 0.00	8.50, 0.00	8.50, 0.00	9.86, 0.00	11.77, 0.00
	4.01, 0.22	4.91, 0.25	5.90, 0.28	7.02, 0.30	8.31, 0.33	8.49, 0.32	8.49, 0.30
	8.49, 0.46	8.49, 0.43	8.49, 0.39	8.49, 0.37	8.49, 0.34	9.85, 0.37	11.77, 0.41
30	3.97, 0.00	4.85, 0.00	5.80, 0.00	6.79, 0.00	7.69, 0.00	8.15, 0.00	8.31, 0.00
	8.63, 0.00	8.66, 0.00	8.71, 0.00	8.85, 0.00	9.26, 0.00	10.36, 0.00	12.15, 0.00
	3.97, 0.22	4.85, 0.25	5.79, 0.27	6.78, 0.29	7.68, 0.31	8.14, 0.31	8.30, 0.29
	8.61, 0.47	8.65, 0.44	8.71, 0.40	8.84, 0.38	9.25, 0.37	10.35, 0.39	12.14, 0.42
60	3.86, 0.00	4.68, 0.00	5.53, 0.00	6.35, 0.00	7.07, 0.00	7.58, 0.00	7.88, 0.00
	9.00, 0.00	9.11, 0.00	9.30, 0.00	9.63, 0.00	10.26, 0.00	11.39, 0.00	13.14, 0.00
	3.86, 0.22	4.68, 0.24	5.52, 0.26	6.35, 0.27	7.07, 0.28	7.57, 0.28	7.87, 0.28
	8.99, 0.49	9.10, 0.46	9.29, 0.43	9.62, 0.41	10.26, 0.41	11.38, 0.43	13.13, 0.46

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1990RMxAA..21..259C

TABLE 2. Coordinates $(\varpi_{min},0)$, $(\varpi_{max},0)$, (ϖ_1,z_1) and (ϖ_2,z_2) in *kpc* for a tube orbit with Z = 50 kms^{-1}

kms^{-1} $\Pi \backslash \Phi-\Phi_0$	-80	-60	-40	-20	0	20	40
0	4.36, 0.00	5.26, 0.00	6.25, 0.00	7.38, 0.00	8.50, 0.00	8.50, 0.00	8.50, 0.00
	8.50, 0.00	8.50, 0.00	8.50, 0.00	8.50, 0.00	8.70, 0.00	10.29, 0.00	12.28, 0.00
	4.19, 1.26	5.08, 1.37	6.08, 1.49	7.21, 1.61	8.33, 1.72	8.35, 1.60	8.37, 1.49
	8.15, 2.42	8.21, 2.20	8.26, 2.01	8.30, 1.85	8.52, 1.76	10.11, 1.93	12.09, 2.15
30	4.32, 0.00	5.18, 0.00	6.11, 0.00	7.08, 0.00	7.86, 0.00	8.20, 0.00	8.33, 0.00
	8.64, 0.00	8.67, 0.00	8.75, 0.00	8.92, 0.00	9.47, 0.00	10.74, 0.00	12.64, 0.00
	4.14, 1.25	5.01, 1.35	5.94, 1.45	6.91, 1.55	7.70, 1.59	8.06, 1.54	8.20, 1.46
	8.29, 2.46	8.38, 2.24	8.50, 2.07	8.71, 1.94	9.28, 1.91	10.55, 2.02	12.45, 2.22
60	4.19, 0.00	4.98, 0.00	5.80, 0.00	6.58, 0.00	7.23, 0.00	7.67, 0.00	7.93, 0.00
	9.04, 0.00	9.17, 0.00	9.38, 0.00	9.78, 0.00	10.51, 0.00	11.76, 0.00	13.64, 0.00
	4.02, 1.21	4.81, 1.30	5.63, 1.38	6.42, 1.44	7.08, 1.46	7.53, 1.44	7.81, 1.39
	8.67, 2.57	8.86, 2.37	9.12, 2.22	9.54, 2.13	10.30, 2.12	11.55, 2.21	13.43, 2.39

ıax respectively intersect the sheet ν_{max} in the region $z \geq 0$. They are given ɔr peculiar velocity components $|\Pi|$, $\Phi - \Phi_0$, adopted Φ_0 = 220 kms^{-1}, at a ɔsition ϖ = 8.5 *kpc* on the B-axis for Z = 10 kms^{-1} and Z = 50 kms^{-1} in tables and 2 respectively.

[I. THE ORBIT OF THE SUN

A more detailed study has been performed of the orbit of the Sun ꞉ound the galactic rotation axis. A position $\omega_\odot$ = 8.5 *kpc* , $z_\odot$ = 0.007 *kpc*

ıd peculiar velocity components Π = - 9 kms^{-1}. $\Phi - \Phi_0$ =12 kms^{-1}, Z = 7 kms^{-1})elhaye 1965) have been adopted for the Sun. Table 3 gives the values of the ırst integrals I_1. I_2 and I_3, the limiting coordinate surfaces λ_{min}, λ_{max} and

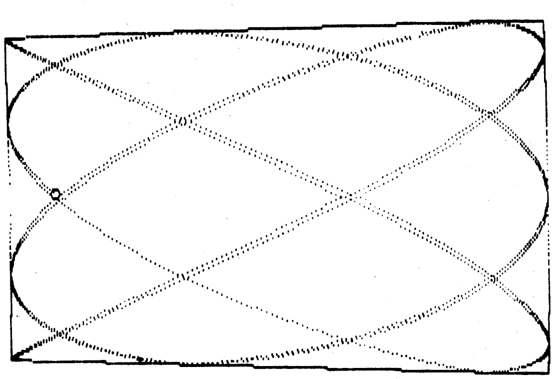

FIGURE 1. Projection of the orbit of the Sun on a meridional plane

TABLE 3. Values of the first integrals and the limiting coordinates of the orbit of the Sun

I_1 = - 141365.19 km^2s^{-2}
I_2 = 1972.00 $kms^{-1}kpc$
I_3 = 3590.76 $km^2s^{-2}kpc^2$

λ_{min} = 71.796 kpc^2
λ_{max} = 86.903 kpc^2

ν_{max} = 0.000557 kpc^2

	kpc	*kpc*
$(\varpi_{min},0)$ =	(8.43,	0.00)
$(\varpi_{max},0)$ =	(9.28,	0.00)
(ϖ_1,z_1) =	(8.42,	0.23)
(ϖ_2,z_2) =	(9.28,	0.25)

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TABLE 4. Position and velocity of the Sun along its orbit

t 10^6 years	ϖ kpc	ϕ degrees	z kpc	Π	Φ kms^{-1}	Z	λ_{min}	λ_{max}	z=0	ν_{max}	revol.
0.00	8.50	0.00	0.007	-9.0	232.00	7.00					
14.71	8.43	23.63	0.11	0.3	234.01	6.25	*				
45.43	8.70	72.52	0.23	14.7	226.79	-0.45				*	
97.63	9.28	146.15	-0.00	0.5	212.49	-6.43			*		
98.20	9.28	146.92	-0.00	-0.2	212.49	-6.43		*			
150.34	8.71	220.30	-0.23	-14.9	226.38	1.23				*	
181.72	8.43	270.17	-0.10	0.4	234.01	6.29	*				
196.46	8.50	294.02	0.00	8.9	232.04	7.01			*		
240.50	9.12	360.00	0.24	11.8	216.33	2.15					*
246.60	9.18	368.29	0.25	9.5	214.78	-0.54				*	
265.21	9.28	393.44	0.20	-0.1	212.54	-3.71		*			
300.42	8.98	441.82	-0.00	-14.7	219.63	-6.64			*		
348.35	8.42	516.07	-0.23	-0.6	234.07	0.88				*	
348.74	8.42	516.70	-0.23	0.4	234.08	0.94	*				
395.76	8.95	589.77	0.00	15.0	220.25	6.66			*		
432.21	9.28	639.94	0.21	-0.2	212.54	3.60		*			
449.98	9.19	663.95	0.25	-9.0	214.52	-1.03				*	
488.60	8.65	720.02	0.08	-13.9	228.00	-6.60					*
499.86	8.51	737.55	-0.00	-9.6	231.96	-7.00			*		
515.73	8.43	763.20	-0.11	0.3	234.01	-6.24	*				
546.32	8.69	811.89	-0.23	14.7	226.84	0.45				*	
598.48	9.28	885.49	0.00	0.6	212.49	6.43			*		
599.21	9.28	886.46	0.00	-0.2	212.49	6.43		*			
651.27	8.71	959.74	0.23	-14.9	226.35	-1.23				*	
682.73	8.43	1009.72	0.10	0.4	234.01	-6.30	*				
697.36	8.50	1033.39	-0.00	8.9	232.06	-7.01			*		
727.80	8.92	1080.10	-0.19	15.1	221.00	-4.44					*

ν_{max} and the intersections $(\varpi_{min},0)$, $(\varpi_{max},0)$, (ϖ_1,z_1) and (ϖ_2,z_2) of the orbit of the Sun. Figure 1 shows the projection of the orbit of the Sun on a meridional plane, being this projection a quasi-periodic motion. Table 4 shows the changes of the position and the velocity of the Sun with time, indicating the consecutive crossings with the coordinate surfaces in its motion around the rotation axis of the Galaxy.

IV. SUMMARY

An Eddington type approximation of the galactic potential, with three first integrals in involution, has been used to determine the major properties of the orbits of stars in the Galaxy. The sizes of the tubes have been determined for different ranges of peculiar stellar velocities at the solar neighbourhood. Finally, a more detailed study of the orbit of the Sun has been performed.

The authors wish to thank Dr. M. Sanromá for his collaboration in making possible this contribution to the VI Latin American Regional Meeting of the International Astronomical Union, and Ms. N. Cutillas for her advisoring with the english version of this paper.

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