

THE METHOD OF EICHHORN WITH NON-STANDARD PROJECTIONS
FOR A SINGLE PLATE

O. Cardona¹ and M. Corona-Galindo^{1,2,*}

¹Instituto Nacional de Astrofísica, Óptica y Electrónica
²Centro de Investigación de Física, Universidad de Sonora.

RESUMEN. Se desarrollan las expresiones para el método de Eichhorn en astrometría para proyecciones diferentes a la estándar. El método se usa para obtener las coordenadas esféricas de estrellas en placas astronómicas cuando las variables contienen errores.

ABSTRACT. We develop the expressions for the Eichhorn's Method in astrometry for non-standard projections. The method is used to obtain spherical coordinates of stars in astronomical plates, when all the variables have errors.

Key words: ASTROMETRY

I. INTRODUCTION.

Eichhorn (1985) has developed a method in focal plane astrometry to obtain the spherical coordinates of stars, that uses the standard coordinates (Tan Projection) as auxiliary quantities only. A great advantage of this method over the older methods is that the positions of the reference stars are considered as observations having errors.

For work with Schmidt telescopes or in radio astronomy one needs to consider other types of projection as are the SIN and ARC projections (Greisen, 1984). Therefore, we have developed the necessary formulae for the method of Eichhorn for these two projections.

II CONDITION EQUATIONS

We consider that there are m reference stars with position estimates $((\alpha_{\nu C}, \delta_{\nu C}), \nu=1, \dots, m)$ from a catalogue, with variances $\rho_{\nu}/\cos^2 \delta_{\nu}$ and σ_{ν} respectively (we are following the notation of Eichhorn, 1985). The measured coordinates on the plate for the ν th star are X_{ν} and Y_{ν} with variances ν_{ν} and ϕ_{ν} respectively. The dimensionless coordinates (SIN or ARC) of the stars written in units of the focal length, are ζ_{ν} and H_{ν} . For each ν the measured and dimensionless coordinates are given by the model

*On sabbatical leave from INAOE

$$\begin{pmatrix} X_\nu \\ Y_\nu \end{pmatrix} = s \begin{pmatrix} \xi_\nu \\ \eta_\nu \end{pmatrix} + \Xi_\nu \underline{a} \quad (1)$$

where \underline{a} is the vector of plate constants, Ξ is the model matrix and s the focal length of the telescope.

The measured positions X_ν, Y_ν and the estimated coordinates $(\alpha_{\nu C}, \delta_{\nu C})$ produce the condition equation $H^T = (F^T, G^T)$ where F and G are defined by Eichhorn 1985 as his equations (2) and (3), with covariance matrix

$$\sigma = \text{diag} (v_1, \phi_1, \dots, v_m, \phi_m; \bar{\rho}_1, \bar{\sigma}_1, \dots, \bar{\rho}_m, \bar{\sigma}_m) \quad (2)$$

with

$$\sigma_{x\nu} = \begin{pmatrix} v_\nu & 0 \\ 0 & \phi_\nu \end{pmatrix} \quad \text{and} \quad \bar{\sigma}_{a\nu} = \begin{pmatrix} \rho_\nu & 0 \\ 0 & \bar{\sigma}_\nu \end{pmatrix} \quad (3)$$

and the residual vector is given by $H_0^T = (d^T, 0)$. The adjustment parameters are the plate constants \underline{a} and the correction to the positions.

$$\beta^T = (\cos \delta_1 d\alpha_1, d\delta_1, \dots, \cos \delta_m d\alpha_m, d\delta_m)$$

III. NORMAL EQUATIONS

Eichhorn has derived the solution to the normal equations for the corrections as

$$\underline{a} = \left(\sum_{\nu=1}^m \Xi_\nu^T J_\nu \Xi_\nu^{-1} \right)^{-1} \sum_{\nu=1}^m \Xi_\nu^T J_\nu d_\nu \quad (4)$$

and

$$\beta = \left(s^2 B \sigma_x^{-1} B + \sigma_a^{-1} \right)^{-1} s B^T \sigma^{-1} \left(d - \Xi \underline{a} \right) \quad (5)$$

where the weight matrix is given by

$$J_\nu = \begin{pmatrix} \sigma_x + s^2 B & \bar{\sigma}_a^{-1} B^T \\ \bar{\sigma}_a^{-1} B^T & \bar{\sigma}_a^{-1} \end{pmatrix} \quad (6)$$

and B is the blockdiagonal matrix whose 2×2 elementary block is

$$B_\nu = \begin{pmatrix} \partial \xi_\nu, \eta_\nu \\ \partial \alpha_\nu, \delta_\nu \end{pmatrix} \begin{pmatrix} 1/\cos \delta_\nu & 0 \\ 0 & 1 \end{pmatrix} \quad (7)$$

IV. COMPUTATION OF THE MATRIX B.

Our problem then reduces to the computation of the matrix B in order to obtain the weight matrix. To do that, we start considering the orthogonal matrix

$$R_1^{-\delta_0} R_2^{\alpha-\alpha_0} R_1^{\delta} = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \quad (8)$$

where (α_0, δ_0) are the spherical coordinates of the center of the plate. We evaluate B first for the SIN projection and then for the ARC one.

a) For the SIN projection the transformation equations are

$$\begin{aligned} \xi &= \cos \delta \sin (\alpha - \alpha_0) \\ \eta &= \sin \delta \cos \delta_0 - \cos \delta \sin \delta_0 \cos(\alpha - \alpha_0) \end{aligned} \quad (9)$$

from equation (8) we have $\xi = -a_3$ and $\eta = b_3$, using the orthogonality of the matrix we obtain that

$$B = \begin{pmatrix} a_1 & -a_2 \\ -b_1 & b_2 \end{pmatrix} \quad (10)$$

and from the inverse transformations

$$\begin{aligned} \sin \delta &= \eta \cos \delta_0 + \sin \delta_0 \sqrt{1 - \xi^2 - \eta^2} \\ \tan \alpha - \alpha_0 &= \frac{\xi}{\cos \delta_0 \sqrt{1 - \xi^2 - \eta^2} - \eta \cos \delta_0} \end{aligned} \quad (11)$$

we obtain

$$\begin{aligned} a_1 &= \cos \delta_0 \sqrt{1 - \xi^2 - \eta^2} - \eta \sin \delta_0 / T = W \\ a_2 &= \xi \eta \cos \delta_0 + \sin \delta_0 + \sin \delta_0 \sqrt{1 - \xi^2 - \eta^2} / T = S \\ b_1 &= -\xi \sin \delta_0 / T = U \\ b_2 &= \xi^2 \cos \delta_0 / T + W = V \end{aligned} \quad (12)$$

where

$$T^2 = \xi^2 + (\cos \delta_0 + [1 - \xi^2 - \eta^2 - \eta \sin \delta_0]^{1/2}) \quad (13)$$

and the determinant

$$R^3 = SU + WV = 1 \quad (14)$$

therefore

$$B = \begin{pmatrix} W & -S \\ U & V \end{pmatrix} \quad (15)$$

is of the same form as equation (32) of Eichhorn, and from here one can use the formalism of Eichhorn.

b) For the ARC projection the transformation equations are

$$\begin{aligned}\xi &= -\frac{\theta}{\sin\theta} \cos\delta \sin(\alpha-\alpha_0) \\ \eta &= \frac{\theta}{\sin\theta} (\cos\delta_0 \sin\delta - \sin\delta_0 \cos\delta (\alpha-\alpha_0))\end{aligned}\quad (16)$$

from equation (8) we have $\xi = -\left(\frac{\theta}{\sin\theta}\right)a_3$, $\eta = \left(\frac{\theta}{\sin\theta}\right)b_3$.

Using the same procedure as with the SIN projection one obtains for the elements of the B matrix, in terms of the orthogonal matrix (8), the following

$$\begin{aligned}W &= -\frac{1}{\sin^2\theta} \left(c_1 a_3 + \frac{\theta}{\sin\theta} c_2 b_3 \right), \\ V &= -\frac{1}{\sin^2\theta} \left(c_2 b_3 + \frac{\theta}{\sin\theta} c_1 a_3 \right), \\ S &= \frac{1}{\sin^2\theta} \left(\frac{\theta}{\sin\theta} c_1 b_3 - c_2 a_3 \right), \\ U &= \frac{1}{\sin^2\theta} \left(c_1 b_3 - \frac{\theta}{\sin\theta} c_3 a_3 \right); \end{aligned}\quad (17)$$

and the determinant of B is given as

$$R^3 = SU + WV = \frac{\theta}{\sin\theta} \quad (18)$$

The expression in (17) can be evaluated using the inverse transformations. The expression in (17) can be evaluated using the inverse transformations.

$$\begin{aligned}|\theta|^2 &= \xi^2 + \eta^2 \\ \sin\delta &= \left(\frac{\sin\theta}{\theta}\right) \eta \cos\delta_0 + \sin\delta_0 \cos\theta \\ \sin\alpha-\alpha_0 &= \left(\frac{\sin\theta}{\theta}\right) \frac{\xi}{\cos\delta}\end{aligned}\quad (19)$$

and the elements of the orthogonal matrix (8) to produce

$$\begin{aligned}a_3 &= -\left(\frac{\sin\theta}{\theta}\right) \xi, \quad b_3 = \left(\frac{\sin\theta}{\theta}\right) \eta, \quad c_2 = \cos\theta \\ c_1 &= \cos\delta_0 \left[\left(\frac{\sin\theta}{\theta}\right) \eta \cos\delta_0 + \sin\delta_0 \cos\theta \right]\end{aligned}\quad (20)$$

$$c_2 = \left[\sin\delta_0 - \left[\left(\frac{\sin\theta}{\theta}\right) \eta \cos\delta_0 + \sin\delta_0 \cos\theta \right] \right] / \sqrt{1 - \left[\left(\frac{\sin\theta}{\theta}\right) \eta \cos\delta_0 + \sin\delta_0 \cos\theta \right]^2}$$

and from here again one can follow the procedure of Eichhorn.

V. CONCLUSIONS

In this work we have shown that in order to apply the method of Eichhorn in Astrometry for the SIN and ARC projections it was only necessary to change the B matrix contained in the weight matrix. The equations for the required changes were derived.

BIBLIOGRAPHY

- Eichhorn, H., 1974, *Astronomy of Star Positions*, Ungar, New York.
Eichhorn, H., 1985, *Astronomy Astrophys.* 150, 251.
Greisen, E.W., 1984, *Astronomical Image Processing*, Circular No. 10, Space Telescope Science Institute, p. 45 February.

Octavio Cardona: INAOE, Apartado Postal 216 y 51, Puebla, Pue. 7200, Mexico.
Manuel Corona-Galindo: Centro de Investigacion en Fisica, Univ. de Sonora, Hermosillo, Sonora, Mexico.