STRUCTURE AND EVOLUTION OF WHITE DWARFS

(Invited Talk)

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RESUMEN. Presentamos una breve revisión de las propiedades de enanas blancas aisladas y se describe como el estudio de las pulsaciones observadas en ellas nos dan información sobre su estructura estelar y su evolución.

ABSTRACT: We give a brief review of single white dwarf properties and describe how the study of the pulsations observed in them give us information on stellar structure and evolution.

Key words: STARS-EVOLUTION – STARS-INTERIORS – STARS-WHITE DWARF

INTRODUCTION

It is important to study the white dwarfs because 80% of all stars evolve to become white dwarfs, so they ve all the history of stellar formation and evolution imprinted on them.

Our work has been primarily the study of the single white dwarfs by means of their pulsations, since h the pulsations we can measure: their total masses; their surface layer masses; their rotation period; their gentic field; even their core composition; and we can get information on: the equation of state of highly condensed matter, including neutrino emission processes; the conductivity opacities; diffusion of matter in the presence of a strong gravitational field; the stellar history through their stratification; and we can even calibrate convection theory.

Most importantly, these objects are the oldest stars in our galaxy, and the compact pulsators provide ans of measuring their ages, giving an independent and accurate determination of the age of the disk of our galaxy, what we mean the time elapsed since the start of star formation in the disk of our galaxy (Winget et al. 1987).

But how can we get all this information from simple light variations? The light curve of these pulsating dwarfs is so rich that they provide many independent constraints on the underlying structure of the stars.

Before we proceed to the pulsators, let us first review the basic properties of white dwarfs and pre-white arf stars.

SINGLE WHITE DWARFS

The best evidences suggest that the single white dwarfs are formed from stars with original masses ween 1 and 8 or 9 M\(_{\odot}\). About 95-98% of all white dwarfs come through the planetary nebulae phase. The other few cent come from the hot subdwarf stars, like those that dominate the Palomar Green (PG) survey.

These compact objects span the entire range of the H-R diagram, from luminosities of \(3 \geq \log L/L_\odot \geq .5\), or effective temperatures of 150 000K \(\geq T_{\text{eff}} \geq 3 700\)K. In spite of their diverse origin and luminosities, they are a narkable homogeneous class of objects. After the initial collapse from the hot planetary nebulae nuclei (PNN) phase, stars not in binaries all have \(\log g \approx 8\), giving a very narrow mass distribution clustered around 0.6 M\(_{\odot}\). Figure shows the mass distribution obtained by Weidemann and Koester for different types of white dwarfs (Weidemann and Yuan 1989). It is reassuring that the mass distribution obtained for the PNN by Schonberger and Weidemann (1987) are also clustered around 0.6 M\(_{\odot}\) for planetary nebulae in our galaxy and in a Magellanic Cloud as nuclei evolve so fast that their number might be underestimated.
The masses obtained by Weidemann and Koster, as well as those obtained from the gravitational redshift by Greenstein and Trumper (1967), Koester (1987) and Wegner (see 1989 review), are all statistical, since they get only the mass/radius ratio, and must use a mass/radius relation (Hamada and Salpeter 1961) to get their masses.

There is only a handful of white dwarfs with masses derived astrometrically: Sirius B: $M = 1.053 \pm 0.028$ (Gatewood and Gatewood 1978); 40 Eri B (triple system): $M = 0.42 \pm 0.02$ (Popper 1954); Procyon B: $M = 0.62$ (Popper 1980); Stein 2051B: $M = 0.50 \pm 0.05$ (Strand and Kallrakal 1989)- but assume companion mass; L870- (DA+DA white dwarf pair with $P_{orb} = 2.5$, d): $M = 0.41$ and $0.46 \pm 0.1$ (Saffer et al. 1988); and other pre-cataclysmic variables have white dwarfs with masses of $M = 0.5M_\odot$, like G107-70 (Harrington et al. 1981).

In terms of composition, the white dwarfs come in two major flavors: those with essentially pure hydrogen atmospheres (DAs) constitute about 80% of all white dwarfs; and those with essentially pure helium atmosphere (DOs when hot and DBs when cool) comprise about 20% of all white dwarfs (Sion et al. 1983). They is a small percentage of DBAs (He dominated atmospheres with traces of H), DCs (continuum only - cool), DOs (carbon - cool, probably He dominated, and the carbon is probably due to the deepening of the He surface convection down to the carbon tail), and DZs (some metal lines, specially Ca - again cool, and the metals probably accreted from the interstellar medium). The ratio of DBs to DAs is however temperature sensitive (Sion 1984), since there are no DAs hotter than $T_{eff} = 80000K$ or cooler than 5 500K, and there are no DBs between 45 000K and 30 000K, indicating the white dwarfs change their spectroscopic type during evolution, probably due to the interplay of diffusion and envelope evaporation (Fontaine and Wesenael 1987).

It is again interesting that the PNN also divide into about 70% hydrogen rich and 30% helium rich, which might indicate a separate channel for DAs and DBs, although again the variable ratio of DAs to DBs with effective temperature presents problems to this interpretation (Shipman 1989).

Models describing the evolution before planetary nebulae phase insists that these objects have C/O core with either a surface hydrogen layer of $10^{-4}M_\odot$, on top of a helium layer of $10^{-2}M_\odot$, or a pure He surface layer depending if the ejection of the planetary nebulae occurred during quiescent hydrogen burning, i.e., on an interpulse phase, or at a shell flash on the asymptotic giant branch (e.g. Iben and McDonald 1985). For example, R. Mendez has classified the planetary nebulae Abell 30 and Abell 78 as having pure He surface layers (Mendez 1987), indicating planetary nebulae ejection is a shell flash. To match the timescales of evolution across the planetary nebulae phase, the models generally require that the PNN have either a H shell burning, or a He shell burning (Schonberrer 1987).

When the stars contract to become white dwarfs, the surface layers are rendered chemically pure by the effect of gravitational settling, due to the strong gravitational field. The timescales for diffusion are of the order of few thousand years, very short compared with the evolutionary timescale (Vauclair 1989; Pelletier et al. 1989). For these models, the only limits on the masses of the surface layers from the observations are that they should be large than $10^{-16}M_\odot$ to be optically thick, and smaller than the previous evolutionary phases produced.

The hottest DA stars observed have effective temperatures of 80 000K, while the hottest non-DA stars have considerably higher temperatures, with the extreme being 11504+65, with $T_{eff} = 160000K$, and belong to the PG1159-035 spectroscopic class (Nousek et al. 1986). To explain the observed gap of the DBs from effective temperatures of 45 000K to 30 000K as the interplay of mixing by convection and diffusion, the model requires that 45 000K all the hydrogen diffuses upward on the star, producing a H layer, i.e. a DA white dwarf, but at 30 000K the helium convection layer will break through the thin hydrogen layer and transform those stars with thin ($10^{-13}M_\odot$) layers into DBs. Several billion years latter, when effective temperatures between 5 000K and 10 000K are reached,
p convection zone in the DAs will turn most of the stars into non DAs, if the hydrogen surface layer is less than $10^{-7}$.

The observation that the ratio of DAs to non-DAs change from 4 to 1 at high temperatures to 1 to 1 at $10^4$ K cooler agree with this hypothesis of thin hydrogen surface layers (Fontaine and Wesemael 1987; Shipman 1989).

The thin H layer model has the advantage of explaining the observations that the photospheric X-emission in DA white dwarfs observed with EINSTEIN and EXOSAT is smaller than that expected from pure atmospheres, implying an extra opacity source which must be due to the presence of small traces of heavier elements (Fontaine and Wesemael 1989). The main reason is that trace elements other than H will produce strong absorption edges in the X-ray/EUV spectrum, caused by ground state absorption edges and resonances lines in the central region (Paerels and Heise 1989). Interpreting the X-ray absorber as He requires the existence of very thin layers ($M_H \leq 10^{-13} M_\odot$) on the surface of hot DA white dwarfs so that we can see the underlying He layer at short wavelengths.

Also, to explain the DBAs, in which traces of H are seen in about 13 helium dominated stars, the H face layers must be almost transparent, i.e., $M_H \leq 10^{-13} M_\odot$ (Koester 1989).

Note that both mass loss and accretion from the interstellar medium are ineffective in hot white dwarfs, in comparison with the strong effect of diffusion. The possibility of mass loss in hot DA white dwarfs has been inferred in the presence of shortward shifted absorption components of highly ionized species (Bruhweiler and Kondo 1983).

The interaction of diffusion and mass loss requires $M \geq 10^{-14} M_\odot$ yr$^{-1}$ to render diffusion ineffective, but if such a mass loss existed, a layer of $10^{-12} M_\odot$ of H would be immediately lost. On the other hand, the rapid mass loss to $10^{-8} M_\odot$ yr$^{-1}$ observed on some planetary nebulae might deplete the H surface layer prior to the white dwarf age (Koester 1989).

Against accretion there is evidence in favor of a weak wind or static halo in hot DA white dwarfs. The retention rate necessary to compete with diffusion (around $10^{-16} M_\odot$ yr$^{-1}$) is much higher than expected for an isolated white dwarf in the interstellar medium. The typical time between cloud encounters, where accretion rates would be higher, is around $5 \times 10^7$ yr, larger than the lifetime of the hottest DA (Koester 1989). Note also that any magnetic field will make the propeller mechanism effective in hot white dwarfs, screening out the light elements from accretion.

Another point against accretion is the discovery by Oswalt et al. (1988) of a binary system containing a DA and a DB white dwarf. Since both stars share a common trajectory through the interstellar medium, there is no accretion history, and therefore accretion has nothing to do with the separation between DA and DB stars.

Similarly, to explain the DQ stars as a result of the convective dredge-up of the carbon, requires that the thickness of the He layer be about $10^{-4} M_\odot$, i.e., a thin He surface layer.

So it seems that there is an inconsistency with evolutionary calculations and the surface layer masses necessary to interpret some of the observations of white dwarfs.

PULSATING STARS

There are four classes of pulsating white dwarfs and pre-white dwarfs:

- PNV, or K1-16 stars, are the PNV which show light variations. They are hot, and we can see the Hα and He I lines.

- DOV, or PG1159 stars (GW Vir stars), with temperatures upward of 120 000 K (McGraw et al. 1979). PG1159-035 spectroscopic class has 10 objects, but only 5 are known to pulsate. Their spectra are similar to those of O VI PNN (Wesemael et al. 1985).

- DBV, or V477 Her stars, discovered in 1982 after the prediction by Don Winget from his model calculations (Winget et al. 1982), has 14 members.

- DAV, or ZZ Ceti stars, were the first pulsating white dwarfs discovered. Landolt (1968) found that the Hγ line of HL Tau 76 has a quasi period of 750 sec and amplitude of 0.3 mag. The class now has 20 members known.

These classes are well spaced in log $T_{eff}$, or log $L/L_\odot$ (Figure 2), and it is important that the stars are discriminable except for their variability, i.e., they don't show any differences from the other white dwarfs except for their pulsation (McGraw 1979). All the pulsating white dwarfs are multiperiodic, with periods ranging from 100 sec to 1500 sec, and with amplitudes of a few percent (Figure 3). The periods observed are all at least one order of magnitude higher than those expected from radial pulsations, and the multiperiodic nature as well as the periods indicate that they are non-radial g-mode pulsators, with the angular form of fluid displacement described by a spherical harmonic.
Fig. 2. Theoretical H-R diagram showing the positions of the compact pulsators instability strips.

![Hertzsprung-Russell Diagram](image)

Fig. 3. Light curve of the pulsating DB star GD358 (upper curve) and a comparison star, obtained with Nather's two stars photometer.

\( Y_{\text{cm}}(\theta, \phi) \) (Figure 4), and with the restoring force been the strong gravity. The radial displacements are only of the order of a few meters \((10^{-4} R_*\)) and therefore grossly exaggerated in these figures. The main cause of the light variations is the temperature variations (Kepler 1984).

With the introduction of the two-star photometer by Ed Nather in 1972 (Nather 1973) where we can observe both the candidate star and a field star simultaneously, to test for variations in the sky transparency, the number of ZZ Ceti stars grew to the present number of 20. McGraw and Robinson (1976) and McGraw (1979) demonstrate that the DAV where normal stars and concentrated in the H-R diagram near the position of the maximum of the opacity, indicating the the excitation mechanism was the same \( k \) and \( \gamma \) mechanisms occurring in the partial ionization zones of the Cepheids. With this discovery, and the success of model calculations for the ZZ Ceti stars (Winget 1981) Don Winget predicted that there should be a similar instability strip near the maximum of the He I opacity for the DAV...
For the ZZ Ceti stars we have already been able to measure their rotational velocities, since rotation troys the spherical symmetry, breaking up the azimuthal degeneracy so the frequency of pulsation of a mode depends on the \( m \) value. Rotation therefore splits a pulsation mode of index \( l \) into \( 2l + 1 \) modes. The rotation periods range n around 17 hr \( (V_{\text{rot}} = 1 \text{ km s}^{-1}) \) to 89 hr \( (V_{\text{rot}} = 0.15 \text{ km s}^{-1}) \) (Kepler et al. 1983; Kepler 1984) confirming the ults that white dwarfs are slow rotators, and showing that the rotational velocities are still lower than those limits und 20 km s\(^{-1}\) deduced from the broadening of the spectral lines (e.g. Milkey and Pilachowski 1985). An upper it on the magnetic field of \( B < 10^8 \text{G} \) could also be derived from the splitting of the pulsations of the ZZ Ceti star 18, ZZ Ceti itself (Jones et al. 1989).

On the surface layer masses problem, the ZZ Ceti stars have also produced some restrictions: Fontaine l his group from Montreal (Fontaine et al. 1982, 1985; Wesemael et al. 1986) and Greenstein (1982), have nonstrated that the ZZ Ceti stars occupy a very narrow instability, less than 2000K wide around 12 000K, and that sibly all the stars in the instability strip are pulsators. This indicates that pulsation is a normal evolutionary phase fe DA stars, which start to pulsate when they cool down to 13 000K, due to formation of the partial ionization zone -1. A similar result for the He ionization zone has been derived for the DB stars (Liebert et al. 1986). The restriction the surface layer mass comes from the fact that to have the blue edge of the instability strip for the ZZ Ceti stars at 800K, the models require that the H surface layer mass be less than \( 10^{-8} \text{M}_\odot \). Even with a thin H surface layer, to tch the temperature of the blue edge the models must use a very efficient convection theory, ML3, i.e., using the ing length as 3 times the pressure scale height (Winget et al. 1988, Koester et al. 1985). The models also indicate t have pulsations at all, the hydrogen surface layer must be smaller than \( 10^{-7} \text{M}_\odot \), otherwise the partial ionization becomes so deep that the work necessary to lift up the overlying layers is much larger than the available energy, the pulsations are strongly damped (Winget et al. 1981, 1982; Bradley et al. 1989).

The lower limit on the surface layer mass comes from the restriction that the layer must be larger than \( -14 \text{M}_\odot \), otherwise the hydrogen layer would be so thin that the DAs should also pulsate around the He I opacity ximum, near 25 000K.

On the other hand, if not all DAs inside the instability strip pulsate, then it is possible that there are some s with larger H layer mass, which are these nonpulsators. We have begun a search for pulsators and non-pulsators und 12 000K, but since the multiperiodic nature causes the modes to beat down for up to a couple of hours, we st observe a star for at least 3 hr to make sure it is not a pulsator. There is also an observational limit, since we not see pulsations with amplitudes less than 1 millimagnitude, and the inclination of the axis of pulsation to the : of sight might hide some pulsators (Pesnell 1985). On our search, we have already found at least 4 non-variables hin the instability strip, but we must determine their temperatures very accurately and observe more stars to make...
a statistically significant statement on the existence of non-variable stars inside the instability strip. To conclude, the pulsation models constrain the H layer to be thinner than at least one-third of the large majority of DA stars.

To really obtain all the information we talked earlier from the pulsations, we must identify each periodicity and identify the index \( \ell \) for each mode, but the very complex nature of the light curves of the white dwarf pulsators makes the problem of deciphering the pulsations very difficult or impossible from a single site. The data introduced on the data set by the interruption of the observation when the sun comes up causes the appearance of six lobes on the Fourier spectra of the light curve. For example, for the star PG1346+082, we have observed the star for 17 consecutive nights from Texas; the best we can do from a single site, due to constraints of weather and telescope allocation; note that white dwarfs are faint and therefore we need telescopes of 1.5m or larger to observe these small amplitude pulsations. The Fourier spectra of the star is seen in Figure 5 and it is a mess. To see what the gaps from daylight introduced in the spectra, we can calculate the Fourier spectra of a single sine curve sampled at the same time we observe the star. The result is what we call a Spectral Window, and it is not a delta function because of the side lobes introduced by the finite length and the gaps in the observations. The obtained power spectra is the convolution of similar spectral window for each of the real periodicities in the star, and all the side lobes combine together to make the mode identification impossible.

Fig. 5. a) Power spectrum of the star PG1346+082 with the data taken at McDonald Observatory. b) Power spectrum of a single sine curve with the same spacing as the data.

To solve this problem, a group of astronomers, headed by Ed Nather from Texas, created what is called the Whole Earth Telescope (Nather 1989), which is a program to observe the same star from a set of telescopes around the Earth (Figure 6), such that when the star sets for one telescope, it can be observed from another telescope, making the observations continuous. The observations are all coordinated in real time from the headquarters in Texas so that they are really continuous and we waste no time by having several telescopes observing the same star at the same time. We have already succeeded in 3 observing runs with this network, and the result can be seen for the same star PG1346+082 (Figure 7), for which we obtained 12 days of almost continuous data (Nather 1989; Provencal et al. 1989). The spectral window of the 'continuous' data is much better behaved, and we deduced that the star has 15 periodicities simultaneously present.

For the DOV star PG1159-035, the prototype, we had an observing run during March 89 with the WET and got 232 hours of almost continuous data on the star. The power spectrum obtained is so rich that it will take many months to analyze it and get all the information contained in it.

The observations have already allowed us to measure its rotation period as 2.7 days, since the splitting of the pulsations with same \( \ell \) are due to the rotational splitting as mentioned before. The observations also lead us to identify \( \ell \) and obtain the mass of the star very accurately, from the spacing between each pulsation band, which has the same \( \ell \) but different \( k \) values. The pulsation model calculations of Kawaler et al. (1985) indicate that the period spacing is very sensitive to mass for these hot stars, being almost independent of the composition of the surface layer or of the luminosity (Figure 8).
Fig. 6. Location of the sites of the Whole Earth Telescope.

g. 7. a) Power spectrum of the Whole Earth Telescope data on the star PG1346+0.82. b) Power spectrum of a single sine curve with the same acing as the data.

![](https://example.com/fig6.png)

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g. 8. Characteristic period spacing from theoretical pulsation models (Kawaler 1987). Dashed lines are for pure carbon models with different masses, and the solid line for a model with a helium-rich envelope. The data point is the 21.1 second spacing found for the $l = 1$ pulsations in 31159-035.
The obtained mass is $M = 0.59 \pm 0.01 \, M_\odot$ (Kawaler 1987), even more accurate than the astrometric masses we mentioned before. PG1159-035 was also the first star to have its evolutionary timescale measured, since Winget et al. (1985) measured the rate of period change for the dominant pulsation of the star to be $\dot{P} = (-2.4 \pm 0.1) \times 10^{-11}$ s/s, or $P/\dot{P} = (6.6 \pm 0.1) \times 10^8$ yr, using a definition of $\dot{P} = \frac{f \, \delta \nu}{dt}$, which differs by an exact factor of 2 due to the integration from the Winget et al. definition of $dP/dt = (2\pi P)^{-1} \, \delta \nu/dt$, and $\dot{P} = f \, \delta \nu$. The definition of $\dot{P}$ is arbitrary, but the one we use throughout this paper is the one consistent with those obtained from the theoretical model calculations.

The measured rate of period change is exactly the one Winget et al. (1983) had predicted from pulsation models. The detection of the evolutionary timescale of the order predicted by the theoretical calculations demonstrated that the pre-white dwarf evolutionary models, including the cooling rates due to neutrino plasmon emissions, were essentially correct (Kawaler et al. 1985).

PG1159-035 evolves fast because it is a pre-white dwarf, still contracting from the planetary nebula phase. Since $P/\dot{P} = -a \cdot T/T + b \cdot R/R$, with $a$ and $b$ being constants close to unity, the negative value observed indicates that the contraction is dominating over the cooling at this stage, when the star is hotter than 120,000K. We have checked the time of maximum of the 516 sec pulsation on these 232 hr obtained in March 1989, and it fits the predicted ephemeris with the rate of period changed derived in 1985 perfectly (Figure 9) even though the modes at 517 and 518 sec present in the 1989 data were not present before 1985 and are now shifting the power of the previous 516 s period (Koupelis and Winget 1987, Winget et al. 1989 - in preparation).

The appearance of these new modes might be due to the smearing by diffusion of the composition layers that resulted from the previous phases of shell burning.

For the PNN variables, the situation is not as simple. As mentioned earlier, all evolutionary models of PNN indicate that they have hydrogen surface layers of $10^{-4} \, M_\odot$, or $10^{-2} \, M_\odot$ He layers, and are either H shell burners or He shell burners, to explain their rate of evolution across the H-R diagram. But the pulsation model calculations indicate that the $e$ mechanism (periodic variations in the rate of nuclear burning) should make all the PNN with shell burning unstable to pulsations with periods between 50 and 200 sec. Butler Hine has looked for these pulsations in 35 PNN, and found none (Hine and Nather 1987), indicating that the PNN are not shell burners, and therefore should have thin hydrogen ($M_H < 10^{-5} \, M_\odot$) or thin He ($M_{He} < 10$ layers). D'Antona and Mazzitelli (1987) have demonstrated that models with thin H layers and therefore not shell burners can still be constructed with the appropriate evolutionary rates across the H-R diagram. Note however that there are two PNN pulsator, K1-16 and Lo 4, but with periods larger than 350 sec, and therefore not the same predicted by the $e$-mechanism.

Again on the measurement of an evolutionary timescale, we have been measuring the pulsations of the ZZ Ceti star G115-B15A for the last 15 years, and have obtained a total of 195 hr of high speed photometry on the star. The star has a very simple period structure, with one mode dominating totally the light variations at 215 sec (Kepler et al. 1982). We have been measuring the variation of the time of maximum of the dominant pulsation and derived a rate of period change of $\dot{P} = (8.3 \pm 5.0) \times 10^{-15}$ s/s, equivalent to $P/\dot{P} = (8.2 \pm 5.0) \times 10^8$ yr. Note that this is still

![Fig. 9. (O-C) diagram for the 516 second pulsation of PG1159-035. Each data point is the seasonal phase for the year indicated. The solid line shows the parabola with the rate of change of period with time ($\dot{P}$) measured by Winget et al. (1985).](image-url)


This rate of evolution, which for a ZZ Ceti star is only due to cooling since the star is not contracting anymore, already limits the core composition of the white dwarf to be O/Ne/Mg or lighter, because a heavier composition would have crystallized prior to 15000K, and therefore would give a much faster cooling rate. Since our limit on the rate of change of period with time, $P$, will continue to improve quadratically with the time baseline the star is served, or about 17% per year nowadays, we expect to reach the predicted timescale for a C/O core of $P/P = 2 \times 10^3$ yr in 10 years. If, on the other hand, the value of $P$ we obtained is real, in about 4 years we will have a 3σ result. This might imply that the star has a core composition like O/Ne/Mg, a particular interesting possibility in view of the notation of these elements in the ejecta of classical novae by Williams et al. (1985) and Ferland and Shields (1978).

But what can we learn from these evolutionary timescales? Since the white dwarfs are the oldest objects in our galaxy, we can measure the age of the disk of our galaxy, by measuring the age of the coolest white dwarfs. The age function of white dwarfs obtained by Liebert, Dahm and Monet (1988) which shows a decrease by an order of magnitude in the number of white dwarfs fainter than $log L/L_\odot = -4.5$, equivalent to $T_{eff}$ around 4000K, indicates at the age of the disk is between 5 and 13 Gyr. The upper limit is for a pure C core and pushing all the uncertainties in the time for main sequence evolution, surface layer masses, conductive opacities and the like to the maximum age, the age depends mainly on the core composition of the white dwarfs, with the best fit at 7 Gyr (Winget and Van Hörn 1987; Wood and Winget 1989). If there is no difference in the time of start of star formation between the halo and the disk, then the age of the disk indicates an age for the galaxy less than 13 Gyr, and of less than 14 Gyr for the universe, assuming that our galaxy formed 1 Gyr after the Big-Bang. Since we expect to measure the core composition of G117-B15A in a few years, as well as to calibrate the evolutionary curves by measurements of the cooling rates, we expect the method of getting the age of our galaxy by measuring the age of the coolest white dwarf to become a firm measurement within a few years.

REFERENCES


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