

NONSPHERICAL RADIATION DRIVEN WIND MODELS  
APPLIED TO BE STARS

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**ABSTRACT.** In this work we present a model for the structure of a radiatively driven wind in the meridional plane of a hot star. Rotation effects and simulation of viscous forces were included in the motion equations. The line radiation force is considered with the inclusion of the finite disk correction in self-consistent computations which also contain gravity darkening as well as distortion of the star by rotation. An application to a typical B1V star leads to mass-flux ratios between equator and pole of the order of 10 and mass loss rates in the range  $5 \cdot 10^{-8}$  to  $10^{-8}$   $M_{\odot}/\text{yr}$ . Our envelope models are flattened towards the equator and the wind terminal velocities in that region are rather high (1000 Km/s). However, in the region near the star the equatorial velocity field is dominated by rotation.

**RESUMEN.** Se presenta un modelo de la estructura de un viento empujado radiativamente en el plano meridional de una estrella caliente. Se incluyeron en las ecuaciones de movimiento los efectos de rotación y la simulación de fuerzas viscosas. Se consideró la fuerza de las líneas de radiación incluyendo la corrección de disco finito en calculos autoconsistentes los cuales incluyen oscurecimiento gravitacional asi como distorsión de la estrella por rotación. La aplicación a una estrella típica B1V lleva a cocientes de flujo de masa entre el ecuador y el polo del orden de 10 y tasa de pérdida de masa en el intervalo  $5 \cdot 10^{-8}$  a  $10^{-8}$   $M_{\odot}/\text{año}$ . Nuestros modelos de envoltente están achatados hacia el ecuador y las velocidades terminales del viento en esa región son bastante altas (1000 Km/s). Sin embargo, en la región cercana a la estrella el campo de velocidad ecuatorial está dominado por la rotación.

*Key words:* STARS-BE -- STARS-WINDS

## 1. INTRODUCTION

In the last years some attempts have been done in order to explain the winds of Be stars within the context of radiation driven winds (see the review by Marlborough, 1987). From an analytical study of the conditions in the critical point, Marlborough and Zamir (1984) concluded that the mass-loss rate is only slightly altered by rotation. Poe and Friend (1986) developed a model which includes the effects of a magnetic field and the finite disc correction (Pauldrach et al. 1986; Friend and Abott 1986). Their model however is spherically symmetric. Therefore, in order to be able to compare their results with the observations they have to let the high rotation models simulate the equatorial regions, while the low rotation models would describe the polar

regions. In this situation they have reached some good qualitative results.

In this work we present the results of our exploratory analysis of a non-spherical (axy-symmetric) model for the envelope of a Be star. Besides incorporating rotation we work with the three equations of motion and we simulate the effects of viscosity. We consider also the distortion of the star due to rotation and the variation of the photospheric temperature as a consequence of the gravity darkening. The mass flux and the velocity field were calculated at different latitudes of the star, in particular, at the equator and poles. This approach enables us to make more direct comparisons between theory and observations. Our model does not take into account time variability, which is likely to be related with nonradial pulsations (see the review by Baade, 1987). However, we believe that there is a continuous background component even for stars with highly variable winds and to this steady component we address our work.

## 2. THE HYDRODYNAMIC EQUATIONS

We consider the equations for the conservation of mass and momentum of a fluid subject to: gravity, gas pressure, radiation force from continuum and lines and centrifugal acceleration. In addition we simulate a viscous force that makes the wind to deviate from angular momentum conservation. The assumptions adopted are: steady state, azimuthal symmetry and meridional flows not significant. We assume also the equation of state of a perfect gas:

$$p = a^2 \rho . \quad (1)$$

Within these approximations the mass conservation can be integrated and we obtain

$$\Phi(\vartheta) = r^2 \rho v \quad (2)$$

where  $\Phi(\vartheta)$  is the mass flux per unit of solid angle. The loss rate is given by the relation

$$\dot{M} = 2 \pi \int \Phi(\vartheta) \sin \vartheta d\vartheta . \quad (3)$$

For the radial component of the velocity we can write

$$v_r \partial v_r / \partial r - v_\phi^2 / r + 1/\rho \partial p / \partial r + G M(1 - \Gamma) / r^2 - 1/\rho F^l = 0. \quad (4)$$

The second term represents the centrifugal force, the fourth is the effective gravitational acceleration and the last term is the radiative force due to line opacity. The parameter  $\Gamma$  is given by

$$\Gamma = \sigma_e L / 4 \pi G M C. \quad (5)$$

All the symbols have their usual meaning. The equation for the azimuthal component reduces to

$$v_r \partial v_\phi / \partial r + v_r v_\phi / r = f . \quad (6)$$

where  $f$  is an unknown viscous force per unit mass. We assume as a solution of this equation

$$v_\phi(r, \vartheta) = \chi [GM(1-\Gamma)/R]^{1/2} \sin \vartheta (R/r)^\beta , \quad -1 \leq \beta \leq 1 , \quad (7)$$

where  $\chi$  is the ratio between the centrifugal acceleration at the equator and

the effective gravity and  $B$  is an adjustable viscosity parameter. Finally, the equation for the meridional component of the velocity leads to the relation

$$-v_{\phi}^2 \cot \vartheta + 1/\rho (\partial p/\partial \vartheta) = 0. \quad (8)$$

We bypass the energy equation assuming a variation of the temperature throughout the envelope of the form

$$T(r, \vartheta) = t_1(r) \cdot t_2(\vartheta), \quad (9)$$

which, if combined with (1), (2), (7) and (8) requires a more restrictive functional relation

$$T(r, \vartheta) = t(R, \vartheta) (R/r)^{2\beta}. \quad (10)$$

This last relation together with equation (7) tells us that a non-viscous flow ( $\beta = 1$ ) and an isothermal envelope ( $B = 0$ ) are inconsistent assumptions. The line radiation force used assumes Sobolev approximation (for discussion see Pauldrach et al. 1986), single scattering (see Puls 1987) and the correction for the finite size of the source of radiation. Therefore the expression is

$$\frac{F^1}{\rho} = \frac{\sigma_e L}{4\pi cr^2} \frac{K}{(\sigma_e \rho v_{th})^\alpha} \left(\frac{\partial v_r}{\partial r}\right)^\alpha \frac{\{1 - [1 - (R/r)^2 + (R/r)^2 \frac{v/r}{dv/dr}]^{1+\alpha}\}}{(1+\alpha) (R/r)^2 [(1-v/r)/(dv/dr)]} \quad (11)$$

where  $K$  and  $\alpha$  are the radiative parameters. If now we combine equations (1), (2), (4), (7) and (11) we obtain the basic equation that describes the wind

$$\begin{aligned} & \left[ v_r - \frac{a_0^2 (R)^{2\beta}}{v_r r} \right] \frac{dv_r}{dr} + \frac{GM(1-\Gamma)}{r^2} [1 - \chi^2 \sin^2 \vartheta \frac{(R)^{2\beta-1}}{r}] = \\ & = \frac{2a_0^2 (R)^{2\beta+1}}{R r} (1+\beta) + f(r, v, dv/dr) \frac{C}{r^2} \left( r^2 v_r \frac{dv_r}{dr} \right)^\alpha; \quad C = \frac{\Gamma GMK}{[\sigma_e v_{th} \phi(\vartheta)]^\alpha} \quad (12) \\ & \left\{ \frac{1 - [1 - (R/r)^2 + (R/r)^2 \frac{v/r}{dv/dr}]^{1+\alpha}}{(1+\alpha) (R/r)^2 [1 - \frac{v/r}{dv/dr}]} \right\} = f(r, v, dv/dr) \end{aligned}$$

This equation is solved using the additional boundary condition  $\tau(R) = 2/3$ .

### 3. PARAMETERS AND RESULTS

#### 3.1 Stellar and wind parameters

In order to model a typical B1 star we adopted  $M = M_\odot$  and  $L = 9700 L_\odot$  (Slettebak et al. 1980). The corresponding radius of a non-rotating star would be  $R \approx 5.3 R_\odot$ , while the effective temperature would be  $T_{eff} = 25000$  K. As we have mentioned we considered the effects of rotation in the star itself obtaining a functional dependence of temperature and radius on latitude. We had also to establish an adequate range of variation for the parameters  $\beta$  and

$\chi$ . For the viscosity parameter we have taken values in the interval 0.00 to 0.49. For the rotation rate  $\chi$ , in view of the uncertainties related to this quantity, we decided to use four different values: 0.5, 0.66, 0.75 and 0.9. Finally, the line radiation parameters  $K$  and  $\alpha$  were taken from the work of Abbott (1982). In our calculations we have used  $\alpha = 0.5$  and  $K = 0.5$  as representative of the conditions prevailing in an early B star envelope.

### 3.2 Results and discussion

Figures 1 to 4 summarize the main results of our computations. In order to emphasize the contrast between equatorial and polar regions we restrict our analysis to these directions. From figure 1 we obtain a ratio between equatorial and polar mass flux of about 10 ( $\chi = 0.9$  model) and a global mass loss in the range  $10 M_{\odot}/\text{yr}$  to  $5.10 M_{\odot}/\text{yr}$ . Figure 2 shows the dependence of the terminal velocity with rotation and viscosity. In summary we could say that polar terminal velocities of the order of 2000 Km/s are obtained while the equatorial velocity decreases from 2000 Km/s to about 1000 Km/s as the rotation rate increases from 0.5 to 0.9. These results agree well with the velocities derived from UV lines but are in conflict with the usual belief of low velocities in the equator. However, in figure 3 we see that the velocity field in this region is dominated by rotation at the inner parts ( $r < 3R$ ) and by expansion at the outer parts. This could possibly be important to understand the extended wings of some  $H\alpha$  profiles that reveal velocities larger than 1000 Km/s (Andrillat 1983). Finally, figure 4 shows the isodensity curves which characterize the envelope. We can see that it possesses an asymmetric (concentrated towards the equator) structure.

### 4. CONCLUSIONS

Some of our results encourage us to make a comparison with Be star observations. For example, our model seems to support the usually assumed ad hoc density enhancement at the equator. In addition the mass loss predicted are in good agreement with the rates obtained from the IR excess and a discrepancy between equatorial and polar mass flux arises naturally. On the other hand the radial velocity laws are rather problematic, particularly the equatorial one. In fact, we do not claim that our model includes all the

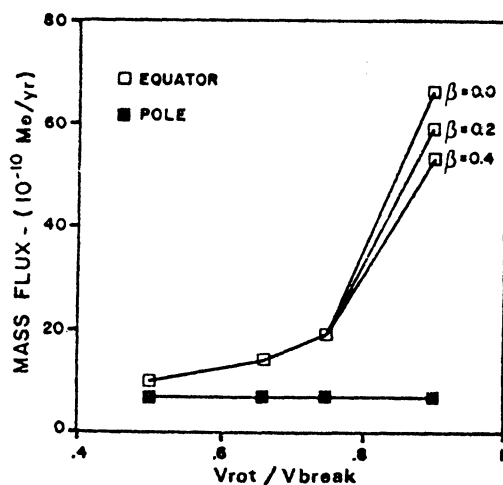


Fig. 1.- Equatorial and polar mass flux as a function of rotation and viscosity ( $\beta$ ).

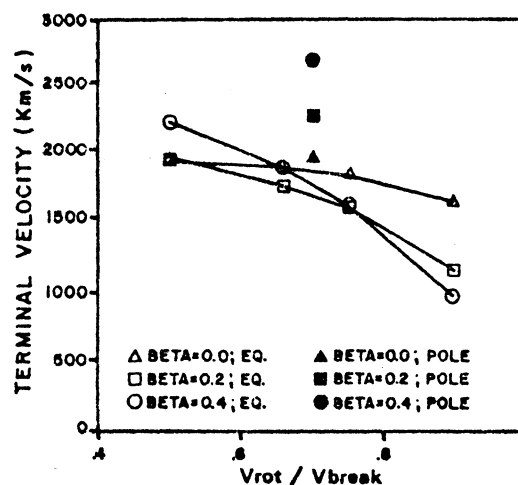


Fig. 2.- Equatorial and polar terminal velocity as a function of rotational viscosity ( $\beta$ ).

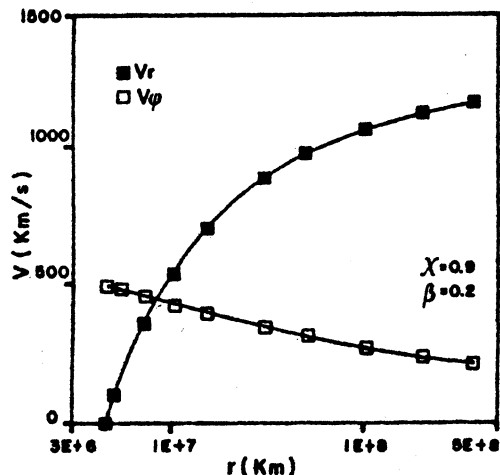


Fig. 3.- Equatorial velocity profiles for the radial and for the tangential component.

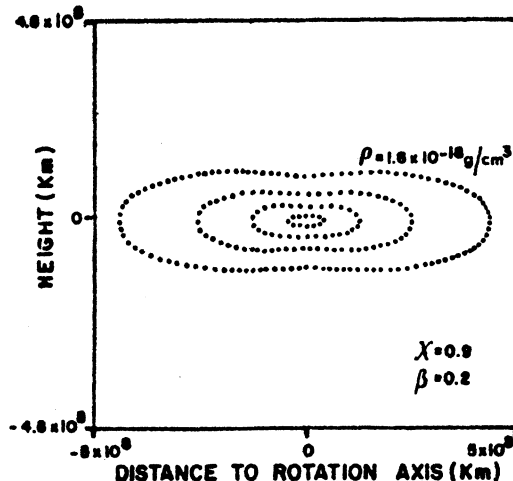


Fig. 4.- Meridional isodensity curves.

physics necessary to explain the Be phenomena. Nonradial pulsations for instance, which are likely to be linked with the observed time variability and may be responsible for periods of enhanced mass loss. The inclusion of such a driving mechanism, among others, would probably greatly improve the present work.

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