

QUASI-PERIODIC OSCILLATIONS IN X-RAY BURSTERS

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RESUMEN: Se estudian fluctuaciones de energía en sistemas estelares binarios, usando un modelo donde la atmósfera de acreción es considerada como una esfera de gas radiante, realizando oscilaciones "termomecánicas". Se obtienen soluciones analíticas para las variables de estado y son comparadas con las soluciones del modelo hidrostático. El modelo describe bastante bien a los "bursters" de rayos X.

ABSTRACT: The energy fluctuations in stellar binary systems, are studied, using the model where the accretion atmosphere is considered as a radiant gas sphere, endowed with "thermomechanical" oscillations. The analytical solutions for the state variables are obtained and compared with the hydrostatic model solutions. The model describe the X-ray burster quite well.

Key words: STARS-ACCRETION — STARS-OSCILLATIONS — x-RAYS-BURSTS

. INTRODUCTION

X-ray bursters are stellar objects that normally belong to binary systems, where one of the components is a collapsed star (a neutron star) and the other one is a normal star. The strong gravitational field of the collapsed star attracts the hydrogen of the atmosphere of its companion. This gas falls forming first as accretion atmosphere, and when the density and the temperature become critical, a nuclear reaction takes place transforming the hydrogen into helium and then to heavier elements liberating a great amount of X-rays. This process, known as a thermonuclear fulguration (Lewin and Clark 1980; Sherwood and Plaut 1975) produces a huge amount of energy in few seconds.

In this paper we will not use this model, we will only consider "thermomechanical" adiabatical oscillations of the gas layer and we will suppose that almost all the energy comes from thermonuclear fulguration zone.

We complete our papers (Aquilano, Castagnino, Lara 1987; Aquilano, Castagnino, Lara 1988) where we neglected the oscillations of the whole gas layer and we only studied the ones of the outmost exterior surface, where we place all the oscillating mass.

. MATHEMATICAL MODEL

We will consider an old binary system with an accretion atmosphere around its collapsed star. This atmosphere is not in hydrostatic equilibrium and perform thermomechanical oscillations where we will only consider the radiation and the gas pressure forces and gravitational forces of the gas and the central collapsed star. Then a generic matter point at distance r from the center of the spherical symmetric body has an acceleration

$$\frac{d^2 r}{dt^2} = \frac{1}{\rho(r)} \frac{dP(r)}{dr} - \frac{G M_{aa}(r)}{r^2} - \frac{G M_{cs}}{r^2} \quad (1)$$

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where, $\rho(r)$ is the density of the atmosphere, $P(r)$ the radiation and gas pressure, G the Newton's constant, $M_{aa}(r)$ all the accretion atmosphere mass contained in the sphere of radius r and M_{cs} the mass of the central star.

In equation (1) we can find a singular stable point, and study the small oscillations around it.

We will consider that the gas oscillates in an adiabatical way according to the equation $T\rho^{1-\gamma} = \alpha = \text{constant}$, with $\gamma = 5/3$.

As the oscillations are small we will use the equation of the stellar models (Motz 1972) for the case of thermodynamics and hydrostatic equilibrium.

Finally we need an equation of state that relates P, ρ, T . We choose an ideal gas equation where the pressure is the gas pressure plus the radiation pressure.

We will adimensionalized the equations to simplify the mathematical treatment. We will take as units the solar mass (M_{\odot}), the solar radius (R_{\odot}), the solar density (ρ_{\odot}), the solar temperature (T_{\odot}), the solar luminosity $L_{\odot} = 4\pi R_{\odot}^2 T_{\odot}^4$, and we will use a unit time parameter obtained from equation (1) i.e.: $t_{\odot} = (R_{\odot}^3/GM_{\odot})^{1/2}$. As we will explain in next section we can take $L = L_C = \text{constant}$. Then we can obtain the analytical equilibrium solution with our adimensionalized parameters,

$$T(x) = \alpha (5/3 D L_C)^{2/5} x^{-2/5}, \quad M(x) = 1/2 (D L_C)^{3/5} x^{12/5}, \quad \rho(x) = (5/3 D L_C)^{3/5} x^{-3/5}$$

where x is the adimensionalized radius, and D is a constant precisely:

$$D = 3K R_{\odot}^{-1} L_{\odot}^{-4\gamma+6} / 16\pi a c^4 (\gamma-1)$$

where K is the opacity (Quinn and Paczynski 1985), $a = \frac{4\sigma}{c}$, σ the Stefan-Boltzmann constant and c the light velocity.

We can substitute these equilibrium solution in the complete non equilibrium dynamical equation (in adimensionalized equation 1), to obtain the equation for small oscillations around the equilibrium state, that reads,

$$x'' = E_1 x^{-7/5} + E_2 x^{-2} - E_3 x^{2/5} \quad (2)$$

where the primes symbolize derivatives with respect to the dimensionless time parameter τ and

$$E_1 = 1/2 (D L_C)^{2/5} B, \quad E_2 = D_C L_C - M_{cs}, \quad E_3 = 1/2 (D L_C)^{3/5}$$

where $B = R\alpha t_{\odot}^2 R_{\odot}^{-2} \gamma \rho_{\odot}^{\gamma-1}$, $C = 4/3 a \alpha^4 t_{\odot}^2 R_{\odot}^{-2} (\gamma-1) \rho_{\odot}^{4\gamma-5}$

From equation (2) we can obtain a first integral

$$x' = \pm (-5 E_1 x^{-2/5} - 2 E_2 x^{-1} - \frac{10}{7} E_3 x^{7/5} + \mathcal{C})^{1/2} \quad (3)$$

where \mathcal{C} is an integration constant. In figure 1 we can draw these trajectories.

We can now, using the small oscillations theory, expand the motion in Taylor series around the stable equilibrium point and obtaining the period

$$P = 2\pi \left[\frac{7}{5} E_1 x_s^{3/5} + 2 E_2 + \frac{2}{5} E_3 x_s^{12/5} \right]^{-1/2} x_s^{3/2}$$

Then we compute the period to be compared with the observational data.

As a check we have expanded, in Taylor series up to the 4th term, equations of the hydrostatic model, around the stable equilibrium point x_s and we have compared with the original analytical function. We show the difference between the analytical curves and the expansion in figure 2. As we can see, the approximation is quite good around x_s and justifies the hypothesis of our model, and also, this expansion establishes the limit where the model is good.

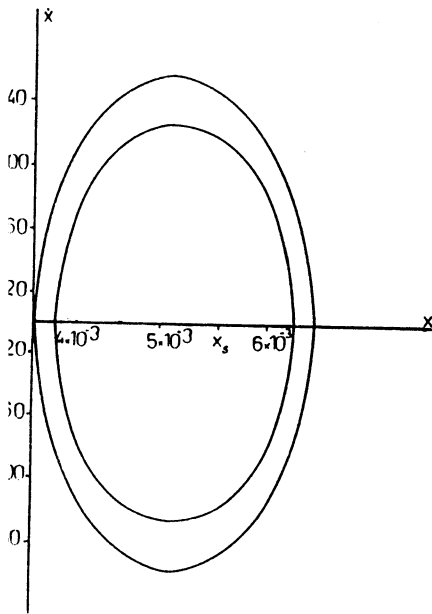


Fig.1: In this phase space we showed two trajectories around the singular point x_s , for SCO X-1 burster.

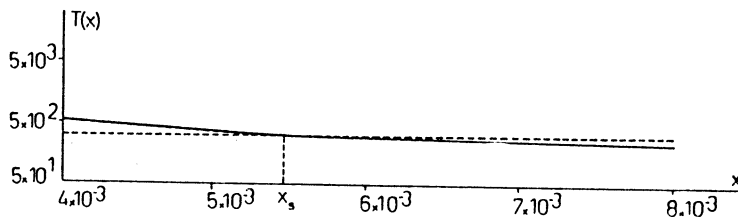
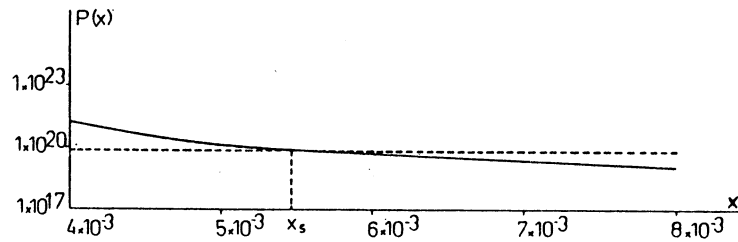
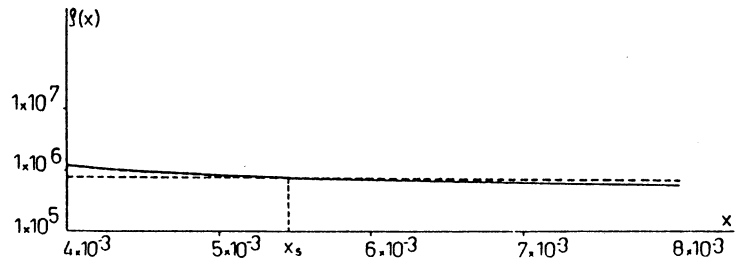
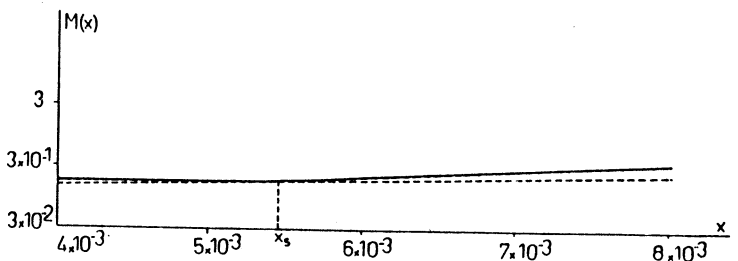


Fig.2: Comparison of the analytical solutions (—), with the equations of the hydrostatic model expanded in Taylor series (---) around the stable equilibrium point x_s ; for SCO X-1 burster.



3. COMPARISON WITH OBSERVATION

We will consider that almost all the energy ϵ is produced in the thermonuclear fulguration zone; thus we will consider that $\epsilon \approx 0$ in the remaining of the accretion atmosphere. Therefore $dL/dr = 0$ and $L = \text{constant}$.

In all cases the neutron star has a mass of $0.8M_{\odot}$. And using our equation and all these data we have computed several examples that are shown in the table,

BURSTER	T ($^{\circ}\text{K}$)	L (erg/s)	ρ (gr/cm^3)	$M_{\text{cr}} + M_{\text{aa}}$ (gr)	r_{g} (cm)	τ (sec)
GX5-1	3×10^6	3×10^{38}	3×10^6	4×10^{32}	4×10^8	3×10^{-2}
SCO X-1	2×10^6	2×10^{38}	2×10^6	3×10^{32}	3×10^8	5×10^{-2}
CYG X-2	3×10^6	3×10^{38}	3×10^6	4×10^{32}	4×10^8	3×10^{-2}
RAPID BURST	1×10^6	1×10^{38}	1×10^6	2×10^{32}	2×10^8	2×10^{-1}
GX3+1	1×10^6	9×10^{37}	1×10^6	1×10^{32}	1×10^8	1×10^{-1}

As in our former paper (Aquilano, Castagnino, Lara 1987) we do not reject the idea that thermonuclear fulguration take place because, most likely they will play a very important role in an eventual final model of X-ray bursters.

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