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THE ORIGIN OF THE FILAMENTS IN THE CRAB NEBULA

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RESUMEN. Tomando en consideración la expansión e historia pasada de la Nebulosa del Cangrejo y los datos observacionales recientes, se estudia la formación de la estructura filamentaria observada por la inestabilidad térmica de la radiación sincrotónica Compton inversa (ITS). Consideramos dos modelos: 1) un modelo de expansión de dos dimensiones en el cual los filamentos de la Nebulosa del Cangrejo se forman por ITS en una envoltura esférica en expansión, teniendo una velocidad radial promedio y un campo magnético en la envoltura de expansión radial (modelo a) o tangencial (modelo b), y 2) un modelo de expansión de tres dimensiones en el cual los filamentos están formados en una nube eyectada, expandiéndose isotrópicamente, con una orientación arbitraria del campo magnético (modelo c). Nuestros resultados indican que hay dificultad en la formación de los filamentos por ITS si el producto del volumen ambiente en expansión ($V_o \propto R^{+\alpha_V}$) y el campo magnético ambiental ($B_o \propto R^{-\alpha_B}$) del plasma, $B_oV_o \propto R^{\alpha_V-\alpha_B}$, está aumentando tan rápidamente como una función del radio R(t) de la nebulosa. Hay formación eficiente de filamentos con α_V comparable a α_B (como en nuestro modelo (a) de dos dimensiones de campo radial magnético en donde $\alpha_V = \alpha_B = 2$).

ABSTRACT. Taking into account the expansion and past history of the Crab Nebula and recent observational data, we study the formation of the observed filamentary structure by synchrotron-inverse-Compton radiation thermal instability (STI). We consider two models: 1) A two-dimensional expansion model in which the filaments of the Crab Nebula are formed by STI in an expanding spherical shell, having an average radial velocity, and a radial (model a) or a tangential (model b) magnetic field to the expanding shell; and 2) A three-dimensional expansion model in which the filaments are formed in an ejected cloud, expanding isotropically, with an arbitrary orientation of the magnetic field (model c). Our results indicate that there is difficulty in forming filaments by STI if the product of the ambient expanding volume ($V_o \propto R^{+\alpha_V}$) and the ambient magnetic field ($B_o \propto R^{-\alpha_B}$) of the plasma, $B_oV_o \propto R^{\alpha_V-\alpha_B}$, is increasing rapidly as a function of the radius R(t) of the nebula. There is an efficient formation of filaments with α_V comparable to α_B (as in our radial magnetic field two-dimensional model(a) where $\alpha_V = \alpha_B = 2$).

Key words: INTERSTELLAR-SUPERNOVA REMNANTS — PLASMA

I. INTRODUCTION

Arcsecond resolution images of the Crab Nebula show a distinctive filamentary structure. Much of the radio flux is concentrated in linear structures with lengths 5-20 times their widths. Clarke et al. (1983) studied the structure of the Crab Nebula in detail using the λ5007 [OIII] line. Their best model is that of two concentric expanding thin surfaces. There is intense synchrotron radiation within the inner shell, and weaker synchrotron radiation between the two shells.

The lack of shock induced X-ray emission is evidence against a strong interaction between the Crab Nebula and the interstellar medium, and one must invoke internal instabilities in order to account for the observed filamentary structure (Woltjer 1958). Additional evidence for an internal origin of the filaments comes from the strong chemical differences among them, suggesting that the individual blobs are formed before the expelled matter and the interstellar medium become mixed up (Bandiera et al.,1983).

The nonlinear development of condensations, created by a thermal instability in outflows emitting primarily synchrotron-inverse-Compton radiation, have been studied by Gouveia Dal Pino and Opher (1989a - GOI; 1989b - GOII). In the present paper we examine whether the observed filaments in the Crab Nebula can be produced by a Synchrotron-Inverse Compton Thermal Instability (STI).

II. BASIC EQUATIONS

A detailed description of our basic assumptions and equations is given in Gouveia Dal Pino and Opher (1989c - GOIII; see also GOIV, this conference). We consider an optically thin expanding plasma in which the major part of the internal energy of the system is due to the relativistic electrons emitting synchrotron and inverse Compton radiation. For the investigation of the development of the thermal instability, we consider transverse perturbations to the magnetic field, since the growth of longitudinal perturbations tends to be inhibited by the thermal conductivity of the relativistic component. We separate out the ambient and the perturbed quantities which we designate with subscripts "o" and "1", respectively.

The time evolution of the synchrotron plus inverse Compton losses in the equilibrium state, $L_{sco}(t)$, is unknown. We assume that the ambient synchrotron loss function $L_{so}(t) = L_{sco}(t)/(\delta_2+1)$ is given by:

$$L_{so}(t) = \left[\frac{R(t_f)}{R(t)}\right]^{3+\alpha_L} L_{so}(t_f) \tag{1}$$

where $\delta_2=U_{pho}(t)/P_{Mo}(t)$, $U_{pho}(t)$ is the energy density of the ambient radiation field, $P_{Mo}(t)$ is the magnetic pressure, $R(t_f)$ is the radius of the source at the present time ($R(t_f) = R_{CN}$), α_L is a constant and $L_{so}(t_f)$ is the present synchrotron loss function. For the evaluation of U_{pho} we use the relation

$$U_{pho}(t) = \frac{L_{sco}(t)R(t)}{3c\ f_c} \tag{2}$$

where f_c is a geometrical parameter which takes into account the deviation of the volume to area ratio of the source from that of a sphere.

For the pressure P_o of the relativistic electrons of the ambient region we also assume a relation similar to (1)

$$P_o(t) = P_o(t_f)\ \left[\frac{R(t_f)}{R(t)}\right]^{3+\alpha_E}. \tag{3}$$

where $P_o(t_f)$ is the electronic pressure today and α_E is a constant.

III. THE EXPANSION MODELS

We consider three possible expansion models for the production of the filaments in the Crab Nebula:

a) The two-dimensional expansion model with radial magnetic field (see Fig. 1a),
b) The two-dimensional expansion model with a tangential magnetic field to the expanding shell (see Fig. 1b),
c) The three-dimensional expansion model (see Fig. 1c).

The equations of motion appropriate to the scenarios above are the standard MHD equations which are described in detail in GOIII. Assuming for the perturbed region a cosine density profile (GOIII; see also GOIV - this conference) we obtained, from the numerical integration of the MHD equations, the nonlinear evolution of the perturbation. The results found for the three expansion models are presented below.

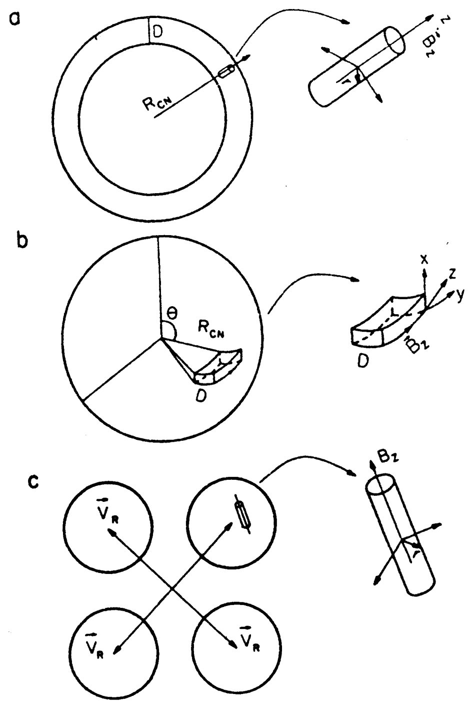

Fig. 1: Schematic configuration of the expanding nebula and the perturbed region: a) We suppose that the filaments originate in a spherical shell expanding with an average velocity V_R. In this case, the magnetic field is assumed to be radial. The volume element of plasma perturbed is a cylinder aligned with the magnetic field ($\vec{B}_z$); b) We suppose that the filaments originate in a spherical shell with a tangential magnetic field. The volume element perturbed is a comoving thin slab aligned with the magnetic field (z-axis) and with the y-axis; c) We suppose that the filaments originate in ejected clouds expanding isotropically. At a time t, the center of the cloud is at a distance R(t) with an outward velocity V_R. The volume element perturbed is a field-aligned cylinder and the magnetic field has an arbitrary orientation in the nebula.

IV. RESULTS

For the evaluation of the expansion rate at the present time $\nu_e(t_f) \equiv 1/\tau_e(t_f)$, we assume $\tau_e(t_f)$ of the order of the age of the source $\tau_e(t_f) \approx 934$ years (Strohmeier 1972). For the mean radius of the Crab Nebula $R_{CN} = R(t_f)$ we take $R_{CN} \approx 5\times10^{18}$cm (Davidson and Fesen 1985)). Using the data given for the whole nebula by Swinbank (1980) we obtained $L_{sco}(t_f) \approx 6.2\times10^{-18}$ ergs cm^{-3} s^{-1} and $P_o(t_f) \approx 9.14\times10^{-9}$ dy/cm^2.

4a) Results for the two-dimensional expansion model with a radial magnetic field

We tested our model for different values of f_c = 2-10, $f_R = R(o)/R(t_f)$ and $\delta_1(t_f) = P_{Mo}(t_f)/P_o(t_f)$ (GOII). As an example, the nonlinear evolution of the density contrast between the perturbed and the ambient regions $\rho_p/\rho_o = (\rho_1+\rho_o)/\rho_o$ (with an initial density perturbation $\alpha_\rho = \rho_1(o)/\rho_o(o) = 0.1$) is depicted in Fig. 2 for curves with $\delta_1(t_f) = 0.04$, $f_c = 10$, and $\delta_2(t_f) = U_{pho}(t_f)/P_{Mo}(t_f) = 0.09$, and different values of (α_E, α_L). The curves were integrated from t = 0 at f_R (= $R(o)/R_{CN}$) = 0.50 to the present time t_f = 467 years ($R(t_f)/R_{CN} = 1$). The results found above suggest that for $-1 \lesssim \alpha_E \lesssim 2$, $2 \lesssim \alpha_L \lesssim 4$, thermal instabilities can grow in the Crab Nebula and explain the formation of the nonthermal filaments in a time less than the lifetime of the source.

4b) Results for the two-dimensional expansion with a tangential magnetic field to the expanding shell

No significant amplification of ρ_p/ρ_o was found in this case.

4c) Results for the three-dimensional expansion calculations

We found that density amplifications of the same order as the 2-dimensional expansion model with radial magnetic field are obtained only for $\delta_1(t_f) \lesssim 0.025$ and an average $f_c \gtrsim 20$. As an example, Fig. 3 shows the density contrast ρ_p/ρ_o for $f_R = 0.50$, $\delta_1(t_f) = 0.02$, f_c = 20 and 30, for different values of (α_E, α_L).

V. CONCLUSIONS

Our results indicate, in general, that there is difficulty in forming filaments by STI if the product of the ambient magnetic field B_o and the expanding volume V_o of the plasma is increasing rapidly as a function of R(t). This is the reason why there is no formation of

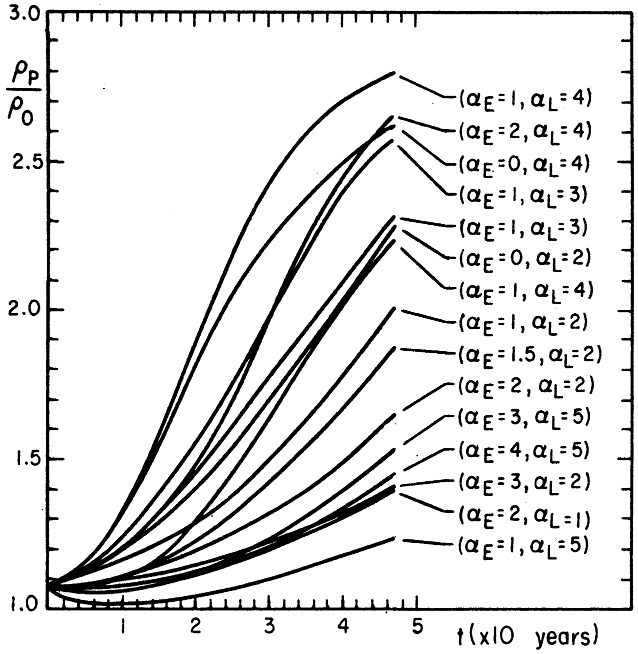

Fig. 2.: Nonlinear evolution of the density contrast ρ_p/ρ_o for the two dimensional expansion model with radial magnetic field for $\delta_1(t_f) = 0.04$, $f_c = 10$, $f_R = 0.50$ as a function of (α_E, α_L).

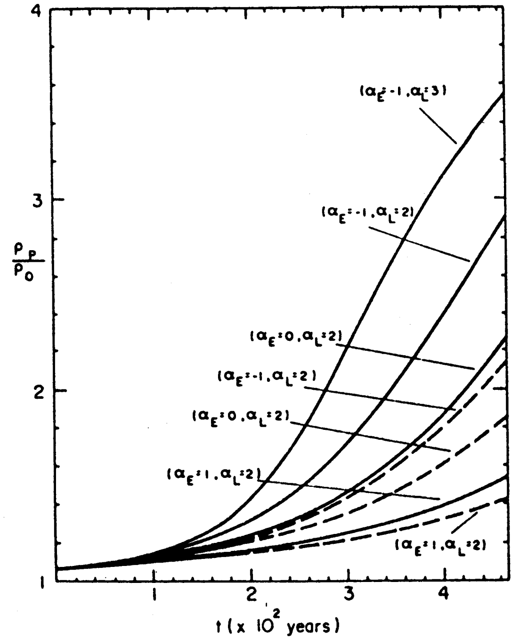

Fig. 3: Nonlinear evolution of the density contrast ρ_p/ρ_o for the three dimensional expansion model for $f_R = 0.50$, $\delta_1(t_f) = 0.02$, $f_c = 20$ and 30, for different values of (α_E, α_L). The dashed curve is for $f_c = 20$ and the solid curve for $f_c = 30$.

condensations in the two dimensional model with a tangential magnetic field to the expanding shell ($B_o \propto R^{-1}$), why condensations formed easiest in our two dimensional model with a radial magnetic field ($B_o \propto R^{-2}$) and only with difficulty in the three dimensional model (also with $B_o \propto R^{-2}$). Generalizing, our results indicate that for plasmas with $V_o \propto R^{\alpha_V}$ and $B_o \propto R^{-\alpha_B}$, (i.e., $B_o V_o \propto R^{\alpha_V - \alpha_B}$) we expect an efficient formation of filaments with α_V comparable to α_B (as in our radial magnetic field two-dimensional model where $\alpha_V = \alpha_B = 2$).

Since we require $f_c \gtrsim 10$ in the models studied, our present results also indicate that the Crab Nebula needs to be appreciably inhomogeneous in order to form filaments by the thermal synchrotron instability.

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