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LIBRATION MOTION OF ASTEROID (279) THULE

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RESUMO. Um modelo analítico baseado no desenvolvimento assimétrico da função perturbadora, e no problema restrito de três corpos planar é aplicado ao asteróiode (279) Thule, com a finalidade de se estudar o movimento libratório resultante da ressonância 4/3 com Júpiter. O período de libração determinado é de 190 anos, e o período do movimento do periélio é de 450 anos.

ABSTRACT. The 4/3 Jovian resonance of (279) Thule is analysed in the framework of planar restricted three body problem, and using the asymmetric expansion of the disturbing function. The libratory character is detected, and its period is 190 years. The period of motion of the perihelion is 450 years.

Key words: ASTEROIDS

I. INTRODUCTION

Nowadays, a very interesting problem in the dynamics of the Solar System is that related with the formation of the Kirkwood gaps, and with formation of groups of minor planets outside the main-belt. Such fenomena are associated with the commensurability between the mean motion of minor planets and Jupiter, but the mechanism of the formation is not still entirely known, in spite of great effort in this direction (see for example Ferraz-Mello (1985), Henrard and Lemaitre (1983), Wisdom (1982)).

The study of this problem presents some difficulties. On the one hand, the analytical treatment is limitted by lack of an adequate expansion of the disturbing function, and on the other hand, the large evolutionary time scale demands a long computing time for numerical analysis.

II. ASYMMETRIC EXPANSION

Despite the large orbital eccentricity and inclination, the study of the motion of the minor planets is performed utilizing the classical symmetric expansion of the disturbing function. Recently Ferraz-Mello (1987) developped a new asymmetric expansion, which is more suitable for high eccentricity libration motion of first order resonance. Later this alternative technique was adapted to resonances of any order (Ferraz-Mello and Sato, 1989).

It is clear from Figure 1 the advantage of the asymmetric expansion in the plane ($k=e\cos\theta$, $h=e\sin\theta$). V_1 represents the neighbourhood of $e=0$ (classical expansion), and V_2 the neighbourhood of a reference value $k_c=e_c$, around which the asymmetric expansion is developped.

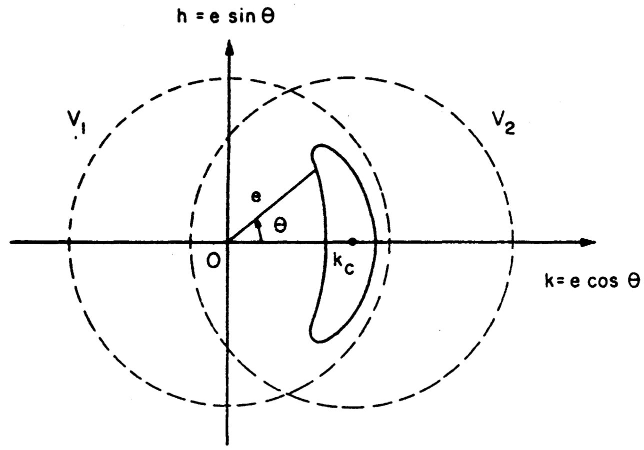

FIG. 1. Neighbourhoods of Classical (V_1) and Asymmetric (V_2) Expansions.

The disturbing function R in the restricted three body problem model is given by

$$R = k^2 m_1 (1/\Delta - \mathbf{r}.\mathbf{r}_1/r_1^3)$$

where k^2 is the square of Gaussian gravitational constant, m_1 the mass of Jupiter, $\mathbf{r}$ and $\mathbf{r}_1$ the heliocentric radius vector of the minor planet and Jupiter's respectively, and $\Delta=|\mathbf{r}_1-\mathbf{r}|$.

The asymmetric expansion of R in the variables (k,h) is

$$R = R_0 + R_k(k-k_c) + \tfrac{1}{2}R_{kk}(k-k_c)^2 + \tfrac{1}{2}R_{hh}h^2 + R_1 e_1\cos\theta_1$$
$$+ R_{1k}(k-k_c)e_1\cos\theta_1 + R_{1h}he_1\sin\theta_1 + \tfrac{1}{2}R_2 e_1^2 + \tfrac{1}{2}R_2' e_1^2\cos 2\theta_1$$

with $\theta = (p+1)\lambda_1 - p\lambda - \varpi$, $\theta_1 = (p+1)\lambda_1 - p\lambda - \varpi_1$, and p being an integer characterizing the first order resonance. The elements e, λ and ϖ are the eccentricity, mean longitude and longitude of perihelion, respectively. The corresponding elements of Jupiter are distinguished by the subscript 1. The coeficients R_0, R_k, ... , R_2' are computed numerically (Ferraz-Mello, 1987).

III. EQUATIONS OF MOTION

Using the canonical variables defined by (Ferraz-Mello, 1988)

$$K = (2J)^{1/2}\cos\theta + \beta e_1\cos\theta_1$$

$$H = (2J)^{1/2}\sin\theta + \beta e_1\sin\theta_1$$

$$G = J_1 - \tfrac{1}{2}\beta^2 e_1^2 - (2J)^{1/2}\beta e_1(\sin\theta\sin\theta_1 + \cos\theta\cos\theta_1)$$

$$\theta_1 = (p+1)\lambda_1 - p\lambda - \varpi_1$$

where $J = L - G$; $J_1 = G + \Lambda/n_1$; $n_1 = \dot{\lambda}_1$, we have

$$dK/dt = \partial\mathcal{H}/\partial H \quad ; \quad dH/dt = -\partial\mathcal{H}/\partial K$$

$$d\mathcal{G}/dt = \partial\mathcal{H}/\partial\theta_1 \quad ; \quad d\theta_1/dt = -\partial\mathcal{H}/\partial\mathcal{G}.$$

The Hamiltonian H is given by

$$H = \mu^2/2L^2 - \Lambda + R$$

with $\Lambda = (p+1)n_1G + \frac{1}{2}(p+1)n_1(K^2+H^2) - n_1J_2$. L and G are Delaunay variables and J_2 is a constant of motion.

IV. APPLICATION TO ASTEROID (279) THULE

The orbit of (279) Thule is situated between the main-belt and Jupiter's orbit, and till now is the unique known minor planet that is captured in the 4/3 (p=3) resonance with Jupiter. The application of the model based on asymmetric expansion was made taking the orbital elements of Thule and Jupiter given in Table 1, which are refered to 14 November, 1973 (for details of application see Tsuchida (1988, 1989)).

Table 1. Orbital Elements of (279) Thule and Jupiter for 11/14/1973

	a (A)	e	λ (rad)	ϖ (rad)
Thule	4.2577	0.0327	4.2529	4.6652
Jupiter	5.2026	0.0485	5.5953	0.2370

The libration motion of Thule in the plane (k,h) is shown in Figure 2, and the libration in the plane (K,H) is done in Figure 3. The curve of Figure 2 is composed by two motions, one corresponding to the libration of the angle θ, and another related with the longitude of perihelion ϖ. The curve completes one cycle in 2200 years, and the variations of semi-major axis and eccentricity in this interval of time are shown in Figure 4 and 5 respectively. Table 2 shows the amplitude Δa of semi-major axis (in units of Jupiter's semi-major axis a_1), the amplitude Δe of eccentricity, and periods of libration (P_L) and of the angle ϖ (P). The results of Nakai and Kinoshita (1986) and Takenouchi (1962) are presented in the same table.

Table 2. Numerical Results of (279) Thule. Δa in units of a_1

	Δa	Δe	P_L(y)	P(y)
Asymmetric Model	0.005	0.068	190	450
Nakai and Kinoshita	0.014/-0.004	0.075	---	470
Takenouchi	---	---	170	460

V. DISCUSSION

Although qualitatively the classical averaged model presents good results, one point of desagreement is the period of libration of asteroids with high orbital eccentricity and inclination (Tsuchida, 1985, 1988). The asymmetric averaged model gives a better result than the classical model (Tsuchida, 1989), but the divergency still persists. As can be seen in Table 2, the error in the libration period P_L is of the order of 10%. In Figure 6 are plotted the variations of averaged (curve (a)) and osculating (curve (b)) semi-major axis. The difference in the period of libration shifts curve (a) from the average of curve (b).

The eccentricity of Thule reachs 0.14 and this fact may contribute to divergency, however this minor planet presents an important particularity, that is, the nearness of its orbit to the Jupiter's orbit. The curve (b) of Figure 6 exhibits a great variation of semi-major axis at each synodic period. Then, in this case, on the contrary to 2/1 resonant case, one necessarily must use averaged elements as initial conditions for analytical averaged models. Nevertheless, applications with averaged elements obtained by Takenouchi (1962) didn't give the expected results. Apparently it is necessary to consider a different technique to determine the averaged orbital elements. On the other hand, the choise of the libration center (Tsuchida, 1989) can affect the libration period. Figure 7 shows the averaged and osculating eccentricity. The smaller error (~4%) in the period of perihelion motion permits to obtain a better fit of osculating eccentricity (curve (b)) by the averaged one (curve (a)).

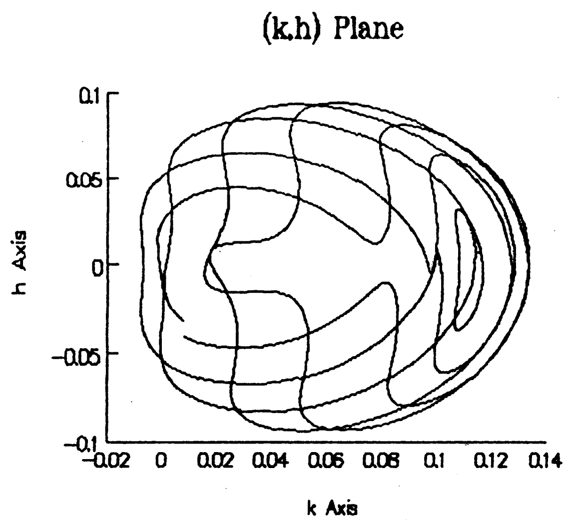

FIG. 2. Libration Motion in the

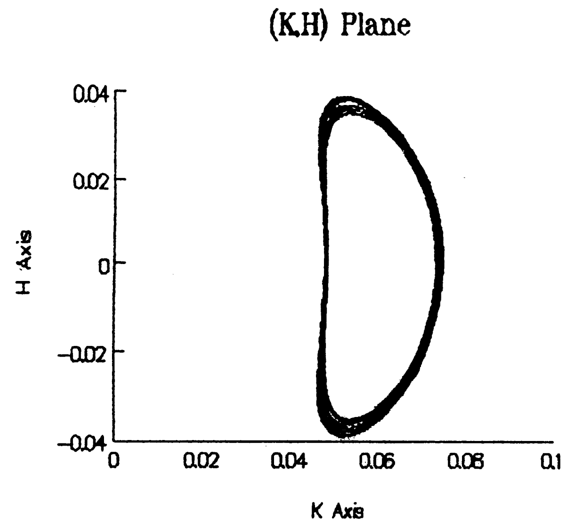

FIG. 3. Libration Motion in the

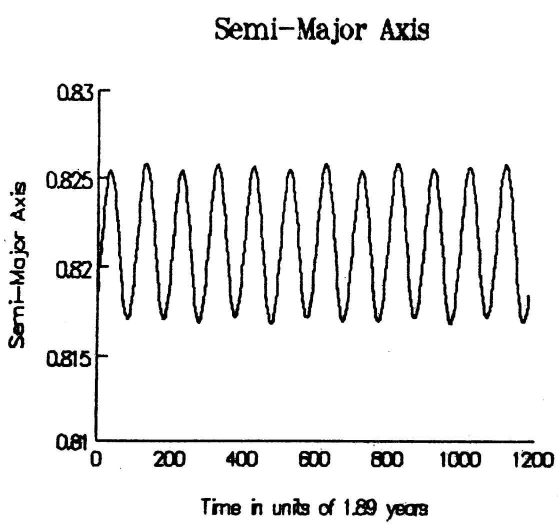

FIG. 4. Variation of the Averaged

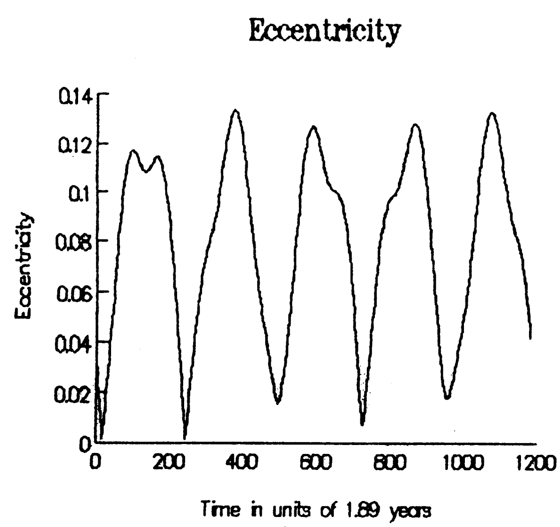

FIG. 5. Variation of the Averaged

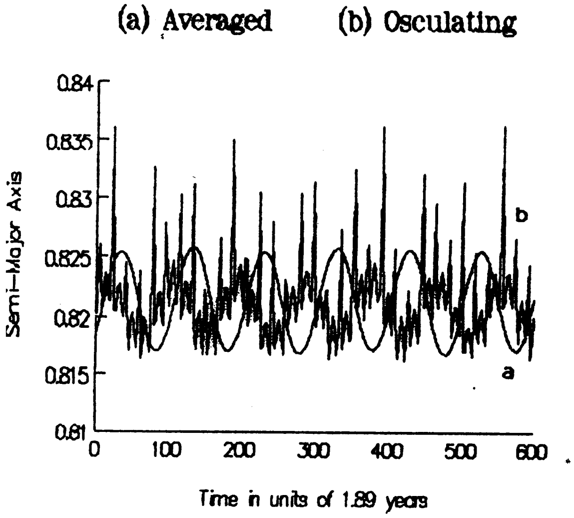

FIG. 6. Variation of Semi–Major Axis

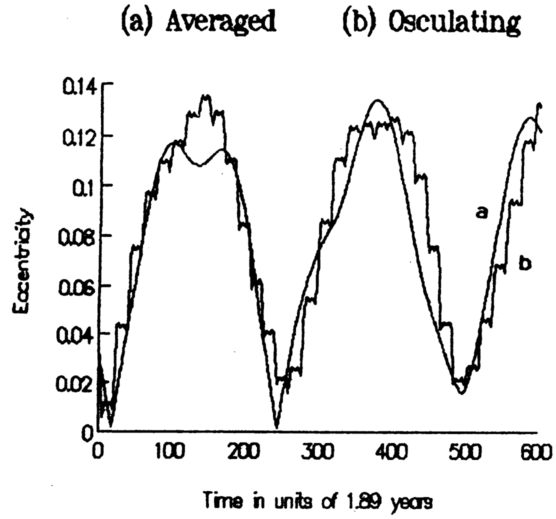

FIG. 7. Variation of Eccentricity

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