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ON THE DOUBLE NATURED SOLUTION OF TWO TEMPERATURE ACCRETION FLOWS

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RESUMEN

Se muestra que efectos no lineales en el disco de fotones suaves-comptonizados de dos temperaturas aparecen cuando se toma en cuenta un acoplamiento variable ($\ln \Lambda$) para protones y electrones. Además de mostrar una nueva forma para obtener la solución convencional a las ecuaciones de disco, este procedimiento muestra que la solución del disco de fotones suaves-comptonizados de dos temperaturas es doblemente valuado. Para acreción esférica habrá soluciones únicamente para tasas de acreción arriba de un valor crítico mínimo independientemente de mecanismos de enfriamiento. Para discos de acreción suave-comptonizados, existen soluciones de dos temperaturas solamente para tasas de acreción abajo de un valor máximo. Debajo de este valor, las soluciones tienen un comportamiento bastante diferente en las ramas superior e inferior. Las soluciones serán dominadas por la presión del gas en la rama superior y por la presión de radiación en la rama inferior. Para Cygnus X-1 encontramos que la turbulencia es cuando menos sónica.

ABSTRACT

It is shown that non linear effects in the two temperature soft photon comptonized disc are exhibited when we allow for a variable coupling ($\ln \Lambda$) for protons and electrons. Besides providing a new way to obtain the conventional solution to the disc equations, this procedure shows that the solution of the two temperature soft photon comptonized disc is double valued. For spherical accretion there will be solutions only for accretion rates above a critical minimum value, independently of cooling mechanisms. For soft comptonized accretion discs two temperature solutions only exist for accretion rates below a maximum value. Below this value, the solutions have quite different behaviour in the upper and lower branches. Solutions will be gas pressure dominated in the upper branch and radiation pressure dominated in the lower branch. For Cygnus X-1 we found that turbulence is, at least, sonic.

Key words: **ACCRETION DISKS – HYDRODYNAMICS – TURBULENCE**

I. INTRODUCTION

Since the mid seventies, two temperature accretion flows have found a large amount of applicability in astrophysical systems, from X-Ray binaries to active galactic nuclei and quasars (Rees *et al.* 1982; Band and Malkan 1989).

Notwithstanding the most recent applications related to pair production in accretion dics (Sikora and Zbyszewska 1985; Kusunose 1987; Tritz and Tsuruta 1989; White and Lightman 1989; Meirelles 1990) and spherical accretion (Kusunose and Takahara 1985; Takahara and Kusunose 1985; Guilbert and Stepney 1985; Begelman, Sikora and Rees 1987; Lightman, Zdziarski and Rees 1987; Parker and Ostriker 1989). All this interest started, as far as we are aware, with the sucessful paper of Shapiro, Lightman and Eardley (1976) explaining theoretically the data of Cygnus X-1. Application of the standard α-accretion disc model to this X-ray source, besides not fitting the data, may lead to instabilities that may cause explosion of the disc. These instabilities are present in the standard model as long as radiation pressure attains 60% of the total pressure. The two temperature disc model of Shapiro *et al.* is ionic pressure dominated in order to avoid this difficulty.

However, it should be argued that these results rely heavily on the assumption of constant Coulomb logarithm, and on the value of the accretion rate.

Besides presenting a new way to obtain the solution of the two temperature soft photon comptonized disc, it will be shown in this paper that non linear effects are exhibited when we allow for a variable coupling (ln Λ) for protons and electrons. The physical meaning is that non linearity comes into play when the number of particles interacting with a test particle varies along the disc. Such effects will make the general solution double-valued, one of them being gas pressure dominated, and the other, radiation pressure dominated. The results we present here differ from others in the literature in the sense that, besides showing the double-value of the solution, we show the conditions under which (depending on the accretion rate) the solution is stationary. To a certain extent, our work here is complementary to that of Rees *et al.* (1982), who exhaustively discussed the conditions under which a steady two-temperature regime is likely to occur. With their main conclusion for the occurrence of this regime being a restriction on the $\dot{M}$ space given by $\dot{m}[= \dot{M}c^2/\mathcal{L}_E] < 50\, a^2$, ($a$ being the ratio accretion velocity to free velocity), we are not allowed to infer if this regime is stationary or not.

The method we employ here has been successfully applied by Meirelles (1990) to study pair production in two temperature accretion discs, cooled by externally produced soft photons, showing a coupled of non linear effects not obtained by the usual procedure (assumptions of constant ln Λ). These effects are confirmed by the work of Kusunose and Takahara (1990).

It should be remarked that we do not resort to any new assumption, other than avoiding the usual assumption of constant Coulomb logarithm. Moreover, it is shown in this paper, that this result applies equally well to two temperature spherical accretion.

This paper extends previous work by Meirelles and Marques (1989) on the consistency of gas pressure dominated two-temperature soft comptonized accretion discs.

II. DISC EQUATIONS

We make the standard assumptions of accretion disc theory (Shakura and Sunyaev 1973; Novikov and Thorne 1973): that the disc half thickness is much smaller than the radial distance, gas travels in approximately circular orbit about a central compact object, viscous stress is proportional to the pressure $t_{r\varphi} = \alpha P$, and all particles have thermal distributions.

To simplify equations we define a set of dimensionless variables: r is the radial distance in units of the gravitational radius, M_{34} is the mass of the central compact object in units of 10^{34} g, $\dot{M}_{17}$ is the accretion rate in units of 10^7 g s^{-1}, N is the proton number density in units of 10^{16} cm^{-3}, $\mathcal{L}_{37}$ is the luminosity in units of 10^{37} erg s^{-1}, the electronic and ionic temperatures, respectively T_e and T_i are in units of 10^9 °K.

The hydrostatic equilibrium equation together with the thin disc assumption gives for the pressure

$$P = \frac{\rho\, \Omega^2\, \ell^2}{3} \quad ; \qquad (1)$$

ρ is the matter density, Ω the angular Keplerian velocity and ℓ is the disc half thickness.

The half thickness of the disc follows directly from equation (1)

$$\ell = 1.22 \times 10^4\, M_{34}\, r^{3/2}\, T_i^{1/2}\, f \quad , \qquad (2)$$

with $f = (1 + P_r/P_g)^{1/2}$, P_r being the radiation pressure and P_g the gas pressure.

Our main concern in this paper are the regions in the temperature spaces T_i and T_e, such that

$$0.45\, T_i \ln \Lambda \gg T_e^2 \quad .$$

This condition guarantees the dominance of unsaturated inverse comptonization over bremsstrahlung or bound-free emission. The reason for that is our interest for systems with X-ray power law spectrum.

We now assume unsaturated inverse comptonization and set the Kompaneetz Y parameter equal to 1 and obtain

$$1 \simeq \frac{4 \times 10^9\, k\, T_e}{m_e\, c^2}\, \tau\, (1 + \tau) \quad , \qquad (3)$$

where k is Boltzmann constant, m_e electron mass, c velocity of light and τ electron scattering optical depth.

For the heat generation we shall use the well known expression from the accretion disc theory,

$$Q^+ = \frac{3}{8\pi}\dot{M}\,\Omega^2\, S$$

$$= 1.94 \times 10^{25} \frac{\dot{M}_{17}}{M_{34}^2\, r^3}\, S \text{ erg cm}^{-2}\text{ s}^{-1} \quad , \qquad (4)$$

with $S = 1 - \delta\, r^{-1/2}$, δ being the ratio of the actual angular momentum of the flow to the Keplerian one at $r = 1$.

From the angular momentum conservation equation together with the definition of the stress tensor $\tau_{r\varphi}$ we obtain for the density

$$\rho = \frac{3}{4\pi} \frac{\dot{M}\, S}{\alpha \ \ell^3 \ \Omega}$$

$$= 0.33 \frac{\dot{M}_{17} \ S}{\alpha \ M_{34}^2 \ r^3 \ T_i^{3/2} \ f^3} \ \mathrm{g\ cm^{-3}} \quad . \qquad (5)$$

The collisional energy exchange term is given by (Spitzer 1962),

$$F_{ep} = 9.24 \times 10^{24} \rho^2 \ell \frac{T_i}{T_e^{3/2}} \ln \Lambda \quad , \qquad (6)$$

where $\ln \Lambda$ is the Coulomb logarithm.

Below we give the formal solution of the system of equations (1) to (6),

$$T_e = 0.24 \left[\frac{M_{34} \, r^{3/2}}{\alpha \ \dot{M}_{17} \ S \ g^3} \right]^{1/6} \frac{(\ln \Lambda)^{1/3}}{f^{2/3}} \quad , \qquad (7)$$

$$T_i = \frac{3 \times 10^2}{\alpha^{7/6}} \left[\frac{M_{34} \, r^{3/2}}{\dot{M}_{17} \ S} \right]^{5/6} \frac{(\ln \Lambda)^{1/3}}{f^{8/3}} g^{1/2} , \qquad (8)$$

$$\ell = \frac{2.1 \times 10^5}{\alpha^{7/12}} \frac{\left[M_{34} \, r^{3/2} \right]^{17/12}}{(\dot{M}_{17} \ S)^{5/12}} \frac{(\ln \Lambda)^{1/6}}{f^{1/3}} g^{1/4} \ , \qquad (9)$$

$$\rho = 6.3 \times 10^{-5} \frac{(\dot{M}_{17} \ S)^{9/4} \ \alpha^{3/4} \ f \ g^{-3/4}}{\left[M_{34} \ r^{3/2} \right]^{13/4} (\ln \Lambda)^{1/2}} \quad , \qquad (10)$$

where

$$g = \begin{cases} 1 \ , & \tau < 1 \\ T_e^{-1/2} \ , & \tau > 1 \end{cases} \quad .$$

To calculate f one needs the ratio of pressures given by

$$\frac{P_r}{P_g} = 3 \times 10^{-3} \frac{\dot{M}_{17} \ S}{M_{34}^2 \ r^3} \frac{\ell}{T_i} \quad . \qquad (11)$$

III. THE SOLUTION FOR THE ELECTRONIC TEMPERATURE

We have written the formal solution of the disc equations, [(7) to (10)], to show explicitly their dependence on the ratio of pressures. We now obtain the solution in a much more tractable form.

From equations (3), (5), and (11) we obtain

$$\ell = 4.3 \times 10^5 \left[\frac{\dot{M}_{17} \ S \ M_{34} \ r^{3/2}}{\alpha} T_e \, g \right]^{1/2} \quad , \qquad (12)$$

$$T_i = 1.23 \times 10^3 \frac{\dot{M}_{17} \ S \ T_e \ g}{\alpha \ M_{34} \ r^{3/2}} - 1.3 \times 10^3$$

$$\left\{ \frac{\dot{M}_{17} \ S}{\alpha \ \left[M_{34} \, r^{3/2} \right]^3} \right\}^{1/2} T_e^{1/2} \, g^{1/2} \quad , \qquad (13)$$

$$\rho = \frac{7.5 \times 10^{-6}}{(T_e \ g)^{3/2}} \left\{ \frac{\alpha}{\dot{M}_{17} \ S \ M_{34} \ r^{3/2}} \right\}^{1/2} \quad . \qquad (14)$$

As it is customary to express the Coulomb logarithm as a function of the temperature, we shall use equations (4), (6), and (14) to write

$$\ln \Lambda = 8.8 \times 10^4 \left[\frac{\dot{M}_{17} \ S}{M_{34} \ r^{3/2}} \right]^{3/2} \frac{T_e^4 \ g^3}{T_i} \quad , \qquad (15)$$

with T_i given by equation (13).

From the definition of the Debye number (Golant, Zhilinsky and Sakharov 1980),

$$\Lambda = \begin{cases} 1.7 \times 10^5 \dfrac{T_e^{3/2}}{\rho^{1/2}} \quad , \quad T_e < 1.5 \times 10^{-4} \\ \\ 3.3 \times 10^3 \dfrac{T_e}{\rho^{1/2}} \quad , \quad T_e > 1.5 \times 10^{-4} \quad . \end{cases} \tag{16}$$

Equations (14) and (15) combined with equation (16) yield

$$1.76 \times 10^5 \left[\frac{\dot{M}_{17}\ S}{M_{34}\ r^{3/2}}\right]^{3/2} \frac{T_e^4\ g^3}{T_i} = \tag{17}$$

$$= 1.45 \times 10^{12} \left[\frac{\dot{M}_{17}\ S\ M_{34}\ r^{3/2}}{\alpha}\right]^{1/2} T_e^{7/2}\ g^{3/2} \sum \quad ,$$

where $\sum$ is given by

$$\sum = \begin{cases} 1 \quad , \quad T_e < 1.5 \times 10^{-4} \quad , \\ \\ 2.65 \times 10^3\ T_e \quad , \quad T_e > 1.5 \times 10^{-4} \quad . \end{cases} \tag{18}$$

For

$$\dot{M}_{17} < \frac{3.1 \times 10^3}{S\ \alpha^{16/5}} \left[M_{34}\ r^{3/2}\right]^{9/5} \quad , \tag{19}$$

equation (17) has two approximate solutions,

$$T_e \simeq \begin{cases} 3.9 \times 10^{-4} \left[\dfrac{\alpha}{\dot{M}_{17}\ S\ M_{34}\ r^{3/2}}\right]^{\nu/7} \\ \\ \dfrac{0.56}{\alpha^{1/3}} \left[\dfrac{M_{34}\ r^{3/2}}{\dot{M}_{17}\ S}\right]^{1/6} \\ \\ \left[1 + 4.5 \times 10^{-2} \ln\left[\dfrac{\left[M_{39}\ r^{3/2}\right]^{13/14}}{\alpha^{10/7}}\right]\right]^{1/3} \quad . \end{cases} \tag{20}$$

In equation (20) $\nu = 1$ for $T_e > 1.5 \times 15^{-4}$ and $\nu = 7/9$ for $T_e < 1.5 \times 15^{-4}$.

IV. SPHERICALLY SYMMETRICAL ACCRETION

To study spherical accretion we shall adopt some simplifications, identical to those of Lightman *et al.* (1987), i.e.,

1. We assume that the inward velocity v at any radius, is a constant fraction of the free fall velocity

$$v = a \sqrt{\frac{2GM}{r}}\ , \text{ where } 0 < a \leq 1 \quad .$$

2. The luminosity per unit radius is given by

$$\frac{dL}{dr} = \eta \frac{G\ M\ \dot{M}}{r^2} \quad , \tag{21}$$

where $\eta \leq 1$ is the constant local efficiency of converting released gravitational energy into radiation.

From assumption 1 and from the equation of continuity

$$\dot{M} = 4\pi r^2 \rho v \quad , \tag{22}$$

we obtain for the density

$$\rho = 4.25 \times 10^{-9} \frac{\dot{M}_{17}}{M_{34}^2\ r^{3/2}\ a} \text{ g cm}^{-3} \quad . \tag{23}$$

Using equation (6) properly modified to the spherical case together with equations (17) and (19) we find

$$\ln \Lambda = 4.1 \times 10^6\ \eta \frac{M_{34}}{\dot{M}_{17}} \frac{a^2}{r} \frac{T_e^{3/2}}{T_i} \quad . \tag{24}$$

In equations (19) and (20), r is expressed in units of 4 GM/c^2.

Assuming the ratio T_i/T_e, constant and equal to b, we obtain

$$8.2 \times 10^6\ \eta \frac{M_{34}}{\dot{M}_{17}} \frac{a^2\ T_e^{1/2}}{r\ b} =$$

$$= 2.6 \times 10^{14} \frac{M_{34}^2\ r^{3/2}\ a}{\dot{M}_{17}} T_e^2 \sum \quad . \tag{25}$$

If the relation

$$\frac{\dot{M}_{17}^{3/4}}{M_{34}^{1/2}} \frac{b\ r^{11/8}}{\eta\ a^{7/4}} > 1.36 \times 10^3 \quad , \qquad (26)$$

is satisfied, equation (25) has two approximate solutions

$$T_{e\mp} \simeq \begin{cases} d \left[\dfrac{\dot{M}_{17}}{M_{34}^2\ r^{3/2}\ a} \right]^{\nu/2} \\ 2.5 \times 10^{-13} \left[\dfrac{\dot{M}_{17}\ r\ b}{\eta\ M_{34}\ a^2} \right]^2 \ln^2 \{\phi \ln \phi\} \end{cases} , \qquad (27)$$

where

$$d = \begin{cases} 4.6 \times 10^{-7} & , \ T_e > 1.5 \times 10^{-4} \\ 1.13 \times 10^{-6} & , \ T_e < 1.5 \times 10^{-4} \end{cases} .$$

$$\nu = \begin{cases} 1 & , \ T_e > 1.5 \times 10^{-4} \\ 2/3 & , \ T_e < 1.5 \times 10^{-4} \end{cases} . \qquad (28)$$

$$\phi = 2 \times 10^{-3} \frac{\dot{M}_{17}^{3/4}}{M_{34}^{1/2}} \frac{b\ r^{11/8}}{\eta\ a^{7/4}} .$$

The subscript $\mp$ in equation (27) stands for lower (−) and upper branches (+).

V. ANALYSIS AND CONCLUSIONS

With the procedure we have adopted here to obtain the solution of the two-temperature accretion flows, some non-linear effects, such as multivaluedness of the solution and critical values for the accretion rate, are exhibited. This procedure essentially consists in avoiding the usual assumption of constant Coulomb logarithm. Physically, this is equivalent to take variable coupling (number of particles in the Debye sphere) along the disk.

First, analysing spherical accretion we see that, despite the lack of a correct solution for the radiative transfer, the approximation we have used, equation (27), offers some qualitative conclusions. One of them concerns the existence of a minimum local accretion rate, below which there is no solution for a two-temperature flow. For accretion rates above this minimum value we obtain two solutions for the temperature, with completely different structure. The lower branch, lower temperature, slightly dependent on the ratio accretion rate-mass of the compact object, decreasing with the inflow velocity and distance to the compact object. There is no dependence on the ionic temperature as well as no dependence on the efficiency of converting gravitational energy into radiation. In the upper branch, the dependence on the ratio of accretion rate to mass of the compact object and on the inverse of the inflow velocity is sharper.

Temperature, here, besides growing rapidly with distance, depends on the efficiency and on the ionic temperature.

Although these results should be interpreted with reticence because we do not know how dependent upon $\dot{M}$ are **a** and **b**, we conclude from our work and those of Rees *et al.* (1982) and Begelman *et al.* (1987) that a steady two temperature flow occurs, as long as

$$300\ a^2 \left\{ \frac{\eta^4\ a}{b^4\ M_{34}\ r^{11/2}} \right\}^{1/3} < \dot{m} < 50\ a^2 \quad .$$

For accretion discs one finds the existence of a maximum accretion rate, above which there is no two temperature flow. However, this does not affect the establishment of a two temperature regime, because accretion rate close to the maximum would imply luminosity 2 or 3 orders of magnitude greater than the Eddington luminosity.

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