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ON THE INITIALLY CIRCULAR MOTION OF AN ORBITER IN THE OBLATE, ROTATING, MARTIAN ATMOSPHERE

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RESUMEN

Usando la ley de distribución de densidad propuesta por Sehnal and Pospišilová (1988), se estudia el movimiento inicial circular de una órbita en la átmosfera de Marte (considerada oblicua y en rotación). Se estiman analíticamente la diferencia entre el período nodal y Kepleriano correspondiente, así como los cambios independientes en cinco elementos orbitales, sobre un período nodal.

ABSTRACT

Using the density distribution law proposed by Sehnal and Pospišilová (1988), the initially circular motion of an orbiter in Mars' atmosphere (considered as oblate and rotating) is studied. The difference between the nodal period and the corresponding Keplerian one, and the changes of five independent orbital elements, over a nodal period, are analytically estimated.

Key words: **ORBITAL MOTIONS – PLANETS AND SATELLITES-ATMOSPHERES – SOLAR SYSTEM**

I. INTRODUCTION

The perturbed motion of an orbiter in the atmosphere of Mars was firstly studied analytically by Sehnal and Pospišilová (1988). Using the data provided by Moroz, Izakov, and Linkin (1988), they approximated the density-height dependence by the formula

$$\rho = \exp(A_0 + A_1/h) \quad , \tag{1}$$

where ρ results in kg/m^3, h is the altitude in km over Mars' surface, A_0 and A_1 are constants separately determined for the minimal, nominal, and maximal density profiles (Sehnal 1990). The nomimal model, used by us for some numerical estimates, gives the values $A_0 = -37.936$, $A_1 = 2376.1$. The expression (1) is valid in the height range 100-1000 km. In the quoted papers the orbiter was considered as moving in a spherically symmetrical, rotationless atmosphere. Only the perturbations in the semimajor axis and eccentricity, over an anomalistic period, were estimated.

In this paper we shall estimate analytically the perturbations caused by the atmospheric drag in both the nodal period:

$$T_N = \int_0^{2\pi} (dt/du)du \quad , \tag{2}$$

and the orbital elements:

$$z \in Y = \{p,\ q = e\cos\omega,\ k = e\sin\omega,\ \Omega,\ i\} \tag{3}$$

of a Mars' orbiter in an initially circular orbit. Here u = argument of latitude, p = semilatus rectum, e = eccentricity, ω = argument of periastron, Ω = longitude of the ascending node, i = inclination (all with respect to a frame originated in the centre of mass of Mars, whose fundamental plane is the Martian equator plane). The atmosphere will be considered oblate (the surfaces of equal density having the oblateness $\varepsilon = 0.005$, namely Mars' oblateness) and rotating with the angular velocity w.

II. BASIC EQUATIONS

Under the influence of a perturbing factor (considered here as depending on a small parameter σ), the motion of the orbiter undergoes a perturbing acceleration (whose radial, transversal, and binormal components are S, T, W, respectively) and the nodal period will differ from the corresponding Keplerian period (T_0) by a quantity ΔT given by

$$T_N = T_0 + \Delta T \quad . \tag{4}$$

Let us denote: μ = gravitational parameter of the planet, r = planetocentric radius vector, A = cos u, B = sin u, C = cos i, D = sin i, and describe the perturbed motion by means of the Newton-Euler system written in the form (Mioc 1980, 1991)

$$\begin{aligned}
dp/du &= 2(Z/\mu)r^3T\,, \\
dq/du &= (Z/\mu)\{r^3kBCW/(pD) + \\
&\quad + r^2T[r(q+A)/p + A] + r^2BS\}\,, \\
dk/du &= (Z/\mu)\{-r^3qBCW/(pD) + \\
&\quad + r^2T[r(k+B)/p + B] - r^2AS\}\,, \\
d\Omega/du &= (Z/\mu)r^3BW/(pD)\,, \\
di/du &= (Z/\mu)r^3AW/p\,, \\
dt/du &= Zr^2(\mu p)^{-1/2}\,,
\end{aligned} \tag{5}$$

where

$$Z = [1 - r^2C\dot{\Omega}/(\mu p)^{1/2}]^{-1}\,. \tag{6}$$

Consider, as usual, that the elements (3), functions of u, have small variations over one revolution, such that they may be taken as constant (and equal to $z_0 = z(u_0) = z(u(t_0))$, $z \in Y$) on the right-hand side of the equations (5), and these ones can be separately considered. So, we can write $z + z_0 + \Delta z$, the variations Δz in the interval $[u_0, u]$ being determined from

$$\Delta z = \int_{u_0}^{u} (dz/du)\, du, \quad z \in Y\,, \tag{7}$$

where the integrals can be estimated from (5) by successive approximations, with $Z \approx 1$, limiting the process to the first order approximation.

Replacing (6) into the last equation of (5) and expanding it in binomial series, we obtain to first order in σ

$$\begin{aligned}
dt/du &= r^2(\mu p)^{-1/2} + \\
&\quad + r^4C\dot{\Omega}/(\mu p) = f(z, \sigma;\ u), \quad z \in Y\ .
\end{aligned} \tag{8}$$

The expansion of the function f in Taylor series on the hypersurface $H = H(z_0, \sigma = 0; u)$, $z \in Y$, with respect to the small quantities Δz and σ leads to

$$f = f_0 + \sum_{z\in Y}(\partial f/\partial z)_0 \Delta z + (\partial f/\partial \sigma)_0 \sigma + \ldots\,, \tag{9}$$

where the index "0" refers to $u = u_0$.

Lastly, consider the orbit equation in polar coordinates

$$r = p/(1 + e\cos v) = p/(1 + Aq + Bk)\ , \tag{10}$$

where v = true anomaly, and replace (10) into (8). Now, calculating the partial derivatives required by (9), neglecting the terms of the form $(\partial(r^4C\dot{\Omega}/(\mu p))/\partial z)\,\Delta z$, $z \in Y$, which contains σ^2, and substituting the resulting f = dt/du into (2), we find (4), where

$$T_0 = \int_0^{2\pi} f_0\, du\ ,$$

and

$$\Delta T = I_1 + I_2 + I_3 + I_4\ , \tag{11}$$

with

$$\begin{aligned}
I_1 &= (3/2)\, p_0^{1/2}\mu^{-1/2} \\
&\quad \int_0^{2\pi} (1 + Aq_0 + Bk_0)^{-2}\, \Delta p\, du\ , \\
I_2 &= -2p_0^{3/2}\mu^{-1/2} \\
&\quad \int_0^{2\pi} (1 + Aq_0 + Bk_0)^{-3} A\, \Delta q\, du\ , \\
I_3 &= -2p_0^{3/2}\mu^{-1/2} \\
&\quad \int_0^{2\pi} (1 + Aq_0 + Bk_0)^{-3} B\, \Delta k\, du\ , \\
I_4 &= \int_0^{2\pi} \{\partial[r^4C\dot{\Omega}/(\mu p)]/\partial\sigma\}\sigma\, du\ .
\end{aligned} \tag{12}$$

These formulae are general from two viewpoints: the nature of the perturbing factor is unspecified, and the orbit may have any subunitary eccentricity. The method, firstly used by Zhongolovich (1960) for a gravitational perturbing factor, was extended and used by us for various perturbing factor (Mioc and Radu 1977, 1979, 1982; Mioc 1980, 1988).

As to the changes of the orbital elements (3) over a nodal period, it is sufficient to perform the integrals (7) between the limits 0 and 2π (see also Mioc 1991; Mioc and Radu 1991).

In this paper we deal only with circular orbits ($e_0 = 0$, therefore $q_0 = k_0 = 0$); so, $r_0 = p_0$, and (12) acquire the form

$$I_1 = (3/2)\, p_0^{1/2} \mu^{-1/2} \int_0^{2\pi} \Delta p \, du \ ,$$

$$I_2 = -2p_0^{3/2} \mu^{-1/2} \int_0^{2\pi} A\, \Delta q \, du \ ,$$

$$I_3 = -2p_0^{3/2} \mu^{-1/2} \int_0^{2\pi} B\, \Delta k \, du \ , \qquad (13)$$

$$I_4 = p_0^3\, \mu^{-1} C_0 \int_0^{2\pi} (\partial\dot{\Omega}/\partial\sigma)\sigma \, du \ .$$

In the following sections, for simplicity, we shall no longer use the index "0" to mark the initial values of the elements (3) and of functions of them. Every further index "0" (except u_0) is a simply notation and does not refer to initial values. In fact, every quantity which does not depend on u (explicitly or through A and B) and appears in the right-hand side of equations (5) or (13) will be considered constant (over one revolution) and equal to its value for $u = u_0$.

III. EQUATIONS OF MOTION

Let the atmospheric drag be the perturbing factor. In this case we have (see Taratynova 1957)

$$S = -\rho\delta\, v_{rel}\, v_r \ ,$$

$$T = -\rho\delta\, v_{rel}(v_n - rCw) \ , \qquad (14)$$

$$W = -\rho\delta\, v_{rel}\, rDwA \ ,$$

where δ = drag parameter of the orbiter, v_{rel} = orbiter speed with respect to the atmospheric flow, v_r, v_n = radial and transversal components of the orbiter velocity with respect to the planet centre, the other notations being already defined. The velocities are (Okhotsimskij, Eneev, and Taratynova 1957)

$$v_{rel} = (\mu/p)^{1/2}(1 + 2Aq + 2Bk + q^2 + k^2)^{1/2} \ ,$$

$$v_r = (\mu/p)^{1/2}(Bq - Ak) \ , \qquad (15)$$

$$v_n = (\mu/p)^{1/2}(1 + Aq + Bk) \ .$$

With these expressions, one finds easily that for circular orbits, the expressions (14) become

$$S = 0 \ ,$$

$$T = \rho\delta(\mu p)^{1/2}(Cw - p^{-3/2}\mu^{1/2}) \ , \qquad (16)$$

$$W = -\rho\delta(\mu p)^{1/2}\, DwA \ .$$

Now we must express the density as function of u (through A and B). From $h = r - R(1 - \varepsilon \sin^2\varphi)$, where R = 3380 km is Mars' equatorial radius, φ = latitude, and $\sin\varphi = DB$, we obtain

$$h = p - R + \varepsilon RD^2 - \varepsilon RD^2A^2 \ ;$$

therefore (1) becomes

$$\rho = \exp\left[A_0 + \frac{A_1}{(p - R + \varepsilon RD^2)(1 - QA^2)}\right] , \qquad (17)$$

where $Q = \varepsilon RD^2/(p - R + \varepsilon RD^2)$. Since h lies between 100-1000 km, it follows that $Q_{max} = 0.15$, hence we expand $(1 - QA^2)^{-1}$ to first order in Q, obtaining easily

$$\rho = X \exp\,(m\, A^2) \ , \qquad (18)$$

with $X = \exp[A_0 + A_1/(p - R + \varepsilon RD^2)]$, $m = A_1Q/(p - R + \varepsilon RD^2)$. It is easy to notice that m does not exceed 3; subsequently, we shall expand the exponential in (18) keeping nine terms

$$\rho = X \sum_{n=0}^{8} a_n A^{2n} \ , \qquad (19)$$

where we denoted $a_n = m^n/n!$

By virtue of the considerations made in §II, we shall separately have in view that first five equations (5). Since at the analytic calculation of the integrals (7) we take $Z \approx 1$, we shall write these equations omitting in advance the factor Z. So, replacing (19) into (16), then substituting these ones and putting r = p in the right-hand sides of the mentioned equations, they acquire the form

$$dp/du = 2Xp^{7/2}\mu^{-1/2}\delta(Cw - p^{-3/2}\mu^{1/2}) \sum_{n=0}^{8} a_n A^{2n} ,$$

$$dq/du = 2Xp^{5/2}\mu^{1/2}\delta(Cw - p^{-3/2}\mu^{1/2}) \sum_{n=0}^{8} a_n A^{2n+1} , \qquad (20)$$

$$dk/du = 2Xp^{5/2}\mu^{-1/2}\delta(Cw - p^{-3/2}\mu^{1/2}) \sum_{n=0}^{8} a_n A^{2n} B ,$$

$$d\Omega/du = -X\, p^{5/2}\mu^{-1/2}\delta\, w \sum_{n=0}^{8} a_n A^{2n+1} B ,$$

$$di/du = -X\, p^{5/2}\mu^{-1/2}\delta\, Dw \sum_{n=0}^{8} a_n A^{2n+2}$$

IV. PERTURBATIONS OF THE NODAL PERIOD AND ORBITAL ELEMENTS

Examining (13), one sees that the integrals (7) must be performed only for p, q and k. Performing them with the integrands provided by the corresponding equations (20), we obtain

$$\Delta p = 2Xp^{7/2}\mu^{1/2}\delta(Cw - p^{-3/2}\mu^{1/2}) \times \times \left[a_0 P_0 u + \sum_{n=1}^{8} a_n (P_n u + + (1/2n) \sum_{j=0}^{n-1} P_{nj} A^{2n-2j-1} B) - F_1^0 \right] ,$$

$$\Delta q = 2Xp^{5/2}\mu^{-1/2}\delta(Cw - p^{-3/2}\mu^{1/2}) \times \times \left[\sum_{n=0}^{8} \frac{a_n}{2n+1} \sum_{j=-1}^{n-1} Q_{nj} A^{2n-2j-2} B - F_2^0 \right] , \qquad (21)$$

$$\Delta k = -2\, Xp^{5/2}\mu^{-1/2}\delta(Cw - p^{-3/2}\mu^{1/2}) \times \times \left[\sum_{n=0}^{8} \frac{a_n}{2n+1} A^{2n+1} - F_3^0 \right] ,$$

where

$$P_0 = 1, P_n \frac{(2n-1)!!}{2^n\, n!} , \; n = 1, 2, \ldots ;$$

$$P_{n0} = 1,$$
$$P_{nj} = \frac{(2n-1)(2n-3)\ldots(2n-2j+1)}{2^j (n-1)(n-2)\ldots(n-j)} , \qquad (22)$$
$$j = \overline{1, n-1} ;$$

$$Q_{n,-1} = 1,$$
$$Q_{nj} = \frac{2^{j+1}\, n(n-1)\ldots(n-j)}{(2n-1)(2n-3)\ldots(2n-2j-1)} ,$$
$$j = \overline{0, n-1} ,$$

while F_s^0, $s = \overline{1,3}$, represents the numerical value of the whole expression (function of u) preceding it inside the square brackets, for $u = u_0$.

Replace now (21) into (13) and perform the integrations; we find $I_1 \neq 0$ (see below its expression), $I_2 = 0$, $I_3 = 0$. As to I_4, using the last equation (5) and the fourth equation in (20), then performing the last integral (13), in which the part of the small parameter σ is played by X, we find $I_4 = 0$. So, by (11) we have

$$\Delta T = I_1 = 6\pi X\, p^4 \mu^{-1} \delta\, (Cw - p^{-3/2}\mu^{1/2}) \times \times (\pi \sum_{n=0}^{8} a_n P_n - F_1^0) . \qquad (23)$$

As to the changes of the orbital elements (3) over a nodal period, replacing (20) into (17), then performing the integrals between 0 and 2π, and denoting the results also by Δz, $z \in Y$, we obtain

$$\Delta p = 4\pi X\, p^{7/2}\mu^{-1/2}\delta\, (Cw - p^{-3/2}\mu^{1/2}) \times \times \sum_{n=0}^{8} a_n P_n ,$$

$$\Delta q = 0, \quad \Delta k = 0, \quad \Delta\Omega = 0 , \qquad (24)$$

$$\Delta i = -2\pi X\, p^{5/2}\mu^{1/2}\delta\, Dw \sum_{n=0}^{8} a_n P_{n+1} ,$$

where P_n, P_{n+1} are given by (22).

V. CONCLUDING REMARKS

i) Examining (23) and (24), we see that the initially circular orbit or an artificial satellite in the Martian atmosphere (whose density distribution between 100-1000 km is described by (1)) returns to the circular form after a nodal period. The new radius is smaller than the initial one. As to the orbit plane, the node comes back to its initial position, but the inclination becomes smaller.

ii) Particular cases can be obtained, for instance, referring the initial values to the ascending node ($u_o = 0$), neglecting the atmospheric rotation ($w = 0$), or oblateness ($\varepsilon = 0$), etc.

iii) The use of u as the independent variable makes possible the study of the perturbations in the period for very low eccentric orbits (even circular, as we considered). Such a study is impossible if one of the anomalies is taken as an independent variable.

iv) One the basis of the changes in (24), expressions for such variations can be obtained for any other orbital element (except, of course, the anomalies).

v) Unlike the similar studies (Sehnal and Pospišilová 1988; Sehnal 1990), our paper, although it deals with circular orbits only, considers less restrictive hypotheses (oblate, rotating atmosphere), and determines analyltic expressions for the perturbations in both the period and the five independent orbital parameters.

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